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# The illustrations effect in Word Problems-solving involving fractions 

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## Introduction

Berends and Van Lieshout (2009) analyzed the effect on the process of solving word problems caused by the illustrations considered in the design of the problems. The authors distinguished between four types of illustrations and concluded that adding illustrations to word problems can have negative effects. The word problems they proposed are focused on natural numbers. However, when fractions are taught to children, these are strongly associated with the partitioning process and are generally taught using the area model (Freudenthal, 1983). Since students have learned fractions in this way, we believe that if the study is carried out with word problems involving fractions instead of natural numbers, the images would help in the resolution process instead of hindering it. Therefore, the aim of our study is to explore the illustrations effect on word problems solving involving fractions.

## Theoretical framework

The framework is constituted by the concept of word problem, classification of illustrations, and the uses of fractions. According to Verschaffel et al. (2022) a word problem is a verbal description of a situation that presents numerical data to which mathematical operations are applied to obtain the answer to questions posed. To define the categories of illustrations, the ideas of Berends and Van Lieshout (2009) and others were considered. "Bare" category includes word problems that do not contain graphical representations, "useless" category considers pictorial illustrations without numerical information, "helpful" category covers illustrations that are schematic representations containing numerical information stated in the problem, and "essential" category also includes schematic representations containing numerical information, but part of this crucial information is not stated in the problem. Finally, Freudenthal (1983) affirms that the fractions have a geometric character, reflected in linear and area models of a whole fractured. He identifies uses and aspects of fractions, in which these models are involved. In this sense, it is important to consider the above in the design of fractions word problems and the use of illustrations.

## Methodology

In the qualitative and exploratory research 42 students between 14 and 15 years old participated. The inquiry was carried out in a private school at Guadalajara, Mexico. The data was collected through the application of a paper and pencil test designed with a fractions word problem. Each student randomly answered one type of the four tests designed. The context of the word problem posed in each one is the same, but the type of illustration provided is different, as shown in Table 1.

Table 1: Test types according to illustrations

|  | Bare | Useless | Helpful | Essential |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \ddot{0} \\ & \tilde{0} \\ & 0 \\ & \tilde{0} \\ & \tilde{0} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | At his party, Daniel shared six sevenths of his cake among his guests. Then Daniel took for himself a quarter of the piece of cake that was left. What fraction of the cake did Daniel eat? |  |  | At his party, Daniel first shared his cake among his guests. Then Daniel took for himself a part of the piece of cake that was left. What fraction of the cake did Daniel eat? |
|  | Without illustration |  |  |  |

The problem is to calculate the part of a part. It is necessary to identify the unit's complement (the piece of cake that was left after taking six sevenths of it), and then get a part of a part (a quarter of a seventh of the cake). On the one hand, the use of the fraction as fracturer underlies in the problem, but on the other hand, it could also be used as an operator, since we have $1 / 4$ of $1 / 7$.

## Results and conclusion

The results of this exploratory study confirmed our hypothesis, the illustrations when using fractions helped the students to be more successful in their answers (Table 2), unlike the study by Berends and Van Lieshout (2009) where natural numbers were used. Therefore, it is important to consider the mathematical content in the design of word problems and illustrations included in them.

Table 2: Performance of the students depending on the type of illustration

| Type of illustration | Bare | Useless | Helpful | Essential |
| :---: | :---: | :---: | :---: | :---: |
| Correct answers | $3 / 10(30 \%)$ | $4 / 10(40 \%)$ | $5 / 11(45.45 \%)$ | $6 / 11(54.54 \%)$ |

Two trends were observed among students who got a correct answer, those who worked with the fractions as fracturer and others as operator. Among the procedures of the students who obtained incorrect answers, in general, three errors were mainly found. Error 1 was related to the equality of the parts; they considered slices of different sizes as representing the same fraction of the cake. Error 2 occurred when they did not identify part of parts, they only consider the data of the statement, without making a relationship between the quantities, or one of the parts into which the whole shown in the illustration is divided. Error 3 was made by students who associated the keyword "took" with the operation of subtraction. The trends and errors were present regardless of the type of illustration.

## References

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