Mathematics teachers' uses of resources in the context of teaching reasoning-and-proving: Insights from a cross-national study

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This paper aims to describe which resources teachers employ to teach reasoning-and-proving in five different countries, what opinions they hold about them, which limitations they see, and what needs regarding these resources they have. Data were collected through a questionnaire and semi-structured interviews. Results suggest textbooks, other books, and teachers' own materials are almost universally the most used resources for teaching reasoning-and-proving. Teachers, however, mention issues with these and other resources. There are also similarities and differences across the nations, which the paper tries to explain.

Keywords: Teaching resources, reasoning-and-proving, cross-cultural studies.

Introduction

Reasoning-and-proving (R&P) is an essential component of mathematics teaching as it underpins mathematical learning (NCTM, 2009). However, teaching R&P across content areas and grade levels in mathematics classrooms remains an elusive goal, which is dependent on teachers and the type of teaching they enact in classrooms (Buchbinder & McCrone, 2022). Despite a long history of research related to the challenges of teaching R&P and vital insight into these challenges, teachers continue to struggle with teaching R&P (Bieda, 2010; Nardi & Knuth, 2017), as well as with the identification of tasks that prompt reasoning (Clarke et al., 2012) or with finding problems/tasks from various resources for assessing students' R&P skills (Sari et al., 2020).

While many studies have analyzed math textbooks in relation to R&P (Cai & Cirillo, 2014), further research is needed to investigate what resources can aid teachers in teaching R&P in their classrooms (Harel & Fuller, 2010; Mariotti, 2006). Textbooks and curriculum guides are considered the most common resources supporting mathematics teaching throughout the world (Remillard & Heck, 2014), also according to Pepin et al. (2017), teachers internationally increasingly rely on digital resources to build their mathematics curriculum. Understanding the role of various resources in mathematics teachers' planning and enactment of lessons focusing particularly on R&P may inform the design of these resources and mathematics teacher education. To foster this understanding, knowledge about the actual use of the various resources is important (Kock & Pepin, 2020). Cross-national insights into the availability and accessibility of resources for mathematics teachers can be useful in many ways. Identifying common challenges and practices can inform researchers and practitioners aimed at supporting mathematics teachers in different countries. Insights from different countries can lead to cross-cultural exchanges and collaborations among educators.

This study was carried out within the scope of the European project MaTeK[1] with partners from five countries (Slovakia - S, Czech Republic - C, Italy - I, Norway - N, and Turkey - T). The project consortium defined a common research study focusing on improving pre-service mathematics teachers' lesson design capacity related to R&P (Slavíčková et al., 2022), specifically of teacher students for grade levels 5 to 10 (age groups 11 to 16). This study was an initial investigation (context and needs analysis) that sought to understand how mathematics teachers in practice use resources to teach R&P. Thus, we seek to answer the following research questions: What resources do teachers use, what are their views about resources, and what do they need for teaching R&P?

Documentational approach to didactics and teachers' use of resources

Trouche et al. (2020) defined the documentational approach to didactics (DAD) as a holistic approach to teachers' work with resources. The interaction with resources is a two-way relationship since, for instance, a resource's affordances influence a teacher's instrumentation, and teacher knowledge influences instrumentalization, a process conceptualized by Verillon and Rabardel (1995). The document is defined as the outcome of teachers' interactions with the resources. In our work with teachers' use of resources for teaching R&P, we conceptualize this process also as an interactive process with resources and consider the product "document" as an outcome of resources (see Table 1 for categories), teachers' usages of those resources, and their knowledge of how to use these resources.

In addition to our framework, the DAD, Pepin and Kock (2021) developed a resource categorisation model for undergraduate students participating in a challenge-based learning context. They identified four types of resources, which are curriculum, social, cognitive, and general resources (Table 1). In our work with mathematics teachers, we used this categorisation to classify and interpret our findings.

Table 1: Categories of resources and examples, adapted from Pepin and Kock (2021, p. 308)

Resource category	Description	Examples
Curriculum	Resources that teachers and students develop and use to engage with mathematics in and out of the classroom. These resources are intended to be aligned with the mathematics curriculum.	Textbooks, teacher curricular guidelines, online professional platforms for mathematics teachers, manipulatives
Social	Formal and causal interactions with colleagues, direct or web-based	Colleagues, online forums
Cognitive	Mathematical and teaching-related concepts, routines, and frameworks teachers use and/or develop	Set of ideas teachers acquire from various sources that are related to mathematics and teaching.
General	Non-curricular resources used by teachers	Wikipedia, YouTube

In this study, we consider R&P as a unified construct (Cai & Cirillo, 2014), as reasoning generally refers to the process of developing arguments, which might result in proof. However, since our focus is on middle school mathematics, where a formal proof is usually not part of the curriculum, we adopted Stylianides' (2007) definition, which states proof as "a mathematical argument, a connected sequence of assertions for or against a mathematical claim" consisting of a set of accepted statements

[1] https://www.projectmatek.eu

by the classroom community, modes of argumentation, and argument representation (p. 291). Therefore, we believe that R&P is accessible and takes various forms of argumentation and representation depending on the classroom community and grade level. Specifically, this paper focuses on mathematics teachers' use of resources for teaching R&P.

National contexts

Mathematics teachers from five countries (S, C, I, N, and T) were involved in this study. Although our aim was not to compare teachers in these countries, insights from different school systems and diverse cultural backgrounds can help the international research community in fostering teachers' development in teaching R&P. Albeit the similarities in the school cultures of these five countries, it should be noted that there are also differences in terms of teacher training, school organization, or the nature of the resources available to teachers.

All five countries have centrally determined national curricula, and R&P is mentioned in all of them to some degree for grades 5 to 10. In Norway, "Reasoning and argumentation" is highlighted as one of six core elements in mathematics but is not explicitly mentioned in the goals of the curriculum. In the four other countries, R&P is mentioned in the general and/or the specific goals. In all countries except Turkey, schools or teachers can select which textbooks to use. In Slovakia and the Czech Republic, the textbooks must be selected from a centrally approved list. In Italy and Norway, there is no such central approval process. Supplementary resources are used by teachers in all countries, and this could include the use of other textbooks than the chosen or approved ones. Most classrooms in the five countries are equipped with one or more digital resources. In all countries, there is a computer or tablet available to the teacher in the classroom. Interactive whiteboards are also found in most classrooms across the five countries. In Norway, pupils often have access to tablets or computers. In some schools, each pupil is given a tablet or laptop for the duration of their time at that school; otherwise, the school has sets of tablets or laptops available for distribution when it is needed in a specific lesson. All five countries have high-stakes testing once or twice in grades 5-10. In all countries, professional development (PD) opportunities are available to teachers.

Method

We collected data through an online questionnaire and follow-up semi-structured online interviews. Data were collected between March-September 2022 in all 5 countries. The anonymous questionnaire asked for participants' email addresses for potential interview involvement. We used the questionnaires to obtain information concerning teachers' use of resources in mathematics lessons, the purpose of their using resources, their conceptions about R&P, and the use of resources related to R&P. The follow-up interviews, aligned with the questionnaire, sought to elicit in-depth insights into teachers' practices and perspectives. The interview had two parts: (1) use of resources in general, (2) use of resources related to teaching R&P. In this paper, we are focusing on the second part only. In total, 954 mathematics teachers of grades 1 to 13 filled out the questionnaire, 77% of which were females. We conducted 30 interviews lasted on average 57 minutes with mathematics teachers in grades 5-10. The interview participants were selected among volunteers to ensure varying levels of experience, and two-thirds of them were females.

The transcribed interviews in national languages were coded separately by 1-2 researchers from each participating country using an open coding method and MaxQDA software. Next, the "local" codes were compared and merged to create a joint code list. Through several online meetings, the consortium refined the code list by merging, regrouping, or deleting codes to agree on the main categories and subcategories. The consortium referred to literature on resource types (Pepin and Kock, 2021; see also Table 1) and teachers' conceptions of R&P in school mathematics (Knuth, 2002)

during this process. The finalized code list was used to recode the transcripts, which were cross-checked by all consortium members.

Findings

Although this paper focuses mainly on the interview data, here, we include a relevant piece of data from the questionnaire conducted in an earlier phase of the research. One of the questions asked the participants to choose from a list of resources that they usually use for teaching R&P. Figure 1 summarizes the results of this question. The most popular resources for R&P among the larger study group were textbooks, books other than textbooks, and teachers' self-created materials.

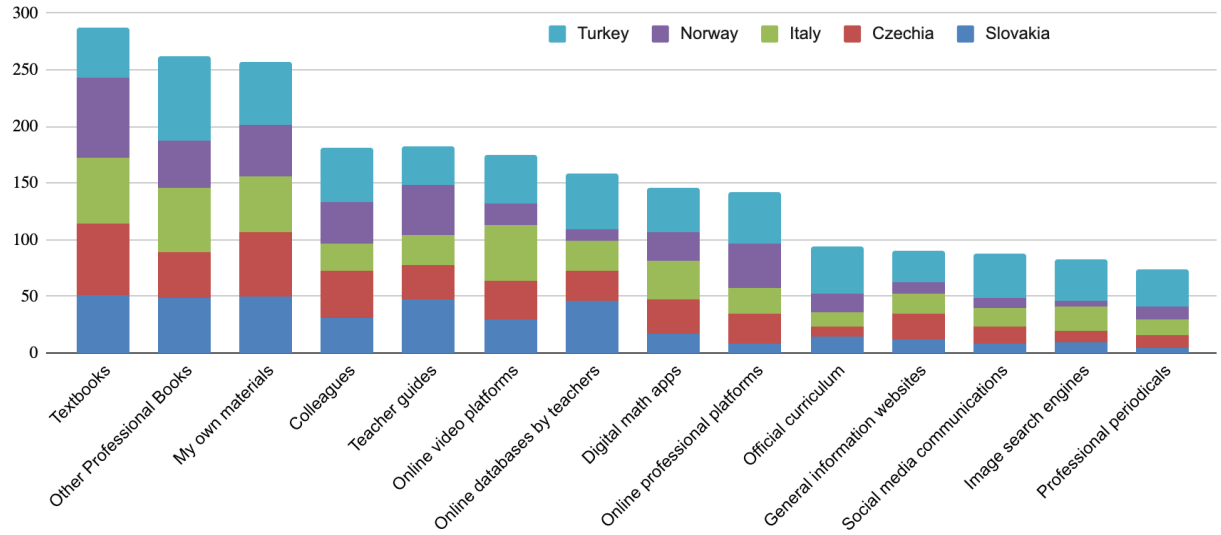

Figure 1: Resources selected by teachers for R&P from a predefined list based on questionnaire data (percentage of participants in each country mentioning the resource)

Analysis of the interviews indicated a similar set of resources that teachers use. We grouped the resources participants mentioned for teaching R&P into four, based on the categorization of Pepin and Kock (2021): Curriculum, General, Social, and Cognitive Resources (Table 2). As seen in Table 2, the resources used by teachers are mostly in the "curriculum resources" category, which are utilized in the teaching-learning processes of R&P. The participants stated that textbooks are one of the first places they look when they are thinking about teaching R&P. When asked about the resources she was using for teaching R&P, one of the Italian participants (I2, female) stated that "Then I would certainly use the class textbook and maybe some resources on the net." The way she articulated her textbook use indicates that a textbook is an obvious place for her to check as a resource, which means that her “document” consists of combining (mainly) a textbook with online resources. Participants stated that they could find places where textbooks include examples of R&P. For instance, a teacher from the Czech Republic (C1, female) stated that; "in our textbooks, there is a section in each chapter . . . where they [students] have to explain why. In those tasks, when someone has calculated something, and it is wrong, students have to justify why". However, this does not mean that teachers can always find what they are looking for in the textbook. There were 27 coded segments indicating that textbooks are not sufficient for them in terms of R&P. For instance, one Turkish teacher stated (T11, female), while talking about a case where she tried to address R&P in her class, "You know,

that's why I first looked at what the textbook did. But I can't always find what I'm looking for in the textbook."

Table 2: Resources participants used for R&P, grouped based on Pepin and Kock's (2021) categories

Curriculum Resources: Textbooks, Manipulatives, Books related to mathematics teaching other than textbooks, Teachers' self-created materials, Dynamic geometry apps–GeoGebra, Curriculum and other official documents, Online professional platforms, Digital math apps, Old exam problems, Teacher guides, Informational classroom posters, Professional periodicals, Academic papers/articles, Postgraduate or PD education
General: General purpose websites, Search engines
Social: Colleagues, Social media
Cognitive resources*: Books related to mathematics teaching other than textbooks, Academic papers/articles, Postgraduate or PD education

* Cognitive resources are the ones where teachers can reach concepts, frames, and routines they use and/or develop.

Teachers access other books as resources or references for new ideas. One Slovak teacher (S1, female) stated, "When I have an interesting book, usually a new one, or even if it's not new, I just have those bookmarks made with coloured stickers, so I'll check it out." Some teachers usually refer to mathematics books for proof or explanations as part of their documents. For instance, S5 (female) said she would use mathematical ideas such as Fermat's Last Theorem from her books, and T4 (male) mentioned that he uses books from his university mathematics classes, such as calculus. Some teachers also use pedagogical books focused on teaching mathematics. In this sense, T6 (female) stated that she uses such a book for finding activities to teach quadrilaterals with a focus on reasoning.

Teachers find GeoGebra as a useful resource for teaching about R&P. When asked about the resources in the school for R&P, an Italian teacher (I1, female) stated, "I love GeoGebra in that sense; it's a great resource for R&P. I use it several times but… it takes time to do it in class, it's not easy." A Norwegian teacher stated that she would use GeoGebra for rather empirical reasoning and developing intuitive ideas; (N10, female) "I think I would use GeoGebra here or have some graphs drawn, and that with also discussing, why doesn't it go the other way? Why does it move like that? So there will be nothing, perhaps no proof, but focus on connections right there." Participants commonly turn to online searches to find relevant information, utilizing both professional platforms for educators and public resources such as Wikipedia.

One aspect of the data we coded was the participants' opinions and reflections on the resources used for R&P. The participants mentioned two types of resources: textbooks and teacher guides. While some teachers in Slovakia and Turkey mentioned that textbooks can provide examples of R&P, most participants across all countries believed that textbooks lack adequate content to support the teaching and learning of R&P. One Slovak participant (S1, female) argued, "There are few argumentative or deriving tasks [in textbooks], only at the beginning of some thematic units." She also noted that the tasks are often covered in an elementary manner and not emphasized further. A Norwegian participant (N3, male) stated, "Textbooks don't have that much if I think about it. ... There are some discussion activities that they sort of recommend, but no, they don't put a lot of emphasis on reasoning-and-proving." Participants from Italy, the Czech Republic, and Turkey shared similar views about textbooks. In general, the teachers argued that there is a minimal number of instances of R&P in the textbook, and the existing examples were not "high-quality" (S5, female). Even in some cases where teacher guides are available, some participants viewed them as insufficient for teaching R&P. Some participants stated that the quality of textbooks in terms of R&P varies based on publishers, mathematical topics, or sections within the textbook.

During the interviews, there were occasions when the participants talked about their needs related to teaching R&P. The codes that emerged in this category, indicating the participants' needs, were; *more knowledge of R&P, to better understand the assessment of R&P, a more specific curriculum guide on R&P, resources on R&P or teaching R&P, professional development for R&P, a school organization allowing teacher development, colleagues with tips on how to better teach R&P.* As seen in the emerged codes, some are directly, and others are indirectly related to resources. Some needs were curriculum-related (e.g., curriculum guides or resources on teaching R&P), some of them were social (e.g., colleagues), and others were cognitive (e.g., more knowledge on R&P). They also expressed their need for professional development in terms of R&P.

One Norwegian teacher (N2, male) stated that they need more examples and clear guidance for teaching and assessing R&P, "but what I have learned now by talking to colleagues at other schools is that no one has a clue [about] . . . what we are actually supposed to do." After commenting on the insufficiency of the textbooks in terms of R&P, T3, (female) said, "the curriculum . . . says we have to do this, . . . but there is no activity that would direct to it." An Italian teacher (I2, female) argued that they do not deal with R&P sufficiently in their classes, especially in lower grade levels. She stresses the teacher's knowledge by stating, "The real problem is the teacher training, the PD! First of all, teachers should be made aware of these aspects." Similarly, C4 (female) stated that it would be good if she could improve herself in R&P, by "[observing] some of those classes where they are doing R&P, during the practice [in teacher education], or observation of other colleagues teaching it. Or simply think about it." A Slovak teacher S1 (female) stated that "A teacher's guide would be good . . . for a teacher who did not adopt reasoning himself or uses it little."

Discussion

Both the questionnaire and interview data confirmed that teachers rely on textbooks a great deal. However, we have 27 coded segments indicating "Textbooks are not good enough." Teachers also intensely use other books and their own materials. In terms of DAD theory (Trouche et al., 2020), the teacher's role as a designer who transforms resources into documents through interacting with resources makes the process of instrumentation highly relevant. Teachers seem to combine various curriculum resources as the affordances of textbooks are leading them to change their practice. In addition, the teachers use their knowledge to use resources to fulfill the educational goals. The instrumentalization process in our case appears to be crucial as well, as teachers interpret or understand R&P in school mathematics differently. This is probably due to the fact that our countries' curricula are not very clear or specific about R&P (see the national contexts). However, in this study, as we pointed out also before, our objective was not to examine the documentational genesis of teachers, nor how the differences between the five countries affect teachers' selection/use of resources, but rather to explore the resources that mathematics teachers generally use for teaching R&P. Further studies are needed in this regard.

Findings from five countries can inform the international research community in developing ideas to support mathematics teachers' development in teaching R&P. Teachers across these countries shared similar needs regarding teaching R&P, specifically, the need for curriculum resources (Pepin & Kock, 2021). The teachers need specific examples, cases, or activities to effectively integrate R&P into their classes. This need might be influenced by their R&P conceptions, which may influence their use of resources. Some participants highlighted their need to better understand R&P before implementing it in their classes, necessitating cognitive resources to aid their development. On an international scale, this research also provides insights for future studies that aim to design interventions for improving pre-service teachers' design capacity. Results indicate that the main resource (textbook) is not sufficient for teachers, and teacher educators need to help them improve their skills in modifying tasks or activities, coming from textbooks or other resources, and making teacher moves to support

pupils' reasoning. Therefore, pre-service teachers need to be able to foster pupils' reasoning, regardless of the task or activity they have at hand. Overall, these common needs of teachers across different school systems suggest a strong potential for international collaboration in producing resources to support teaching R&P.

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