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Solutions in open-ended tasks by elementary school students with different achievement levels in mathematics

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The diversity of students in elementary school poses a challenge to the teaching and learning of mathematics. Differentiation is discussed as a useful approach to dealing with a heterogeneous group of students in a class. In this paper, we report on a part of a study that investigates a concept of differentiation for mathematics education in heterogeneous classes. This concept is based on open-ended tasks that can be solved on different levels of difficulty and therefore allow all students to work on the same tasks. For our study, we developed the open-ended task "Inventing Equations and Equation Sets" (IE-IES) and investigated the solution of 153 third graders with different achievement levels in mathematics. The exploratory cluster analysis reveals three groups that differed in terms of processing levels. Furthermore, data analysis suggests that students' achievement level influences the solution.

Keywords: Diversity in elementary mathematics classrooms, differentiation, open-ended tasks, inventing equations and equation sets, students' solutions.

Introduction

In elementary school, classes are characterized by the diversity of students. Diversity not only refers to different learning abilities and achievement levels but also to dimensions such as native language, race, ethnicity, age, and physical abilities. Teaching mathematics in elementary school comes with the challenge to cope with heterogeneous learning groups and to provide a learning environment for students with different learning conditions and achievement levels. To offer learning opportunities for all students based on their individual dispositions, differentiation is discussed as a meaningful approach (Demo, Garzetti, Santi & Tarini, 2021). Referring to the literature review of Graham, de Bruin, Lassig, and Spandagou (2021), there is no common understanding of differentiation, but rather "so many misconceptions and definitional inconsistencies that it is difficult to know what is being enacted in the name of differentiation" (Graham et al., 2021, p. 161). The spectrum ranges from differentiation as a teacher's strategy to provide students with specific needs with anything different in the class to differentiation as a framework for teaching and learning which is based on social constructivist ideas.

In mathematics education, the concept of differentiation as an approach to teaching and learning in heterogeneous or inclusive classrooms is broadly discussed in connection with tasks that allow differentiation (Bardy, Holzäpfel & Leuders, 2021). If there is a narrow perspective on differentiation as a special treatment for one student with special needs, the task is supposed to be adapted to their achievement level (Demo et al., 2021). If differentiation is rather understood as a framework for teaching and learning in heterogeneous classrooms, one of the most important goals is to respond to the different needs of learners with tasks that allow various approaches, solution processes, and strategies (Small, 2017) and to enable students with different achievement levels to participate and work on these tasks. In the literature, these tasks are named differently. Small (2017) uses the term

“open questions” for the questions which are “framed in such a way that a variety of approaches or responses are possible” (p.7). Hashimoto and Becker (1999) distinguish three types of problems regarding the characteristic of openness. Firstly, problems for which the process of solution is open, and therefore a variety of approaches are encouraged. Secondly, problems in which the end products are open, and therefore various solutions are allowed. Thirdly, problems are formulated by students independently. This means that after a problem is solved, the students invent a similar one. Bardy et al. (2021) use the term adaptive task and emphasize that these tasks intend to cognitively activate all students and facilitate the work on their achievement level. In this paper, we use the term open-ended task with a broader understanding of open-endedness than Hashimoto and Becker (1999). Referring to Bardy et al. (2021) and Small (2017), we define open-ended tasks as tasks that bear the potential to cognitively activate students, allow different starting points, various approaches, and multiple solutions, and enable students with different ability levels to participate and work on the same tasks. Our definition of open-ended tasks is in line with a concept of differentiation which is broadly known among German researchers in mathematics education and called – literally translated – “natural differentiation” (Scherer, 2013, p. 102). Natural differentiation means that differentiation is naturally given by the characteristics of the task, and students can choose their level of working. The concept of natural differentiation is a framework for teaching and learning mathematics in heterogeneous classrooms, which is built upon ideas of social constructivism and based on the assumption that open-ended tasks allow all students to work on the same task according to their learning and achievement level. So far, few qualitative studies suggest that students exhibit different levels of performance when solving open-ended tasks (e.g., Weskamp, 2019; Scherres, 2013). To our knowledge, the assumption that students work on open-ended tasks according to their learning and achievement level has not been tested systematically and empirically yet. With our research, we intend to contribute to filling this gap by investigating students’ solutions to an open-ended task in arithmetic.

In our broad study, we analyzed whether third-grade students worked on the open-ended task “Inventing Equations and Equation Sets” (IE-IES) according to their learning potential, including cognitive and affective prerequisites (Friedrich, 2023). In this paper, we report a small part of the whole study by focusing on the following research questions: *(RQ1) To what extent do third graders from heterogeneous classrooms show different levels of processing and solutions when completing the open-ended task IE-IES? (RQ2) To what extent do third graders with different achievement levels in mathematics show differences in solving the open-ended task IE-IES?*

The design of the study is built on the supply-use model of Helmke (2009) which describes the complex processes of teaching and learning and all influencing factors in a simplified way (for the English equivalent see Brühwiler and Blatchford, 2011). To explain the effects of teaching and learning success, the model gives a compact overview of the most important variable clusters. In our study, we investigate two variable clusters: the provided open-ended task which represents the offer, and the students’ solution which represents the use.

Methods

To answer the research questions, we designed a quantitative study and collected data in the third grade of elementary school using different instruments. In the following sections, we describe our sample, the utilized instruments, and materials, the process of data coding, and data analysis.

Sample

The sample includes 153 third-grade students from different heterogeneous classes of four different schools. The schools were randomly chosen and are in the city center of Kassel and the county. Due to the different locations, we ensured the sample comprises students with multilingual backgrounds (29%) and various socioeconomic statuses. The classes used different math textbooks and the students were not explicitly familiar with open-ended tasks.

Instruments and Materials

We used a standardized math test for third graders (May & Bennöhr, 2021) to measure students' achievement levels in mathematics and to ensure the heterogeneity of our sample in this field. The math achievement test is timed, presented in a booklet, and comprises 74 items. The test is divided into two subtests: one for basic knowledge of arithmetic and computational abilities (56 items) and one for abilities in solving word problems (18 items). The reliability of the test is excellent with $\alpha = 0.93$ for the whole test, $\alpha = 0.91$ for the first subtest, and $\alpha = 0.81$ for the second subtest (May & Bennöhr, 2021).

To measure students' solutions to an open-ended arithmetic task, we developed a task that encourages students to invent and systematize equations. This task which we subsequently call "Inventing Equations and Equation Sets" (IE-IES) includes two different activities. The first activity consists of inventing various equations in which the equal sign is relationally used (IE: e.g., $2 + 7 - 3 = 10 - 4$) (Harbour, Karp & Lingo, 2016). The second activity was about inventing a set of equations (IES: $2 + 7 - 3 = 10 - 4$, $2 + 7 - 2 = 10 - 3$, $2 + 7 - 1 = 10 - 2$). Here, we ask students to take one already invented equation, try to vary it to get another equation that is related to the first one, and then vary the second equation, etc. The students had a set of cards with numbers from 0 to 9 and operation signs for addition, subtraction, multiplication, and division (see Figure 1).



Figure 1: Material for the open-ended task IE-IES

All cards were multiple times available. The open-ended task IE-IES allows various approaches and solutions and enables students to work on their individual learning and achievement level. Students can invent equations and equation sets of different complexity by varying the size of the numbers, the arithmetic operations, and the operation steps. Additionally, they can combine numbers that stay

inside the ten boundaries (e.g., $15 + 12$, $26 - 14$) or cross the ten boundaries (e.g., $15 + 17$, $26 - 18$). Similarly, they can vary systematically within an equation to come from one side to the other side (e.g., inverse change such as $10 + 15 = 12 + 13$) or between equations to come from one equation to another equation (e.g., from $10 + 15 = 12 + 13$ to $12 + 15 = 14 + 13$). With this open-ended task IE-IES we intended a deeper understanding of arithmetic and algebraic relationships.

Data collection

In each class, data were collected at two different times. First, we conducted the math test with all students in one class. Together with the introduction and instruction, this lasted about 45 minutes which equals the period of one lesson in German classrooms. Secondly, we conducted two math lessons (90 minutes) using the open-ended task IE-IES. The lesson was carried out along a script by one of the authors (Silke Friedrich). In this way, we were able to ensure a similar course of lessons in all classes. The lesson started with an introduction of the first activity IE for all students. For visualizing the process of inventing equations, we used a scale and colored cubes (see Figure 1). This was followed by an individual work phase which gave all students enough time to invent equations independently. This phase of students' work was followed by a short communication with the whole class, in which the second activity was introduced. Afterward, the students had enough time to invent sets of equations independently. During each work phase, we invited students to record their work with paper and pencil. Each activity (IE and IES) was documented on a separate sheet of paper; worksheet one (WS1, $n=153$) and worksheet two (WS2, $n=153$). The worksheets provided the database for our analysis.

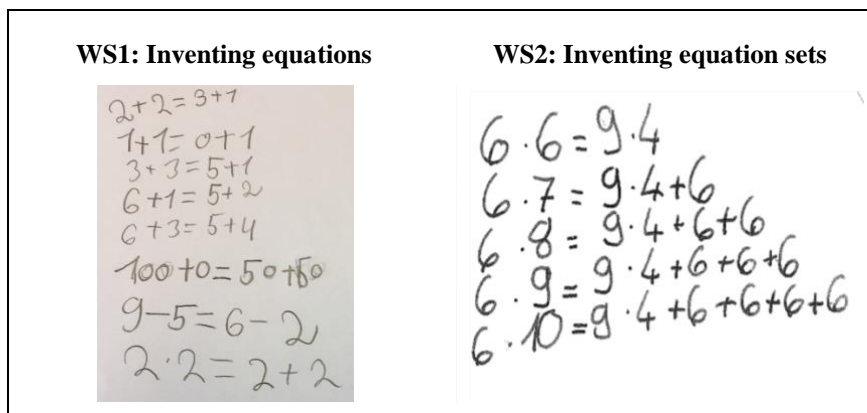


Figure 2: Students' worksheets

Data Analysis

To assess whether our sample is heterogeneous in terms of math achievement, the standardized math test was analyzed descriptively. We determined the total score of correctly solved items and computed the solution frequencies, mean values, and standard deviation.

The students' solutions to the open-ended task IE-IES were analyzed based on the method of structured qualitative content analysis. The aim was to develop a coding system that allows to convert the mathematical content of the students' worksheets into numerical measurement values. For the coding process, we defined three units of analysis: the document with all notations of one activity, one correct equation, and one equation set with at least two equations in a row that shows a

mathematical relationship (e.g., $6 \cdot 6 = 9 \cdot 4$; $6 \cdot 7 = 9 \cdot 4 + 6$; $6 \cdot 8 = 9 \cdot 4 + 6 + 6$; the set was invented by a student). We coded both activities (IE and IES) separately. For the analysis unit “document”, we captured all invented equations/equation sets (incorrect and correct separately) and all equations/equation sets with the relational use of the equal sign. For the units of analysis equation and equation sets, we deductively developed a coding system to capture the complexity regarding the components and the used systematic (Table 1). The codes were the same although the examples differed in the coding guide.

Table 1: Codes for components within an equation and for systematics within an equation

Codes for components within an equation			Codes for systematics within an equation		
Code	Code names	Examples	Code	Code names	Examples
NOS	Number of operational steps (Steps are counted)	$3 = 3$ (0)	Co	Commutativity	$8 + 7 = 7 + 8$
		$20 + 30 = 10 + 40$ (2)	Ic(1)	Inverse change of numbers in addition (+1 and -1)	$5 + 7 = (5 + 1) + (7 - 1) = 6 + 6$
		$2 + 3 = 11 - 2 - 4$ (3)	Ic(>1)	Inverse change (+>1 and ->1)	$5 + 7 = (5 + 5) + (7 - 5) = 10 + 2$
		$5 \cdot 9 - 7 = 35 : 5 + 31$ (4)	Cc(1)	Concurrent change of numbers in subtraction problems (+1 and +1 or -1 and -1)	$13 - 6 = (13 + 1) - (6 + 1) = 14 - 7$ $13 - 6 = (13 - 1) - (6 - 1) = 12 - 5$
TO	Type of arithmetic operation (Different types are counted such as addition, subtraction, etc.)	$3 = 3$ (0)	Cc(>1)	Concurrent change (>1)	$13 - 6 = (13 - 3) - (6 - 3) = 10 - 3$
		$20 + 30 = 10 + 40$ (1)	Pw	Part-whole	$4 + 5 + 1 = 9 + 1$
		$2 + 3 = 11 - 2 - 4$ (2)	Mr	Multiplicative relationship	$5 \cdot 6 + 6 = 6 \cdot 6$
SN	Size of numbers (All digits ≥ 1 are counted)	$20 + 30 = 10 + 40$ (1)			
		$5 \cdot 9 - 7 = 35 : 5 + 31$ (2)			
TR	Transitions in addition and subtraction	$5 + 8 = 6 + 7$ (2)			
		$23 - 8 = 34 - 19$ (2)			

In the process of coding the complexity of an equation or an equation set, we applied the rule to look at the one equation respective equation set in which the coding feature occurred most frequently. Hereby, we received a maximum value for each code related to a student’s solution. Finally, we summed the individual values of the codes for the components and the systematics used into two totals. For example, the student’s solution to IE (see Figure 2, WS1) was coded as follows:

- The number of correctly invented equations (NIE_WS1): 7
- The maximum sum of components (MSC_WS1): 5 (NOS 2, TO 2, SN 1, TR 0)
- The sum of different systematics (SVS_WS1): 3 (Co 0, Ic(1) 3, Ic(x) 1, Cc(1) 0, Cc(>1)1, Pw 0, Mr 0)

To answer RQ1 and to identify different groups in terms of task processing and level of processing, we decided on an explorative cluster analysis. For this purpose, we used variables that represent the processing level of the open-ended task IE-IES such as the number of invented equations/equation sets, the number of components, and the number of systematics. Finally, the cluster analysis was conducted based on eight variables which partly include various codes:

- The NIE_WS1: Number of invented equations (correct and relational)
- The MSC_WS1: Maximum sum of components (includes all codes from Table 1, left side)
- The SVS_WS1: Sum of different systematics (refers to all codes from Table 1, right side)
- The NIE_WS2: Number of invented equations (correct and relational)

- The NIES_WS2: Number of invented equation sets
- The MNES_WS2: Maximum number of equations in one set
- The MSCS_WS2: Maximum sum of components in one set (all codes from Table 1, left side)
- The SVS_WS2: Maximum sum of different systematics (all codes from Table 1, right side)

The conditions for a cluster analysis were given in the data since the eight variables showed a normal distribution in the students' solution (this was tested by Shapiro-Wilk-Test). To make the variables comparable, the z-score was calculated before the cluster analysis. For the cluster analysis, we used three methods: The Nearest-Neighbor-Method to identify outliers, the Ward-Method to determine the number of clusters, and the K-Means-Method to evaluate the determined number of clusters (Schendera, 2010).

To answer RQ2 and gain insight into the relationship between math achievement and the level of processing when solving the open-ended task, we also calculated correlation (Pearson's r ; two-sided test). Here, we referred to the same variables as in the cluster analysis.

Results

The analysis of the standardized math test (max. test score 74) confirmed the heterogeneity of our sample regarding achievement levels in mathematics. Based on the raw values (test scores), the entire sample ($n = 153$) reached a mean value of $M = 36.16$ with a standard deviation of $SD = 14.16$. The Shapiro-Wilks test showed normal distribution for math achievement in our sample.

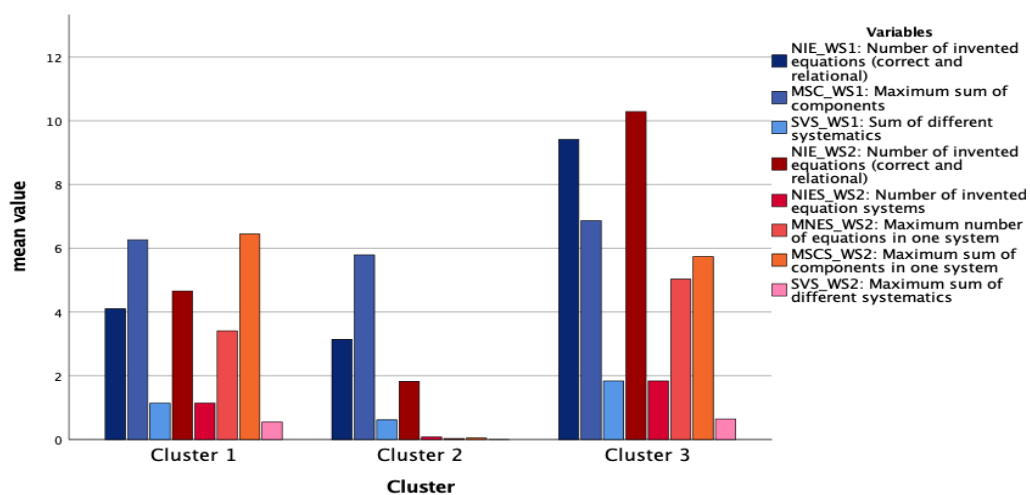


Figure 3: Graphical representation of the clusters

Figure 3 visualizes the result of the cluster analysis revealing that students' solutions to the open-ended task IE-IES can be clustered into three different groups. Thereby, analysis of the composition of the groups suggests three different levels of processing. Group 3 displays the highest level of processing regarding all variables except for the maximum sums of components in one set (MSCS_WS2). Groups 1 and 2 cannot be classified in the hierarchy. However, we assume the processing level of Group 1 to be higher than the processing level of Group 2. Group 1 shows significantly more extensive solutions, especially regarding the second activity IES. The interpretation of the statistical and content-related observation suggests a hierarchy of the processing levels from Group 2 via Group 1 to Group 3.

Table 2: Correlations between mathematics achievement and variables of students' solution

		NIE_WS1	MSC_WS1	SVS_WS1	NIE_WS2	NIES_WS2	MNES_WS2	MSCS_WS2	SVS_WS2
MA	<i>r</i>	0,29**	0,39**	-0,05	0,32**	0,20*	0,21**	0,20*	0,11
	<i>p</i>	< 0,001	< 0,001	0,576	< 0,001	0,011	0,009	0,012	0,178
	<i>N</i>	153	153	153	153	153	153	153	153

** correlation is 2-sided significant at the level of $p < 0,01$; * correlation is 2-sided significant at the level of $p < 0,05$

Table 2 displays the results of the correlation analysis. The first activity (IE) shows a highly significant but relatively weak correlation between math achievement (MA) and the number of invented equations (NIE_WS1, $r = 0,29^{**}$). On the other hand, a medium correlation can be observed between math achievement (MA) and the maximum sum of components in the invented equations (MSC_WS1, $r = 0,39^{**}$).

For the second activity (IES) significant but relatively weak correlations exist between math achievement (MA) and the number of invented equation sets (NIES_WS2, $r = 0,20^*$), the maximum number of equations in one set (MNES_WS2, $r = 0,21^{**}$), and the maximum sum of components in one equation set (MSCS_WS2, $r = 0,20^*$). As with the first activity, a medium correlation appears between math achievement (MA) and the number of invented equations (NIE_WS2, $r = 0,32^{**}$).

Conclusion

In general, our results confirm the assumption that the concept of “natural differentiation” with open-ended tasks is suitable for mathematics education in heterogeneous classes. In this vein our results are in line with the qualitative results of Weskamp (2019) and Scherres (2013) that suggest students exhibit different levels of performance when solving open-ended tasks. Concerning the investigated open-ended task on inventing equations and equation sets (IE-IES), data analysis suggests that all students from our heterogeneous sample were able to work independently on this task and to develop profound solutions which can be clustered in three different groups of processing and achievement level (RQ1). Furthermore, our results reveal differences in solutions depending on students' achievement levels in mathematics (RQ2). Even if the correlations are not high, the results show that students with higher achievement levels in mathematics worked more intensively on the task and invented not only a higher number of equations (single equations or in a set) but also more complex equations in terms of the inherent components. There is no difference in students' solutions regarding the use of systematics. This was surprising since we assumed that students with a higher achievement level are more likely to show systematic approach.

The presented study provides promising results regarding the use of open-ended tasks as a differentiation concept in heterogeneous mathematics classrooms. However, there are also limitations. Only one open task is examined in the study. Even though this task was designed as an example for many arithmetic content areas, it is not possible to conclude from the results to all arithmetic tasks and other content areas. Therefore, further research is needed.

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