Semiotic function: music notation vs fraction knowledge

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This study examined the effects of an academic music intervention on the conceptual understanding of music notation and the representation of fractions of seventh grade students from a multicultural and mixed socioeconomic public school environment. Students (N = 15) had prior concepts about music instruction, as well as representation of fractions. This is an observational study in which a battery of four tasks was administered before and after instruction. The instruction included 9 sessions of 50 minutes. The results after the intervention show an acquisition of the discrete model of representation of fractions, as well as an association with musical symbols.

Keywords: Fraction representation, semiotic function, music notation, interdisciplinary intervention.

Introduction

Although the impression that music and mathematics are very different subjects, the reality is very different. In ancient Greece, Pythagoras (ca. 580-500 BC) realized the relationship between different sounds and the proportion of the strings that emitted them. This discovery would end up giving rise to a tuning system called Pythagorean tuning. But the relationship between mathematics and music is not limited exclusively to the relationship between different physical aspects of the vibrating object and the sound emitted. From harmony to times, passing through the timbres of the instruments; everything on which music is built has a mathematical component.

Thus, if these two disciplines are so related to each other, it seems logical to try to take advantage of this link to enhance learning. This concept that tries to promote learning by correlating these two subjects is not new. The European Union financed projects such as Mall et al. (2016) in order to promote interdisciplinary.

This context defines the need to continue research on the incorporation of music and its characteristics in the teaching and learning process of mathematics, and vice versa. In particular, the present work intends to evaluate the effects of an academic music intervention on the conceptual understanding of music notation and the representation of fractions of seventh grade students from a multicultural and mixed socioeconomic public school environment.

Theoretical Framework

According to Veraksa et al. (2022), symbolic play represents the child's ability to link concrete experience with abstract thought by connecting the significant with the signifier. In this process, the child develops the semiotic function, an ability to represent an absent object or an event that is not directly perceived through symbols or signs (Veraksa et al., 2022).

In the modern teaching of mathematics, the interaction between perception and action also becomes increasingly important. Different levels of communication work together in semiotic sets and students use the cycles of action and perception to develop mathematical understanding. The identification of

patterns and their interpretation as a system of signs constitutes a fundamental mathematical task. The repetitions, and therefore the regularities, can be found by observing written symbols. These regularities are the basis of mathematical understanding (Mall et al., 2016).

In this context, the use of musical notation as a fraction representation tool is within what is called representation by discrete models and symbolic notation (Azaryahu et al., 2020). Regarding representations, it has been found that fractions can be represented discretely, symbolically (numerically or literally) and geometrically (Rico, 1997).

In discrete representations, the unit is made up of a discrete set of objects. Regarding numerical representations, there are different ways of using numbers to indicate a fraction that is based on the different uses of the fraction (Kieren, 1980): as a quotient (3/5), as a ratio (3:5), with a writing in decimal number (0.6) and as a percentage (60%). In the case of literal representations can be distinguished in different ways: three fifths, three out of five, and a ratio of three to five.

Being the geometric, area or linear model, the one used during fraction instruction (Rico, 1997), as it is a common way to support children's understanding of fraction concepts (Siegler et al., 2013).

The literature is extensive on the use of music as a multimodal approach to fraction instruction. Azaryahu et al. (2020) or Lovemore et al. (2022) demonstrate that there is a deeper understanding of fraction reasoning because students are introduced to fraction concepts in fun and engaging ways.

Methods

This work is part of an experimental and observational study. Within observational studies, the present study is defined as a short-term longitudinal study. We have worked on the same sample, students belonging to 7th grade in the middle school for a period of 2 months, obtaining data at the beginning and at the end of it.

This section gives rise to three subsections in which the sample studied, the method used, the instrument and the research variables and finally the intervention carried out are presented.

Samples

Regarding the study population, it is a small group of 15 students belonging to a public institute in Valencia, Spain. Among the students there is a high number of immigrants and students at risk of school absenteeism. In turn, there is great diversity with respect to the economic and cultural levels of the families of the students that make up the classroom. The ages of the students are between 12 and 14 years old. Note that, the complete intervention and pre-test and post-test were done by 12 students.

Methodology

The study has three different sections. First, a battery of four questions was asked, prior to the intervention (pre-test). After this, an intervention in Mathematics class is carried out with an approximate duration of two months (9 sessions of 50 minutes each). Finally, a second battery of four questions (post-test) is carried out. Both sets of questions contain questions related to the contents of fractions that the students have already seen in the previous course (6th Grade in the middle school) and that will be extended in their current course.

This work is part of a larger study, so the first and second sessions are the ones that focus, to a greater extent, on graphic representation. Although the rest of the sessions the representation through music continues to be present, but it is used for the operation, equivalence and order of fractions, as well as problem solving. Thus, details of Session 1 and Session 2 are given.

Session 1

The first intervention begins with the introductory part of the fractions, as well as a brief introduction of how these have a great importance in the world of music.

To carry out this introduction to the mathematical reality, the students are introduced to the concept of fraction from the musical environment of the orchestra. For it a portrait of a famous cello performer is used as support Figure 1a, a culturally known instrument, as well as the musical symbols Figure 1b. It should be noted that in the Spanish curriculum the students, in courses prior to 7th Grade, have already been introduced to both musical symbols and string instruments.

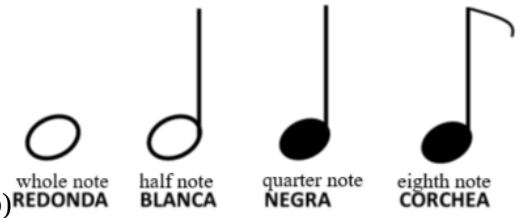
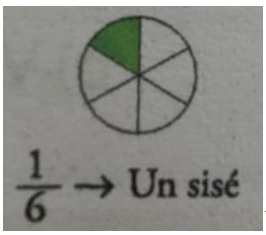
a)  b) 

Figure 1: Introductory images, a) Portrait of Guilhermina Suggia; b) Name of music symbols

Session 2

This session takes place during the first few bars of the conventional fractions unit. In the manual used in the classroom (Colera & Gaztelu, 2020) the continuous and linear representation models are presented (Figure 2). For the first, use is made of typical flat figures (circles and rectangles), the linear model is presented in the usual way using a line and placing the corresponding fraction on it.

a) 

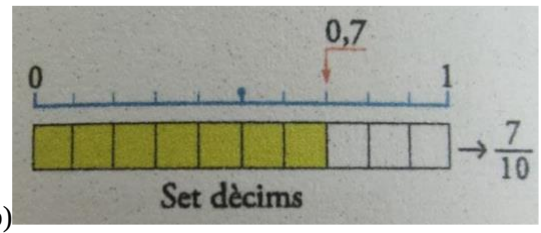
b) 

Figure 2: Representations of the manual (Colera & Gaztelu, 2020), a) Continuous model; b) Linear model

This session tries to directly address the main objective of the work. To do this, after introducing the models seen in the manual, a linear model and a discrete model for the representation of fractions from a musical point of view are presented to the students. This is done using the images in Figure 3 as support.

Therefore, in the intervention, the linear representation model is introduced through the cello strings, Figure 3a. This part is contextualized in a historical way, exposing how Pythagoras discovered the relationship between the lengths of a string and the frequency of the sound they produced.

Next, the discrete model is introduced through the musical symbols Figure 3b. With this, the usefulness of this symbols is exposed as a codifying element that is used to increase the speed with which the interpreters are able to read the scores.

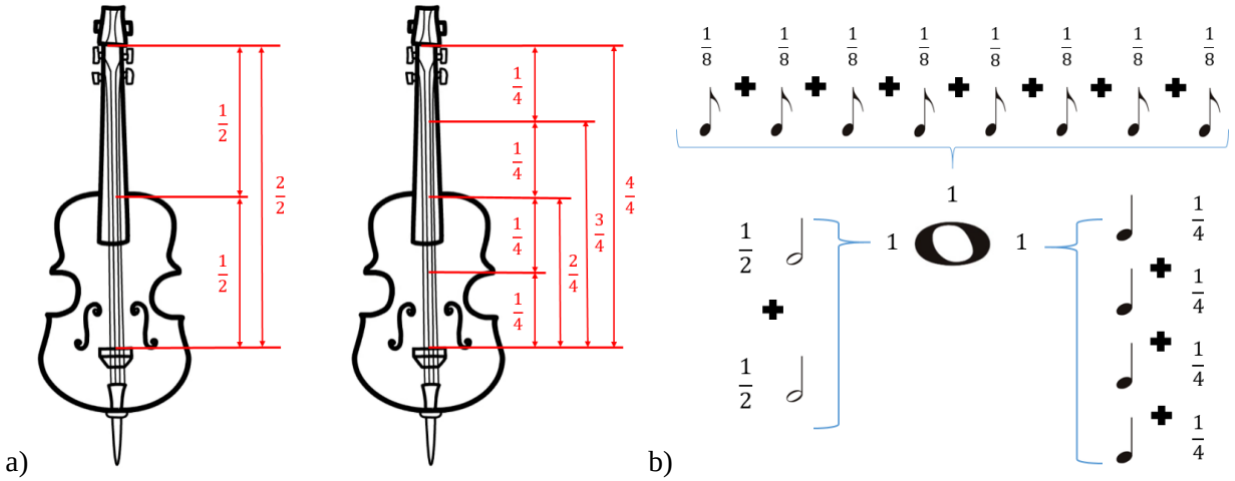


Figure 3: Representation of fractions using music, a) Linear model; b) Discrete model

Consequently, this session covers, among others, the part of the curriculum corresponding to graphic representation that corresponds to Q1 of the pre-test and post-test.

Instrument and Research Variables

To obtain research data, two pencil and paper batteries with 4 questions each are used. Although some data changes in the questions between the pre-test and the post-test, the isomorphism between the two is preserved to make possible an evaluation of the evolution of the students.

Q1. Represent, in as many ways as possible, the following fraction: ¼

Q2. Order, from smallest to largest:

Pre-test: 2/2,1/4, 7/8, 5/4. *Post-test: 1/2, 3/4, 2/8, 2/2*

Q3. Write and represent two equivalent fractions.

Q4. Calculate, you can rely on representations:

Pre-test: a) 1/2+3/8; b) 2/2-1/4. Post-test: a) 2/4+1/2, b) 6/8-1/4.

Since the work, due to the extension allowed, focuses on the representation of fractions, only the analysis and the research variables of Q1 will be determined.

Question Q1 allows you to check the fraction representation skills of the students. In particular, it allows us to analyze how many representation models the students are familiar with. On the other hand, possible errors made in carrying out this process will also be highlighted, if any.

In order to carry out a descriptive analysis of this issue, the research variables defined are three:

- Representation Type: this field corresponds to the representation model used in the response: 1) area, 2) linear, 3) discrete or 4) numerical (decimal).

- Success: this variable allows determining if the students have solved Q1 correctly, so 1 corresponds to correct and 0 to incorrect.
- Errors: this field introduces the errors exposed in the theoretical framework that appear in the students' answers to this question: a) blank question (Null), b) answer by making equivalent fractions, and c) random answer.

Note that, the categories of the Representation Type are obtained when the students' solutions are reviewed.

Results and Discussion

First, a descriptive analysis of the results obtained in the pre-test is carried out, followed by the analysis of the results obtained in the post-test. Subsequently, the evolution of the individual student is presented.

Q1 Results

As can be seen in Table 1, 7/12 students, in the pre-test, have shown an area model. Thus, as discussed in Siegler et al. (2013), it seems that the area model continues to be the protagonist in the teaching of fractions, at least in the initial stages. Likewise, it should be noted that only two students have chosen to leave the answer blank.

Table 1: Question 1 (Q1) results

		Pretest	Post Test
Success		7/12	10/12
Representation type	Area	7/12	10/12
	Linear	0	1/12
	Discret	0	5/12
	Decimal Representation	0	2/12
Null		2/12	0
Error Type	Equivalent Fraction	2/12	4/12
	random answer	1/12	1/12

Regarding the pre-test errors, it is interesting to mention that they are not intrinsic to the representation. Two students have answered the question by building equivalent fractions when it is not what is requested, while another student has chosen to write a random answer. This last error occurs when the student simply does not know the procedure to arrive at the correct answer and, instead of leaving the space empty, writes anything (Ciscar & García, 1997). It is not, therefore, an error caused by defects in the understanding of the procedures or by carelessness or distractions.

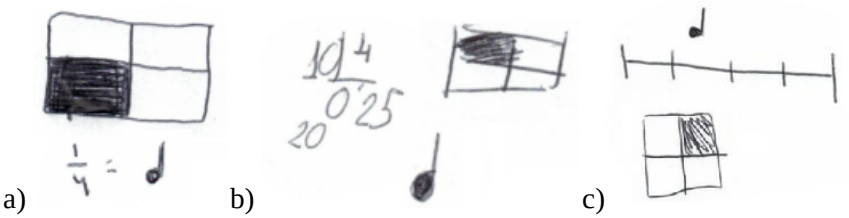
When analyzing what happened in the post-test, 10/12 students have answered correctly. Of these, all have used the area model again, but it is detected that one student uses the linear model and 5/12 of the students use the discrete model.

In addition, 2 students are included who have expressed the fraction in decimal notation, a notation that falls within the symbolic representations (Rico, 1997). This is not considered an error in this matter, it is accepted as a symbolic representation, because the statement does not specify that the representation must be graphic and can be misinterpreted. Regarding the errors, it can be seen that there are 4 students who identify equivalent fractions as a way of representing another fraction. In this work, the equivalent fraction is considered an error, because we have considered the numerical representations like the different ways of using numbers to indicate a fraction and it is based on the different uses of the fraction (Kieren, 1980).

Q1 Evolution

As commented in the descriptive part, in the pre-test, the students exclusively represented the fraction using the area model. It is in this aspect where evolution is most significant.

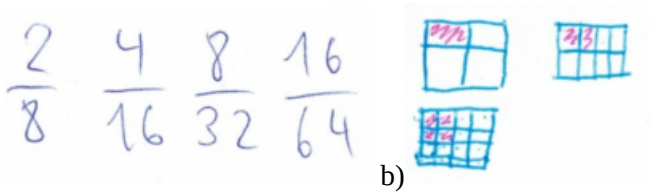
Thus, new models of representation appear among the answers of the students in the post-test. As can be seen in the images in Figure 4, the discrete representation through musical symbols has been successfully introduced through the interventions carried out.



a) b) c)

Figure 4: Extracts of answers from different students, a) Models: area and discrete; b) Models: area, discrete and decimal; c) Models: area, discrete and linear

On the other hand, with respect to the evolution of the errors and the blank answers, in Table 1 itself, it can be seen how no student has once again left the question unanswered. However, there are two more students in the post-test, compared to the pre-test, who have the idea that equivalent fractions are a form of representation of a proposed fraction offering answers like those shown in Figure 5.



a) b)

Figure 5: Extracts of different errors extracted from the answers: a) Equivalent fractions; b) Representations of equivalent fractions

As can be seen in Table 2, the students who used the area model in the pre-test continue to use it in the post-test (Siegler et al., 2013). However, 3 of them (S1, S3, S7) have expanded their responses by adding other models, including the discrete model (Azaryahu et al., 2020; Lovemore et al., 2022). In addition, half of the participating students (S1, S3, S4, S7, S12, S15) have used at least two different models of representation.

Table 2: Question 1 (Q1) evolution by students

Student	Pretest			Post- Test		
	Success	Representation Type	Error type	Success	Representation Type	Error type
S2	0	-	null	0	-	Equivalent Fraction
S15	0	-	null	1	Area/Decimal	-
S9	0	-	Equivalent Fraction	1	Area	Equivalent Fraction
S4	0	-	Equivalent Fraction	1	Area/Discrete	-
S12	0	-	Random	1	Area/Discrete	-
S6, S13	1	Area	-	1	Area	
S10, S16	1	Area	-	1	Area	Equivalent Fraction
S1	1	Area	-	1	Area/Discrete	-
S3, S7	1	Area	-	1	Area/Discrete/Decimal	-

Regarding the errors, it is verified that the idea of using equivalent fractions as a representation is still present, and that more students use it after the intervention, related to the continued graphic use for the construction of equivalent fractions. In particular, it is observed that the S9 student has made this error both in the pre-test and in the post-test. In turn, the S2 student seems to have not evolved at the same rate as the rest of his classmates, having not made any representation and, in turn, having made the mistake of answering the question by constructing fractions equivalent to the proposal.

Conclusion

As a conclusion to this work, it must be remembered that the main objective was to examined the influence of music on the process of learning fractions. Thus, after what was stated in the analysis of the results, it can be said that the interventions with musical content applied to mathematics, in effect, have caused a positive effect on the learning of fractions, particularly in the graphic representation, in the case of this students.

It should be mentioned that, throughout the interventions in which musical elements and their relationship with mathematics were introduced, the students followed the classes with great interest, as well as showing curiosity in the mathematical resolutions of the activities proposed in the last interventions, in the same way than Azaryahu et al. (2020) or Lovemore et al. (2022).

Specifically, the influence of music has been shown in a very direct way in the analysis of graphic representation (Azaryahu et al., 2020). In this question, answers have appeared in the post-test in which the students made use of the discrete representation using the musical symbols to develop their answers. In this way, the use of music as an interdisciplinary element allows, in addition, to favor a

collaboration between different subjects allowing the development of collaborative projects (Mall et al., 2016).

Regarding the limitations of this work, it should be noted that it is a Final Master's Project, in which access to more students was limited, which is why, in subsequent investigations, this intervention will be replicated with a larger number wide of students. Likewise, there are no data from a control group that would allow for a comparison between students with different methodology, which will be considered in a future.

Acknowledgment

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