



**HAL**  
open science

# The potential of educational resources for teaching practice in Telesecundaria in Mexico: An exploratory study

Alexandra Angel, Olimpia Figueras, Carlos Valenzuela García

► **To cite this version:**

Alexandra Angel, Olimpia Figueras, Carlos Valenzuela García. The potential of educational resources for teaching practice in Telesecundaria in Mexico: An exploratory study. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfred Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04417155

**HAL Id: hal-04417155**

**<https://hal.science/hal-04417155>**

Submitted on 25 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The potential of educational resources for teaching practice in Telesecundaria in Mexico: An exploratory study

Alexandra Angel<sup>1</sup>, Olimpia Figueras<sup>1</sup> and Carlos Valenzuela García<sup>2</sup>

<sup>1</sup>Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, México;  
[alexandra.angel@cinvestav.mx](mailto:alexandra.angel@cinvestav.mx)

<sup>2</sup>Universidad de Guadalajara, México

*This paper concerns an analysis of educational resources on multiplication and division of fractions, used in the Telesecundaria modality in Mexico, of which Excel is a part. For that reason, the design of activities for multiplying and dividing fractions using Excel is reported. Part of the results of an exploratory study involving trainee teachers are described. Finally, the use of this software and its limitations are discussed, as well as the areas of opportunity that its use allows. As a main result, the hypothesis that the Telesecundaria's Teaching Model is a potential environment to enrich in-service teachers' knowledge about the concepts of multiplication and division of fractions, their teaching, and the integration of digital technologies in their practices, is strengthened.*

*Keywords: Multiplication and division of fractions, mathematics educational resources, use of textbooks, use of spreadsheets, Telesecundaria.*

## Introduction

Secondary education in Mexico is provided in several modalities: General, Technical, Telesecundaria, Community and For Workers. Each modality was created with specific objectives to address different educational problems in the country. Telesecundaria was integrated into the Mexican educational system in 1968, and was designed based on the Italian educational model, the *Telescuola*. "Both [models] sought to provide a low-cost alternative to the demand for secondary education in areas where there were no formally established schools" (Secretaría de Educación Pública [SEP], 2010, p. 36). By the 2022-2023 school year, more than 1.3 million young people (12-14 years old) attended a Telesecundaria school, representing 21.3% of the country's secondary school population (SEP, 2023). In the Telesecundaria modality, a single teacher is responsible for teaching all subjects in each school grade. However, the lack of specialisation of teachers has contributed to their difficulties in mastering the content they teach (Flores and Rebollar, 2008), particularly mathematical content. As part of the teaching and learning processes in Telesecundaria, teachers have teacher's books for each subject they teach, as well as audio-visual resources dealing with the didactics of those subjects. They also have textbooks for students in each of the subjects, as well as various digital, audio-visual, and computer-based educational resources that are referred to in the books (SEP, 2010). All these educational resources are publicly available on the Telesecundaria web page (<https://telesecundaria.sep.gob.mx>).

## Research problem and motivation

By the 2020s, many textbooks include references to other digital educational resources, such as apps and videos. All these resources are important because, through their creation, ideas about how mathematics should be taught and learned are being expressed and shared among different

communities such as teachers, students, education authorities and educational mathematics researchers. In the case of Telesecundaria in Mexico, textbooks have been and are part of those tools of the teachers' daily practice, however, these are no longer structured and used in the same way as before, for example the textbooks used in the 2020-2021 school year are linked to digital resources. In addition, a considerable part of the content of pupils' mathematics textbooks for grades 1 and 2 includes the study and use of fraction arithmetic. Topics that are difficult for both primary and secondary school students and teachers (Ball, 1990).

Rezat (2012) recognises the importance of educational resources for expressing and sharing ideas in terms of curriculum development, teacher training and daily practice. The researcher states that in the face of the diversity of artefacts, the understanding of their use is not evident and highlights that this issue is an important research problem.

As part of an ongoing research project, an analysis has been made of Telesecundaria's textbooks available since 2018, on mathematical texts about multiplication and division of fractions, including the content of the digital resources referred to in those books. The first-grade student's mathematics book refers to an audio-visual resource describing how to use the Excel spreadsheet to multiply fractions and how to set up the format of the cells to write these numbers. The analysis of that textbook included a review of the audio-visual resource and the spreadsheet for multiplying fractions. As part of the analysis of the use of Excel, different multiplications of fractions were proposed, with numerators and denominators up to four digits, proper and improper fractions, and equivalent fractions. From this scrutiny, it was concluded that the calculations made by the software can be potentially confusing for both students and teachers in Telesecundaria. More information is given in the results section. Based on this analysis, a sequence of activities was designed for teachers in which hypothetical scenarios are posed about the use of Excel in classroom and the questions that may be raised by students. The purposes of this report are to expose: 1) the results of the use of Excel as a mediating artefact for teachers to study multiplication and division of fractions, and 2) the potential of Telesecundaria's educational resources in terms of learning in the teachers' practice, but not during their teaching in front of the group, but in terms of the review and analysis of the educational resources that he can do, recognising that such activity is part of his practice. The overall research hypothesis is that the Telesecundaria's Teaching Model for multiplication and division of fractions is a potential environment for teachers to enrich their knowledge of these concepts, their teaching, and the integration of digital technology into their teaching practice.

## **Theoretical framework**

The framework of Local Theoretical Models (LTMs) proposed by Filloy (1999) is the theoretical framework within which the research being developed, and of which this report is a part, is structured. According to Puig (2008, p. 88) "the LTMs are developed to account for ... phenomena that occur in teaching and learning situations, but we also conceive teaching and learning situations as communication and production of meaning situations". The LTMs are constructed considering four components and their interrelationships, which are briefly described in terms of this research: 1) the Formal Competence Models' component - which is abbreviated Formal Component - relates to an analysis on multiplication and division of fractions as mathematical objects, 2) the Teaching Models'

component involves, among other things, the analysis of mathematical texts that are among the educational resources used by Telesecundaria teachers, these texts characterise the meanings of multiplication and division of fractions, the tasks and contexts used for their teaching, i.e. their didactic treatment, 3) the Communication Models' component refers to the decoding that teachers do of the messages issued by the creators of the educational resources, and finally, 4) the Cognitive Processes Models' component, deals with the teachers' performances, but also with their ways of producing meaning from the mathematical activities they carry out.

## **Methodology**

The LTM framework, in addition to being a theoretical framework, also serves as a methodological framework. In the structure of this methodology, two stages can be distinguished. The first consists of a theoretical analysis of the problem to construct their components. The construction of the first two components is reported in Angel et al. (2022). The second stage is experimental and involves the interaction of Telesecundaria teachers with the selected educational resources. But prior to this, and as part of the second stage, the development of the experimentation was designed, and the validation of the designed instruments was planned. This validation was done through an exploratory study that was carried out in October 2022 during the 55th National Congress of the Mexican Mathematical Society and formed part of the teaching workshops that are offered at this event. Prior to the Congress, a call for secondary school teachers of all modalities was published. In the case of the workshop: 'Multiplication and division of fractions, more than just teaching algorithms', ten participants voluntarily attended: most of them were trainee teachers.

The workshop was structured through three two-hour face-to-face sessions that took place in a classroom where each participant had access to a desktop computer. Each session was organised in two moments. In the first one, participants were given worksheets on which they could write down their personal reflections on the proposed activities. The worksheets referred to some Telesecundaria digital resources, which were reproduced. In the second moment, in plenary, participants discussed about: 1) their reflections, 2) the suitability of the activities for teaching multiplication and division of fractions, and 3) the use of the educational resources referred to. This report deals with session 1. Data collection during the workshop was done through worksheets, and notes taken by the three researchers - the authors of this paper - who coordinated the workshop during and after the plenary discussions. The researchers exchanged observations after each session about the notes they took and made a triangulation of the interpretation of the participants' performances. The data are presented in a descriptive way, and their interpretation is done based on the components of the LTM, focusing on aspects that relate to each of those. The validation of the instruments in the workshop was done to observe whether the proposed activities promote the reflection processes proposed in their design.

## **Results**

In session 1, each participant received worksheets, the content of which was organised in two parts: 13 items for multiplication of fractions and 5 items for division of fractions. The participants were asked to use Excel to build a calculator for multiplying fractions, like the one shown in Figure 1. Different Telesecundaria's resources were used in the other sessions.

	A	B	C	D
1				
2			=A2*B2	
3				

**Figure 1: Calculator for multiplying fractions in Excel**

The audio-visual resource from Telesecundaria: *Tutorial for calculating products of fractions in a spreadsheet* (SEP, 2019, p. 42) was reproduced. In it is mentioned that, in Excel, it is necessary to change the format of the cells to 'fraction category', to avoid, for example, that when typing '2/5', the program writes '02-may'.

The activities were designed based on the previous analysis of the use of Excel in which it was observed that the software simplifies the fractions that the user enters, for example: 1) if  $2/4$  is typed, then  $1/2$  is displayed, and 2) when  $5/5$  is typed, 1 is displayed. The software also rewrites the expressions that the user enters, for example: 3) if  $7/3$  is typed, then  $2 \frac{1}{3}$  is displayed. In addition, 4) if the multiplication refers to an empty cell, then the software interprets it as if it had a 0. Finally, Excel approximates the fractions that the user types when they are written with numerators and denominators that are numbers with more digits than the format allows, namely when the cell has a fraction format of 'up to one digit', 5) if the user types the fraction  $11/20$ , then the software displays  $5/9$ . As a result of this analysis, it was concluded that the calculations made by the software can be potentially confusing for both students and teachers in Telesecundaria.

Below are some of the items raised in the worksheets, as well as some of the participants' responses.

*Item 1. Ana wrote in the spreadsheet the examples shown in Figure 2, row 2 and 3, and asks why in row 2, where  $1/2$  and  $1/3$  are multiplied, numerator by numerator and denominator by denominator are multiplied, but in row 3 they are not? What would you answer?*

	A	B	C
1	Fracción a	Fracción b	a*b
2	$1/2$	$1/3$	$1/6$
3	$1/2$	$4/5$	$2/5$
4	$2/5$	$2/3$	$1/4$
5	$2/5$	$2/3$	$4/15$
6	$1/4$	$1 \frac{1}{3}$	$1/3$

**Figure 2: Examples included in the worksheets**

Most of the participants mention that the result has been simplified by the spreadsheet. The users' answers show notions of equivalent fractions. They mention that because of simplifying  $4/10$  they obtain the equivalent fraction  $2/5$ . However, other questions also emerge linked to the idea that the simplification process "makes the fractions smaller", this is observed in the expressions written in the worksheets of 3 participants, for example: "because excel simplifies the fraction. The result without simplifying would be  $4/10$  but excel divides it by 2 to make it a smaller fraction"; "the fraction is simplified, i.e., it doesn't show such big numbers. Equivalent fractions". It is worth mentioning that in the collective discussion these misconceptions went unnoticed as users did not mention it.

*Item 2. Without showing his work, Rodrigo asks the following: -Teacher, calculate the product of  $1/2$  and  $11/20$ . How much do you get? What result do you get? How would you explain to your student what the spreadsheet is doing?*

Most of the users set the cell formatting using the 'up to one digit' instruction, so when they typed  $11/20$  the spreadsheet could not display that fraction, but instead displays  $5/9$ . This surprised most of the participants, however, there were some who did not even notice it. In addition, there were users who chose the format 'up to three digits' and did not notice that phenomenon. After some participants commented on how they configured the cell formatting, there was a consensus that it was convenient to select the 'up to three digits' fraction cell format in the spreadsheet. The relationship between the fractions was also mentioned in the discussion. Some participants expressed that, if they looked at the decimal representation of both fractions, it was possible to notice that  $5/9$  approximates  $11/20$ . Participants were asked how to know how good Excel's approximation was? Two techniques were mentioned to assess the approximation: 1) make a subtraction, the closer the result is to 0, the better the approximation, 2) make a division, the closer the quotient is to 1, the better the approximation. On the other hand, in the worksheets it was observed that one participant makes an indistinct use of the words 'simplify' and 'approximate', referring to: "Result is  $2/7$ . When configuring the cell format, 2 digits should be chosen to ensure that the application does not simplify the fraction, since, if not configured, it will automatically look for the fraction of 1 digit + [more] close".

*Item 3. Laura proposed the examples shown in Figure 2, rows 4 and 5, and asks why the products of cells C4 and C5 are different if they have the same factors? Which result is correct? How would you explain to the student what the spreadsheet is doing?*

Most of the participants answered in their worksheets that this is due to the configuration of Excel, and they refer that this approach is like item 2, and that  $1/4$  approximates  $4/15$ . In fact, some show in their worksheets how  $1/4$  is equivalent to  $4/16$ . However, there are two users who consider that both results are correct: "both results are correct, but it can be seen in two ways as the cell configuration decides whether the result will be two or one digit". The user attributes the fact that the two-digit denominator cannot be written to the cell formatting configuration, so in essence they are both correct, and one could argue why both results are correct by making a trial-and-error approximation.

*Item 4. Alejandro comments that the spreadsheet is doing something strange, because in cell B6 shown in Figure 2, he has put a fraction and the spreadsheet has written other numbers. What is going on? Which fraction has your student originally written?*

Most of the participants mention that Excel writes improper fractions in the form of mixed fractions. In the discussion one user mentioned that if it is necessary to display improper fractions on the screen, one could use the spreadsheet cells as boxes to write the numerators and denominators of the factors and add the multiplication sign between the cells as shown in Figure 3. However, a discussion arose among the participants that one multiplication sign is enough, because otherwise the idea that the numerator and denominator of a fraction are separate unrelated numbers is fostered among the students and this can be counterproductive, so this proposal was discarded. Seven users mention "the computer is doing" or "Excel is doing". This shows that they are interpreting how the software works.

	A	B	C
1	numerator	x	numerator
2	denominator	x	denominator

**Figure 3: Proposal for displaying improper fractions in Excel**

*Item 6. How would you use the spreadsheet to show your students that every fraction multiplied by 0 is 0?*

Users proposed different strategies to show in a teaching situation that the property is true "for every fraction". For example: "Let each student perform the multiplication  $x0$  [by zero] using the fraction he/she wants to use, without restriction. -Leave one cell empty-. In this case, the notion of arbitrariness resides in the fact that the property is fulfilled whatever the fractions that the students propose. Another answer is: " $a/b \cdot 0 = 0$ ;  $a \cdot 0/b \cdot 1 = 0/b$ . Using the property that any integer times zero gives zero", in this case the arbitrariness is expressed using letters. In addition, there was one user who expressed this idea as follows: "give them a paper with a fraction written on it and you just happen to give a 0 and then we open the introduction to the topic". In the latter case, the notion of arbitrariness is communicated to the students through choosing a random fraction written in the pieces of paper that will be placed in a container. Finally, others expressed the idea that a calculator can be built in Excel to multiply fractions, in which one of the factors is zero, and the other factor can be proposed by the students, and in this way, it will be seen that the product will be zero in all cases. All these examples show the users' production of meaning for the expression "any fraction". Users realized that the software interprets the value of an empty cell as zero.

Regarding the activities of division of fractions, among other questions, participants were asked: *What activities would you propose to extend the study of the algorithm of division of fractions?* Some of the answers were: "make manual or illustrated representations to be later shown on the spreadsheet"; "use the graphical method"; "propose different cases: divide by 1, by 0, by the same number"; "what happens if you divide a fraction by zero? And zero by a fraction? Is it commutative? How do you divide an integer by a fraction? And a fraction by an integer?" Finally, in the discussion it was mentioned that in Excel division is not commutative like multiplication. However, it was clarified that when we consider solving arithmetic problems, in some cases changing the order of the factors can even modify the word problem. There was discussion about the difficulties students might encounter when dividing fractions in Excel, users mentioned that, with previous use of Excel, they had a better understanding of what to expect in case of dividing fractions, but they were concerned that students would question them about division by zero.

In summary, the main questions that arose in the discussions were about how the spreadsheet processes the information it receives. Specifically, about how the software works, how it changes the representation it makes of fractions, and how participants would use this resource in a classroom. In terms of the Formal and the Teaching Models' components, users were able to realise the limitations of Excel and at the same time identify areas of opportunity to teach: 1) equivalent fractions, 2) whole numbers as fractions, 3) the relationship between improper and mixed fractions, 4) properties of fraction multiplication and division, and finally, more broadly, 5) the resources they draw on to study fraction multiplication and division: their own knowledge about fractions, particularly their decimal representation, as they recognised how it was employed in item 2.

In terms of the Communication Models' component, it was observed that participants used expressions such as "Excel is doing". This can be seen as a situation of communication between Excel and the users. A situation in which a decoding process is identified in which users develop a process

of meaning production by interpreting what Excel does. It is possible to consider Excel as an entity that users interact with as a tool to extend their reasoning skills on fractions. The collective reflection about the spreadsheet as a very particular calculator, in which not only the format of the cells can be configured, but also the visualisation of the numbers used, took place. In addition, it is possible to use other Excel elements to emphasise some relationships, as users used colours to illustrate the commutativity property of multiplication. In the discussion, they extended this idea, mentioned that when you combine two different colours of paint, no matter in which order you mix them, the colour of the paint you get is the same. This shows that users explored the potential of what Excel has to offer beyond its intended use. This promotes the broadening of their knowledge in the use of technology for teaching fractions.

Participants pointed out that Excel rewrites the improper fraction as a mixed fraction, which could hinder students from seeing that the factors are necessarily reciprocal fractions. In the discussion, they mentioned strategies they would employ to guide students to observe this relationship, including modifying aspects of the Excel visualisation and writing mixed fractions as improper fractions using paper and pencil, and other strategies already mentioned above. In terms of the Cognitive Processes Models' component, this is an example where elements related to the concept of multiplication of fractions and with respect to teaching processes were mentioned; because of the reflection on the possible difficulties students may face when doing these activities. Also, regarding the use of digital technology, since the design of tasks based on the properties of the software is mentioned.

## **Conclusions**

Many digital resources that have not been designed for educational purposes are brought into the classroom, such as Excel. This report describes a sequence of activities involving the use of this spreadsheet to multiply and divide fractions. Using other spreadsheets leads to different opportunities for reflection, as the treatment of fractions is different. In the case of Excel, at least three processes were discussed in this report: 1) simplifying, 2) approximating and 3) rewriting.

In the exploratory study it was observed that these processes induce the users to decode the way the spreadsheet works, which emerges spontaneously in the reasoning of some of them, but there are other users who did not even check Excel calculations. However, after collective discussions, everyone was able to participate in that decoding process. The data collected in the worksheets have allowed us to identify other elements that did not emerge in the discussions, mainly related to participants' misconceptions. In the development of the sequence of activities, situations of communication and production of meaning were observed. The results of the exploratory study show that it has been possible to promote the reflection processes proposed in the design of the activities, and Excel can be used as a tool to promote reasoning not only about the concepts of multiplication and division of fractions, but also about fractions in general. It was observed that, in the case of division of fractions, users were more familiar with the way Excel works, so that the design of the activities also promotes learning to use this digital tool. The sequence of activities that was designed is a proposal of how to overcome the difficulties that arise when using the Excel spreadsheet to multiply and divide fractions. Reflection and discussion about how Excel works, as well as the



extension of the content towards the concept of division of fractions, seem to be relevant in terms of the production of meaning respect to various concepts linked to fractions.

In accordance with the LTM methodological framework, to conclude the second stage of the research methodology, the workshop should be conducted with the participation of in-service Telesecundaria teachers. It is considered that the misconceptions that were identified with the group that participated in the exploratory study, concerning simplification and approximation of fractions processes, should be included in discussions with Telesecundaria teachers. From the exploratory study it is concluded that the overall research hypothesis is plausible.

## References

- Angel, A., Figueras, O., and Valenzuela, C. (2022). Una herramienta para analizar recursos educativos de Telesecundaria: el caso de la división de fracciones [A Tool for Analyzing Telesecundaria Educational Resources: The Case of Division of Fractions]. In T. F. Blanco, C. Nuñez-García, M. C. Cañadas and J. A. González-Calero (Eds.). *Investigación en Educación Matemática* XXV (pp. 139–147). SEIEM. <https://www.seiem.es/docs/actas/25/ActasXXVSEIEM.pdf>
- Ball, D. L. (1990). The Mathematical Understanding that Prospective Teachers Bring to Teacher Education. *The Elementary School Journal*, 90(4), 449–466. <https://doi.org/10.1086/461626>
- Fillooy, E. (1999). *Aspectos teóricos del álgebra educativa* [Theoretical Aspects of Educational Algebra]. Grupo Editorial Iberoamérica.
- Flores, R. C. and Rebollar, A. M. (2008). La telesecundaria ante la sociedad del conocimiento [Telesecondary in the Knowledge Society]. *Revista Iberoamericana de Educación*, 44(7), 1–11. <https://doi.org/10.35362/rie4472187>
- SEP. (2010). *La telesecundaria en México: un breve recorrido histórico por sus datos y relatos* [Telesecundaria in Mexico: a Brief Historical Overview of its Facts and Stories]. SEP.
- SEP. (2019). *Matemáticas. Telesecundaria. Primer grado* [Mathematics. Telesecondary. First grade] (2<sup>nd</sup> ed.). SEP.
- SEP. (2023). *Principales cifras del sistema educativo nacional 2022–2023* [Key Indicators for the National Educational System 2022-2023]. SEP.
- Puig, L. (2008). Sentido y elaboración del componente de competencia de los modelos teóricos locales en la investigación de la enseñanza y aprendizaje de contenidos matemáticos específicos [Meaning and Elaboration of the Competence Component of Local Theoretical Models in Research on Teaching and Learning of Specific Mathematics Subject Matters]. *PNA. Revista de Investigación en Didáctica de la Matemática*, 2(3), 87–107. <https://doi.org/10.30827/pna.v2i3.6199>
- Rezat, S. (2012). Interactions of Teachers' and Students' Use of Mathematics Textbooks. In G. Guedet, B. Pepin and L. Trouche (Eds.), *From Text to 'Lived' Resources* (pp. 231–446). Springer Science & Business Media. <https://doi.org/10.1007/978-94-007-1966-8>