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An introduction to TWG22: Curricular resources and task design in mathematics education

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During CERME13, TWG22 continued to combine macro level research on curricular resources and micro level research on more granular units of analysis in curriculum design, namely tasks. The presentations were organized around the following themes מרכז: how curricular resources function within and shape a broader curricular system, and how these resources and textbooks provide a lens to study and compare education systems; design research focusing on various units of analysis of curriculum resources such as digital environments, tasks, and units in books; empirical research on teachers' and students' interactions with curriculum materials; and the role of resources (material and non-material) from the teachers' perspective. Finally, along sessions, we noted the ideas shared by the participants about the many connections TWG22 has with other topics addressed in other thematic working groups, urging interest for further collaboration.

Keywords: Curricular resources, task design, task implementation, textbook research, curricular systems.

Scope and focus of the working group

TWG22 on curricular resources and task design in mathematics education carries within its scope a twofold connected perspective. At the macro level, teachers, lecturers, and students work with mathematics curriculum resources, both digital and traditional, inside and outside any formal or informal learning settings. Individually or collectively, teachers and instructors select, (re)design, modify, and interact with such resources for lesson preparation, student assessment, and the planning of their teaching. These resources (e.g., educative curriculum materials) often are the focus of professional development sessions, where mathematics teachers, often with education researchers, design and transform curriculum resources, including print, digital, and hybrid materials, and in the process develop design capacity and valuable knowledge for teaching. At the micro-level, curriculum resources contain mostly tasks derived from textbooks or other sources. The representation of these tasks in resources, their sequencing, and the teachers' actions during their enactment can limit or broaden the cognitive demand the tasks impose and affect students' views of the subject matter. Thus, they can influence the opportunities afforded to students to make mathematical connections and to develop mathematical concepts, skills, or habits of mind. The literature indicates that tasks play a key role in effective teaching. There has been an upsurge in publications on various aspects of task design (e.g., task features that can help generate certain forms of mathematical activity); methods of task analysis (e.g., analyses of the learning affordances of certain kinds of tasks); and principles for task implementation, including digital learning environments (e.g., factors affecting the fidelity of implementation of tasks in the classroom). Students can also be involved in task design activities to foster their reflections about what they know, understand, and do.

We summarize the papers presented at the conference in the way that they were presented in the different sessions, which connected several presentations around a common theme according to the following predefined themes:

- Empirical research on teachers' and students' interactions with curriculum materials/resources/tasks, and influences on this interaction over time (e.g., longitudinal studies);
- Studies on the use of carefully designed curriculum materials/resources/tasks to support the implementation of particular learning goals connecting trajectories and curricular resources;
- The collaboration among teachers, between teachers and researchers, and possibly also students for designing tasks/resources and analyzing their implementation and their added value;
- The study of how curricular resources function within a wider curricular system, and how they shape that system and provide a lens through which curricular systems can be studied and compared;
- Theoretical foundations and methodologies of task analysis informing task design and scaling-up.

Issues addressed during the sessions

An important issue in TWG22 was how curricular resources function within and shape a broader curricular system, and how these resources provide a lens to study and compare education systems. Two papers focused on international comparative studies of textbooks:

- Sikai Wang, Carl Winsløw, & Binyan Xu: Mathematical praxeologies in the Chinese and Singaporean senior high school textbooks: the case of probability
- Hendrik Van Steenbrugge & Ayşe Yolcu: The inscription of desired images of children through contextualized tasks in mathematics textbooks from Flanders and Turkey

In these two papers, textbooks from different countries were the focus of international comparative studies. The papers reflected a combined focus on the methods of selection of the countries, the textbooks, and the topics being compared. The main unit of analysis were the tasks, and the investigated aspects were not limited to the mathematics topics or practices, but also expanded to contextual aspects related to both student's image as well as teacher's practices. Wang, Winslow, & Xu present a praxeological analysis on probability in Chinese and Singaporean textbooks. In their analysis of the first chapter in both textbooks, they showed that the textbooks share three types of tasks and provide similar techniques to solve them. There is a shared focus on calculation of random events in both textbooks. The main differences were in the number of examples and exercises, and also the different types of tasks that are more diverse in the Chinese textbook. Another difference was found on the theoretical level. While both textbooks deal with explanations for the basic properties of probability, the Singaporean textbook does it mainly in an inductive way, while the Chinese combines induction and deduction, providing corresponding proofs. This focus on praxeologies leads to the question of how these differences would be reflected in the practice of teachers and students. Insight into how two different countries' desired images of children are inscribed through mathematical tasks was provided in the paper by Van Steenbrugge and Yolcu. Elementary school textbooks from Flanders and Turkey were compared, analyzing tasks which included additional

context on top of the pure mathematical content. Analyzing the different contexts, including personal, occupational and others, as well as the focus on exactness or flexibility in the tasks, resulted eventually in two perspectives on mathematics: a pragmatic perspective that underlines the usefulness of mathematics and an awareness perspective seeing mathematics as a tool to perceive and understand the world. The authors note that in some cases the inscriptions are more explicit than others, leaving open questions regarding the cultural role of mathematics in inscribing different models of modern citizenship, and the extent in which it should be mediated by textbooks.

The issue of design (of digital environments, of tasks, of units within textbooks) represents the focus of the following papers:

- Hassan Ayoob & Shai Olsher: Curriculum-aligned digital formative assessment (DFA) for elementary school geometry
- Jonas Jäder & Lotta Wedman: Using different versions of a mathematical problem for visualizing students' conceptions of perimeter
- Jane-Jane Lo, Lili Zhou & Jinqing Liu: Stories of congruent triangles in two geometry textbooks

The three papers addressed the investigation of the design process from different perspectives: the designers' intentions and the methodological approach to be adopted, the students' interaction with the tasks, and the role of theoretical frameworks in supporting the researchers' investigation. In addition, the three studies presented in these papers highlight the influence of both the national assessment curriculum and textbooks on the design of tasks and resources. The issue of identifying design criteria is addressed in the two papers by Ayoob and Olsher and by Jäder and Wedman. The paper by Ayoob and Olsher presents a study aimed at identifying guiding principles for the design of digital formative assessment, within a platform for elementary school geometry, aimed at informing teachers about the learning of geometry by 5th graders aligned with the Israeli curriculum. The research study, which follows a design-based approach, suggests two main phases, which are presented and illustrated in the paper: the formulation of essential components for digital formative assessment through the mapping of the Israeli curriculum, reviewing the research literature and analysing textbooks; and the design and implementation of specific tasks through a specific platform. The paper by Jäder and Wedman focuses on criteria for designing and modifying tasks to investigate students' conceptions. Specifically, it presents the design of different versions of the same problem to investigate students' conceptualisation of perimeter in primary (age 11-12) and secondary school level (age 15-17) in Sweden. The reflections on the effectiveness of the task design process are developed through the analysis of students' solutions, which shows that the chosen problem provides conceptual challenges to enable the investigation of students' conceptions. Furthermore, the analysis presented suggests that the discussion on task design should be deepened by focusing on the properties and definitions of concepts, the use of non-typical examples and the relationship between processes and concepts. The third paper, by Lo, Zhou and Liu, presents a framework for analysing the design of units within textbooks that is useful for comparing textbooks. The authors present the results of a comparative analysis of a unit focusing on the introduction and development of congruent triangles in a U.S. high school textbook and a Chinese Grade 8 mathematics textbook. The comparative analysis is developed using Dietiker's (2015) Mathematics Curriculum as A Story framework, which conceptualises mathematics textbooks as stories, according to a narrative

perspective that includes four elements: the mathematical characters, the mathematical actions, the mathematical setting and a mathematical plot. The results of the analysis highlight this framework's potential in textbook analysis research.

Another aim of TWG22 was focusing on tasks and their design. Three papers addressed challenging didactical goals highlighted in mathematics tasks – flexibility, reflection, and interaction:

- Menucha Farber & Boris Koichu: Teaching situations designed by experienced teachers to foster students' mathematical flexibility
- Edith Schneider: Tasks stimulating reflections on mathematics
- Vilma Mesa, Eric Khiu, Saba Gerami & Thomas Judson: Conceptions of spanning sets and linear independence emerging from examples and student responses to reading questions in an interactive linear algebra textbook

In these three papers, the emphasis is put on issues of how to make students think and how to make their thinking visible through requirements such as flexibility, reflection, and interaction. In addition, these papers highlight bringing together practitioners and researchers, and involving teachers and students in the process of designing or using such tasks. Farber and Koichu investigated the teaching situations designed by experienced teachers to foster students' mathematical flexibility, i.e. switching between problem-solving strategies and between multiple representations in various tasks for students across the curriculum. The designed teaching situations regarding flexibility in problem-solving tasks were classified into multiple-solution situations, "find a better strategy" situations, and dead-end situations. The study puts emphasis on the development of mathematics teachers' task knowledge and design skills. Reflection is another important activity/process in mathematics classrooms that can be encouraged through various tasks. Based on a literature review, Schneider brings description and categorisation of reflection tasks as a basis for designing tasks that should encourage students to reflect on mathematics: mathematics-oriented, model-oriented, context-oriented, and subject-oriented reflection. The paper highlights a strong need for teacher in-service training for identifying, implementing, and designing reflection tasks in the classrooms. Encouraging students to engage with mathematics may be also effectively activated using digital resources. Mesa, Khiu, Gerami, and Judson used interactive textbook features to engage students to think about the content to be learned. In this way, the teachers and researchers could better understand how students interpret the information in the textbook task, and how they rely on their own prior knowledge to tackle the problem. Furthermore, such analysis of student responses may be helpful for teachers to alter their lesson. These issues again highlight the important interplay between the research community, teachers, students, and resources.

The following papers address the issue of resources, focusing on their role from the teachers' perspective:

- Dubravka Glasnović Gracin & Goran Trupčević: Time as a resource in mathematics education: comparing teacher and student perspective
- Erdiñ Çakırođlu, Iveta Kohanová, Işıl Işler-Baykal, Mária Slavíčková, Benedetto Di Paola, Jakub Michal & Siro-Malén Høyne: Mathematics teachers' uses of resources in the context of teaching reasoning-and-proving: Insights from a cross-national study

- Alexandra Angel, Olimpia Figueras & Carlos Valenzuela García: The potential of curricular resources for teaching practice in Telesecundaria in Mexico: an exploratory study
- Giovanna Bimonte, Francesco S. Tortoriello & Ilaria Veronesi: Topological approach to historical analytic narrative in the “Game Theory lab” activities

The studies presented in the four papers make it possible to reflect on the interplay between different resources, highlighting in particular the influence of cultural and social differences between countries on the way teachers use and perceive resources, and the role of teachers' personal resources (beliefs, goals, knowledge) on their interpretation and use of specific resources (time, tasks and activities from textbooks, digital tools...). Furthermore, the results of these studies highlight the role of digital technology as a “friend” for teachers, helping them to save and manage their time (first paper), to promote mathematical processes (second paper), reflect on their knowledge (third paper), and realise interdisciplinarity (fourth paper). The paper by Glasnović Gracin and Trupčević presents a study focusing on a teacher's and her students' reflections on the role of time as a resource in mathematics teaching and learning. By adopting Adler's (2000) framework, which categorises resources into basic, human, material, and social and cultural, time is interpreted as a non-material resource. The study presented in this paper was conducted to investigate how the teacher and her students experience time in mathematics education according to different aspects (mathematical content, lesson planning, enactment, and homework) and the interaction between time and other resources. The analysis of the data collected highlights the high value placed on time by the participants and the consistency between views and experiences of the teacher and her students. The issue of the teachers' perspectives on the role of resources in supporting specific mathematical processes (in this case, reasoning and proving) is addressed in the paper by Çakıroğlu et al. These scholars investigated the resources used by pre-service teachers to teach reasoning and proving in five countries (Slovakia, Czech Republic, Italy, Norway, and Turkey) and their views on these resources. The study presented in the paper was carried out by means of a questionnaire, which involved 954 pre-service teachers from grades 1 to 13, and a series of follow-up semi-structured online interviews with 30 of them. The data analysis was developed with reference to Pepin and Kock's (2021) classification of resources (curriculum, general, social, and cognitive) and through the theoretical lenses provided by the Documentational Approach to Didactics framework. The results highlight the key role of textbooks, other books and teachers' own materials as fundamental resources for the teaching of reasoning and proving, but also that teachers perceive the main resource available, textbooks, as insufficient. This suggests the need to promote international collaboration aimed at designing professional development to support the development of teachers' skills in modifying textbooks' tasks and activities to support their pupils' reasoning. The exploratory study documented in the paper by Angel, Figueras and Valenzuela García is an example of exploring of the role of specific digital resources and tools in a context of limited availability. The study involved a small group of teachers in a workshop designed with reference to Filloy's (1999) framework of Local Theoretical Models. The workshop aimed to make teachers reflect on the use of Excel as an educational resource for teaching multiplication and division of fractions according to a teaching model introduced in the context of Telesecundaria education in Mexico. The study shows the role played by the analysis of specific digital resources used in Telesecundaria in making teachers become aware of possible misconceptions and in enriching their knowledge of both specific mathematical contents, their teaching and the integration of digital resources in their practice. The paper by Bimonte, Tortoriello and Veronesi presents the design of an

educational activity for students. The context of the study is an Italian national project (the “Mathematical High School” project), which aims to promote the restoration of the interdisciplinary dimension of mathematics in the teaching curriculum. The designed activity, involving mathematics, history and economics, focuses on use of game theory models to reinterpret past events with the aim of students developing awareness of the role of mathematics as a tool for analysing the interactions between agents.

The following four papers related in particular to the theme of empirical research on teachers’ and students’ interactions with curriculum materials:

- Sotirios Katsomitros & Konstantinos Tatsis: Teachers’ problem posing for facilitating mathematical transition from primary to secondary school
- Sümeyra Tütüncü, Jeremy Hodgen & Jennie Golding: Proposing educative features of the curriculum materials that can enhance teachers’ noticing
- Ingolf Schaefer & Erik Hanke: Reflecting from the start – curriculum design to foster continuous reflective practice in mathematics pre-service teacher education
- Luciana Vieira Andrade, Hélia Jacinto & Rúbia Barcelos Amaral: The relationship between teacher education and the choice of Mathematics textbooks

Together, these four papers and the related discussions addressed teacher knowledge and capacities in relation to resource use, as well as supporting these knowledge bases and capacities through curriculum resources, professional development initiatives, and teacher education. Methodologically and conceptually, these studies also exemplify and underline the need for our field of study to continue developing existing frameworks. Katsomitros and Tatsis focus on three grade six teachers’ modification of tasks from a grade seven textbook as a way to smoothen transition from elementary to secondary school in Greece. Their study reveals how tasks got modified by the teachers in terms of presentation, content, and solution, and how teachers’ views related to these modifications. Tütüncü, Hodgen, and Golding provide us with an analysis of five sets of teacher’s guides from the UK and USA with a focus on the potential in these materials for teacher learning about noticing student learning. As such, the study adds to the research field with specific design features to support teacher learning for teaching mathematics. Both Schaefer and Hanke’s and Andrade, Jacinto, and Amaral’s studies relate curriculum resources and teacher education. Schaefer and Hanke reflect on an attempt to have German student teachers reflect about content and didactics throughout a four-year period via an e-portfolio. Andrade, Jacinto, and Amaral inquire in the knowledge Brazilian teachers draw upon when selecting a mathematics textbook. These scholars have traced connections between the related knowledge teachers draw upon and teacher education.

The three papers below had a specific focus on the design of print and digital curriculum resources.

- Saba Gerami, Shi Qi Lim & Vilma Mesa: Undergraduate students’ use of interactive mathematics textbooks: A study of user’s action paths
- Antoine Julien, Mari Jensen, Alexander Schmeding & Abolfazi Rafiepour: An analytical framework for programming tasks in mathematics textbooks
- Nils-Jakob Herleiksplass, Viktor Freiman & Raymond Bjuland: Exploring the structure of multiplication tasks in Norwegian textbooks within a developmental approach

Whereas Julien et al. and Herleiksplass et al. carried out an analysis of a print textbook, Gerami and colleagues drew on data of students' use of digital textbooks. In a USA-based study, Gerami, Lim, and Mesa aim to better understand how students use digital textbooks by drawing on visualizations of students' use of digital textbooks and on students' descriptions of textbook use based on these visualizations. In so doing, the authors want to inform textbook designers to improve the features of digital textbooks. Following the recent incorporation of programming into in the mathematics curriculum in Norway, Julien, Jensen, Schmeding, and Rafiepour report about the development of a framework to analyze programming tasks in mathematics textbooks. The framework builds on and modifies existing frameworks in the field, resulting in a proposed four-level analytical approach, which is exemplified with programming tasks from a grade 10 textbook. The authors see promise in the framework to support educators and textbook authors to evaluate the potential of programming textbook tasks. Also relating to the Norwegian educational context, Herleiksplass, Freiman, and Bjuland analyzed multiplication tasks in a grade two textbook with Russian roots. In so doing, these scholars identified a number of multiplicative aspects, such as repeated addition and properties of multiplication, and a specific structure to prompt students to observe and/or explore, reflect, and synthesize, which the authors trace back to the Russian roots of the textbook.

One session of TWG22 during CERME 13 was dedicated to poster presentations, followed by a discussion for each poster submitted to this group. In this way, the poster presenters had the opportunity to get feedback on their ideas and to reflect on the issues they posed in their posters. As such, this session served as a support to poster presenters prior to the following conference poster session. The presented posters were as follows:

- Emília Mit'ková & Monika Dillingerová: Wilma as a way of connection between maths educators in universities and teachers in practice
- Tandeep Kaur, Eilish McLoughlin & Paul Grimes: Design principles for tasks to support student learning in mathematics across primary-secondary transition: An iterative approach
- Kristin Parve & Mart Laanpere: Granularity of CT tasks embedded in secondary mathematics lessons
- Rebekka Will: Textbooks in interaction in primary school mathematics lessons
- Adi Eraky, Ronnie Karsenty & Alon Pinto: On the bumpy road to decisions: Reconciling different agendas and perspectives in cross-community collaborative task design

The poster topics also focused on two main foci of the TWG22, namely the curriculum resources on the macro level, and tasks as resources on the micro level. Both foci highlighted the important interaction between researchers and teachers. Looking at tasks as resources that play a key role in effective teaching, three posters presented different aspects of task design. Parve and Laanpere developed different types of computational thinking tasks that can be embedded in mathematics education. This poster discusses the reciprocal relationship between mathematics and computational thinking, which is reflected in tasks as well. In their poster, Kaur et al. offered a design principle for developing and exploring rich tasks with the aim of facilitating the student transition from primary to secondary level. This design principle was brought out as a result of the collaboration of both researchers and teachers. Collaboration in task design of different stakeholders (teachers, teacher educators, researchers) was also highlighted in the study by Eraky and Karsenty. The analysis of such a collaborative task design process showed that all the participating stakeholders contributed to the

accumulation of both dilemmas and suggestions, that could be grouped into expansion and convergence processes. The collaboration between university professors and teachers can also be established on the macro level of curricular resources. Such a collaboration is presented in the poster by Mitřková and Dillingerová. As a part of a wider project, the authors developed a website with teaching materials with the aim to better connect research and practice in the field of resources and to investigate its effects. But besides teachers, it is important to explore how different resources are used by students. The poster by Will highlights the importance of investigating students learning mathematics with textbooks as part of everyday school interaction. Textbooks are important potentially implemented curriculum resources, and the author is preparing the study on the interaction with textbooks as artefacts. These last two posters, although referring to resources on the macro level, contain tasks and explore teachers' or students' tasks use.

Possible directions for future research

TWG 22 discussions emphasized the rich network of connections with studies from other TWG's: papers are not restricted to a particular school level, so the papers cover working with resources and developing them from early grades to university level; papers are not restricted to a particular mathematics topic, so the papers cover a wide range of content (e. g. geometry, algebra, and more); curricular resources are not limited to pencil and paper, they can be offered in digital environments, games, and authentic realistic settings; and many other connections as well. We envision exploring different modes of interaction among the different TWGs, during and between conferences, to cater to these common interests. Specifically related to TWG 22, the following directions for future research emerged from the discussions: (a) studies extending the unit of analysis from a specific topic to a broader curriculum band; (b) studies regarding the role of material and non-material resources that come into play in task design and implementation; (c) framing and discussing the role of students in task design; (d) continue to develop and utilize existing theoretical constructs in studying and communicating around task design; (e) studies on different actors and the influence of the context in the enactment of curriculum; (f) studies on the design and implementation of authentic activities that are connected to realistic phenomena; (g) the potential of cross-contextual curriculum resource studies.

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All the other papers mentioned in this document are within the references lists of the papers presented at TWG22.