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# **Eighth grade students' proportional reasoning: Strategies used in missing value and comparison problems**

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*The present study investigated 8<sup>th</sup> grade students' proportional reasoning through the strategies they used in different types of problems - missing value, missing value followed by inference and comparison problems. One hundred fourteen 8<sup>th</sup> grade students at five middle schools in Ankara, Türkiye participated in the study. They solved the 10 proportion problems in the test prepared by the authors. Students' proportional reasoning was analysed considering the set of strategies and the types of problems. Findings revealed that students were flexible in using different strategies to solve proportional problems and different problem types promoted different strategies so different proportional reasoning.*

*Keywords: Proportional reasoning, word problems, middle school students.*

## **Introduction**

Proportional understanding is the cornerstone of elementary school arithmetic and essential for learning several mathematics concepts (Lesh et al., 1988). It constitutes a major component of the middle school mathematics curriculum and is required to access high school mathematics like algebra, geometry, probability, and statistics (Çopur-Gençturk et al., 2022). Yet, students have difficulties understanding ratios and proportional relationships. They generally rely on rote algorithms such as cross-multiplication to get correct answers, fail to explain the reasoning behind the algorithm, or identify that there are often better ways to solve these problems (Ayan et al., 2023).

Considering student difficulties in proportional reasoning and the importance of adapting different approaches in proportional reasoning problems, this study aimed to investigate 8<sup>th</sup> grade students' proportional reasoning through the strategies they used in the proportional reasoning test (PRT) including different types of problems - missing value, missing value followed by inference and comparison problems. We argue that an investigation of 8<sup>th</sup> grade students' proportional reasoning strategies along with the types of problems could reveal additional information regarding how they reasoned proportionally. Accordingly, the following questions were sought: (1) What are the frequency and the diversity of the strategies used to solve the problems in the PRT? (2) How do students' performances and use of strategies change in different types of problems?

## **Proportional reasoning**

Proportional reasoning is defined as one's "ability to understand, construct, and use the multiplicative relationship between the two co-varying measure spaces... or within the measure spaces" (Van Dooren et al., 2018, p. 14). It necessitates distinguishing multiplicative relationships from additive ones and adopting different strategies while solving problems (Çopur-Gençturk et al., 2022; Lamon, 2007).

Distinguishing multiplicative relationships from additive ones is one of the developmental leaps needed for students to reach the ability to reason proportionally (Sowder et al., 1998). Students should notice that as the size of the quantities changes, the difference between two varying quantities stays constant in additive relationships but does not stay constant in a multiplicative relationship. Utilising a broad range of strategies while solving ratio and proportion problems is important to understand proportional reasoning (Pişkin-Tunç & Çakıroğlu, 2022). Prior work revealed different strategies that students may use to solve ratio and proportion problems (Bart et al., 1994; Ben-Chaim et al., 1998; Brakoniecki et al., 2021; Cramer & Post, 1993; Cramer et al., 1993; Parker, 1999). Accordingly, students may use the following seven strategies:

- cross-product algorithm: establishing a proportion, making a cross product and solving the resulting equation by division
- factor of change: comparing the quantities, deciding the factor of change between two quantities, and multiplying the factor with the value of the given quantity
- build-up: establishing a relationship within a ratio and then extending it to the second ratio by addition
- equivalent fractions: considering the ratios in the problem as equivalent fractions.
- unit rate: computing one unit of a quantity and then multiplying the result with another quantity to generate the wanted answer
- part-to-part or part-to-whole: considering the individual parts or the part and the parts of which the whole is composed
- percentage and decimal: comparing converted percentages or decimals of the part-whole ratios

The studies carried out to evaluate and develop proportional reasoning of Turkish middle school students revealed that the solution strategies used by the students were limited (Avcu & Doğan, 2014; Toluk-Uçar & Bozkuş, 2018). Students mostly use the cross-product algorithm, a formal strategy that does not emphasise multiplicative relationships (Lamon, 2007). In addition, students had difficulties differentiating multiplicative relationships from additive ones (Mersin, 2018). We argue that there is a need to understand more about Turkish students' proportional reasoning to plan instructional activities and remedial programs related to developing proportional reasoning.

## **Methods**

### **Participants**

The participants of the study were 114 eighth-grade students from five middle schools in Ankara, Türkiye. Ratio and proportion were covered in the 6<sup>th</sup> and 7<sup>th</sup> grades in the Turkish middle school mathematics curriculum. Therefore, all of the participants were familiar with the concepts.

### **Data collection instrument**

To investigate students' proportional reasoning strategies, we reviewed the literature on proportional reasoning and constructed a set of problems for the PRT based on the adaptation of the most widely known problems. The set of problems was sent to three experts (one mathematics teacher educator and two middle school mathematics teachers) and revised based on the expert opinions. The revised PRT included 10-word problems in total including different problem types.

There were 5 missing value problems (in which three values for two quantities are given and the students are asked to find the missing value) and 3 comparison problems (in which multiple complete ratios are provided and students are asked to compare them). Moreover, 2 of the problems were like missing value problems but also required making an inference after finding the missing value. Therefore, this type of missing value problem was called as an inference problem in this study. Sample problems for each problem type were given in Table 1.

Types of problems	Sample problem
Missing value	The ingredients of onion soup for 8 people are as follows: 8 onions, 4 cups of water, 4 cups chicken soup, 2 tablespoons butter. If I will cook onion soup for 4 people, how much water do I need?
Inference (missing value followed by inference)	Two toothed wheels move by touching each other. There are 72 teeth in the large and 18 in the small wheel. If the small wheel turns 1 time, how many times will the big wheel turn? Explain your answer.
Comparison	Mr. Emre is planning to paint the walls of his house. However, the colors of the paint sold in the store are not according to Mr. Emre's wishes. To obtain the desired color, Mr. Emre mixes the blue and white paints. Mixture A consists of 3 cans of blue paint and 2 cans of white paint, while mix B consists of 4 cans of blue paint and 3 cans of white paint. Each paint can is the same size. Emre Bey will choose the lighter blue color between these two paint mixtures. Which one should he choose? Explain your reason.

**Table 1: Sample of proportional reasoning problems**

### Data collection and analysis

The data was collected in the fall semester of the 2022-2023 education year. The authors informed the middle school teachers about the PRT and its application procedures.

To analyze students' proportional reasoning strategies, we first created a set of seven strategies based on existing literature. Each problem in the Proportional Reasoning Test (PRT) was then solved by two experts (authors) to identify the diversity of possible strategies for each problem. It was found that each problem could be solved by five or six different strategies.

Next, we used these strategies to score a subsample of 20 students' responses to test the comprehensiveness of the set. Once we agreed on the set of strategies, we coded the rest of the data individually. For problems where students used multiple strategies, we coded each strategy. After the individual coding process was complete, we compared the coded strategies for each problem, discussing and deciding on the final category in case of disagreement. If students used the correct solution strategies, any calculation errors they made were ignored.

Students' solutions to the different types of problems in the PRT were examined by focusing on the strategies they used. A sample of 114 students was included, and their correct solutions (including simple calculation errors) were analyzed. Out of 1140 possible answers (10 questions for 114 students), 828 (73%) were correct. Among the correct answers, 80 consisted solely of

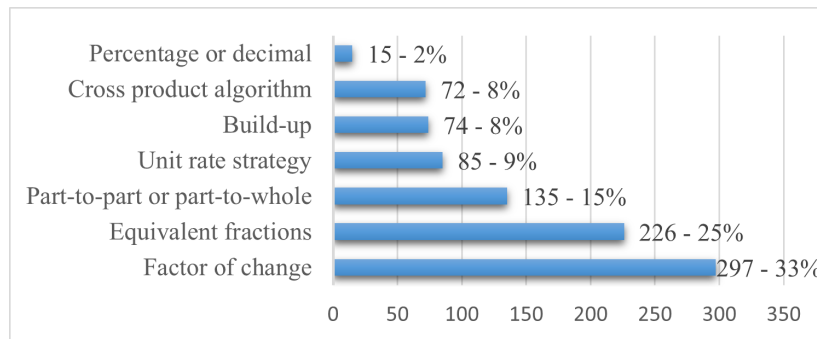
answers without any solution. Therefore, strategies were identified by analyzing the remaining 748 (66%) answers.

The score of each student was calculated based on the number of correct answers they provided. Out of all the participants, 21 answered all the problems correctly. The number of participants who answered 9 problems and 8 problems correctly was the same, with 20 students in each category. Additionally, 15 (13%) participants answered fewer than 5 problems correctly or left them blank. No participants provided incorrect answers or left all 10 problems blank. To calculate the average score, all individual scores were added up and divided by the total number of participants. The average score achieved in the PRT was 7.2 out of 10 points.

## Findings

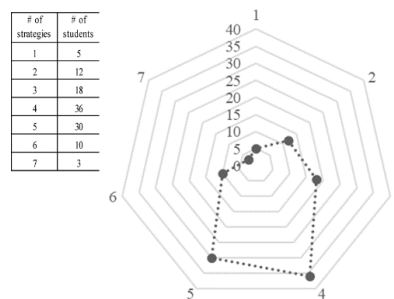
### Frequency and diversity of proportional problem-solving strategies (RQ-1)

A total of 904 strategies were used by 114 participants. However, since some students used more than one strategy for some problems, the total number of strategies used (904) exceeds the number of correct answers, including solutions (748). The frequency and percentage of each strategy used are given in Figure 1.



**Figure 1. Frequency and percentage of proportional problem-solving strategies**

The diversity of the strategies pointed to the number of different strategies used by each student through the PRT (Figure 2). Accordingly, most of the students used four ( $n=36$ , 32%) or five different strategies ( $n=30$ , 26%) to solve the problems in the PRT.

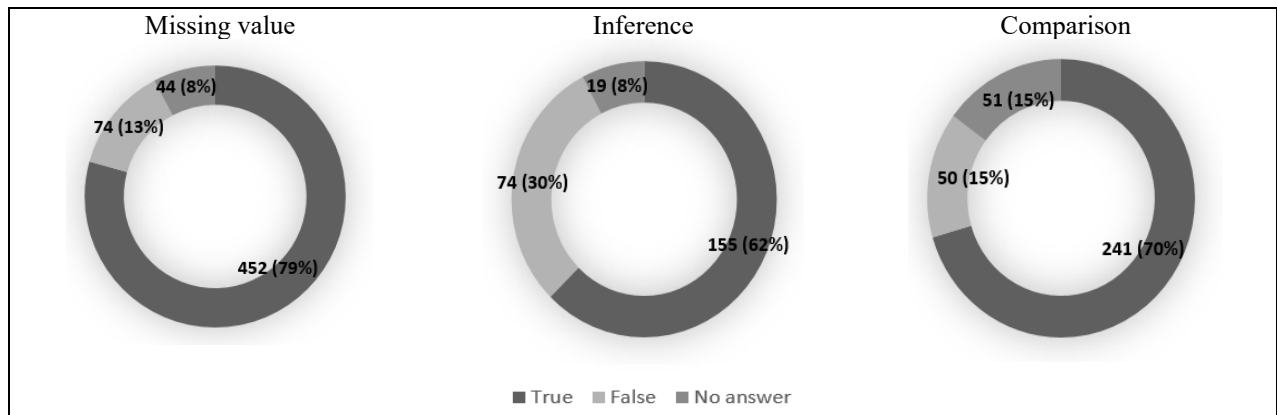


**Figure 2. Number of different strategies used by each student**

### Students' performances and strategies based on problem types (RQ-2)

Figure 3 shows the performance of students on three types of problems. The missing value problems had the highest correct answer rate at 79%, while the inference problems had the lowest

correct answer rate at 62%. The inference problems also had the highest rate of false solutions at 30%. Upon examination of the false solutions provided for inference problems, it was found that the missing value was calculated correctly in most answers, but the inference was incorrect. The comparison problems had the highest rate of unanswered questions at 15%.



**Figure 3. Students' performances in 3 types of proportional problems**

Table 2 provides the frequency and percentage values of strategies used in three types of problems. A total of 904 strategies were used by 114 participants, resulting in correct solutions. Out of 5 missing value problems, 440 strategies were correctly applied. The most frequently used strategy was the factor of change strategy (57%), followed by the cross-product algorithm (15%) and equivalent fractions strategy (12%). For 2 inference problems, 123 strategies resulted in correct solutions. Among these, the most frequently used strategies were equivalent fractions (46%) and factor of change (35%). In 3 comparison problems, 341 strategies were applied correctly. The most frequently used strategies were the part-to-part or part-to-whole strategy (39%) and equivalent fractions strategy (34%). The findings indicate that the preferred strategy varies depending on the type of problem. As inference problems are based on missing value problems, the most frequently used strategies in each problem type were similar. Differences in comparison problems shed light on the differences in students' proportional reasoning.

There were multiple solutions available that involved more than one strategy. Out of the 748 correct answers, 150 solutions indicated the use of multiple strategies. Among these, the part-to-part or part-to-whole strategy was the most commonly used accompanying strategy, appearing in 100 (67%) of the 150 solutions that used multiple strategies. The part-to-part or part-to-whole strategy was used alone in 35 (3%) of the solutions.

Types of Strategies	Missing Value (n=5)		Inference (n=2)		Comparison (n=3)		Total	
	f	%	f	%	f	%	f	%
Cross-product algorithm	66	15	3	2	3	1	72	8
Factor of change	251	57	43	35	3	1	29	33
Build-up	34	8	17	14	19	6	70	8
Equivalent fractions	52	12	57	46	117	34	22	25
Unit rate	25	6	0	0	64	19	89	10
Part-to-part/part-to-whole	1	0	2	2	132	39	13	15
Percentage/decimal	11	3	1	1	3	1	15	2
Total	440	100	123	100	341	100	90	10

**Table 2. Frequency and percentage of strategies based on problem types**

## **Discussion**

The present study investigated 8<sup>th</sup> grade students' proportional reasoning in terms of the strategies they used in different types of problems - missing value, missing value followed by inference and comparison problems. Findings revealed that students were flexible in using different strategies to solve proportional problems in the test. Although the frequencies were different from each other, all of the seven different strategies appeared in the answers. Considering the whole test, the factor of change and equivalent fraction strategies were the most preferred strategies. It was not expected since the cross-product algorithm was reported as the most preferred strategy in the previous studies (Lamon, 2007). Considering that the cross-product algorithm is a formal strategy that does not emphasise multiplicative relationships, using other and various strategies in the present study could be considered as a sign of having proportional reasoning.

Further analysis based on the problem types showed that students' performances and use of strategies changed according to the problem types. The correct answer rate was higher for missing value problems compared to inference and comparison problems. Previous studies found similar findings when they compared the performances for missing value and comparison problems (Silvestre & da Ponte, 2012). The difference in the present study stems from including another type of problem- missing value followed by inference. In contrast to the missing value problems, students' performances were worse in inference problems since finding the missing value is not enough for inference problems. Students have to reason about the calculated missing value considering the direct or inverse proportion situations or the context of the problem. Examination of false answers in the inference problems showed that the students could not make reasonable inferences on the proportion they calculated correctly. Although there were studies investigating students' proportional reasoning process in missing value and/or comparison problems in the literature (Kaput & West, 1994; Silvestre & da Ponte, 2012), there is a need to conduct studies examining students' solutions to inference problems. Classroom observations and task-based interviews might be integrated into the studies to understand how students reason or why students cannot reason when they need to make an inference after finding the missing values.

Analysis of strategies based on problem types yielded a different picture from the whole test. Preferred strategies were changed based on the problem types. Therefore, different problem types prompted different proportional reasoning. We can conclude that missing value problems require basic proportional strategy calculations whereas the other problem types necessitate reasonable proportional inferences and comparisons in accordance with the authentic situation given in the problem. We thought that integrating different problems is important to develop proportional reasoning.

The findings presented in this study are limited to the participating middle schools and students in Türkiye. It is possible that high school students, primary school students, middle school students in different grades, or students from different cultures may prioritize different strategies to solve different problems. Additionally, if the problems were asked in different ways, such as through task-based interviews, more in-depth data about students' proportional reasoning processes could be obtained.

Despite these limitations, the study's findings revealed information about the students' flexibilities in thinking proportionally based on different problem types, which could provide new directions for further studies. To develop algebraic reasoning, problems that require different algebraic thinking, such as comparison and missing value problems, should be provided to students. Encouraging students to use various strategies is also important. In this study, students were asked to solve problems in only one way and provide detailed explanations. However, if students are asked to present multiple solutions, they will be motivated to use different strategies and find different associations. This approach will provide a suitable environment for them to explore a variety of strategies.

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