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► **To cite this version:**

Sonia Yvain-Prébiski. When horizontal mathematization enters the in-service training of primary school teachers: a case study in France. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04416568

HAL Id: hal-04416568

<https://hal.science/hal-04416568>

Submitted on 25 Jan 2024

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When horizontal mathematization enters the in-service training of primary school teachers: a case study in France

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This paper is a progress report which presents the context, the starting point and the outline of an ongoing research project. The initial research aim of this project is to study what are the training issues involved in teaching and learning mathematical modelling in primary school. Through the study of a training device which proposes to solve some real problems in video form, I propose to show how and why horizontal mathematization is progressively invited in the training contents of this device. To do this, I rely on the first results of the analyses of a pre-experiment (analyses still in progress).

Keywords: Horizontal mathematization, primary school' teacher, training on modelling, extra-mathematical situation.

Introduction and context

This paper is a progress report which presents the context, starting point, and outline of an ongoing research focused on the issues of training in mathematical modelling for primary education teachers. Several examples of training content on modelling have been presented over the years with the aim of supporting teachers in implementing modelling activities, including at previous CERMEs (Eyrich Jessen, 2019; Barquero & al., 2019). This ongoing research is a continuation of previous research conducted in secondary education (Yvain-Prébiski & Chesnais, 2019), but in a different context, that of primary education. One specificity of this project is to address a topic that has not been widely studied, as the case of mathematical modelling experience for practising primary education teachers through video-based problems. Our experimental field is the “Rallye Maths”, proposed by the district educational trainer Jean Discours, which is a device implemented in a rather isolated French town (Bagnols Sur Cèze). About 50 volunteer primary teachers are involved, each year, in this device. Discours develops a problem in video form and presents it to teachers in his district. The main specificity of this problem is that it is rooted in everyday life to promote mathematical modelling. Moreover, in the video, the question to be addressed is not given, it is up to the pupils to identify the question(s) that can be addressed mathematically. Once they have agreed on the question to be investigated, they must solve the problem. Discours first offers training to volunteer teachers to help them implement the problem and then a second training to help them analyse the pupils' modelling skills. It is for the preparation of these training time that Discours contacted me to share his questions: what content should be proposed for this training time to support primary school teachers in the implementation of mathematical modelling activities? How can we help them to analyse the modelling approaches of their pupils? This has become the collaboration between the trainer and the researcher in didactics of mathematics, leading to a research project that is still under construction. In this paper, I present at first the theoretical framework underlying this project. I then show how my pre-experiment contributed to the research project. Finally, I explain how I envisage the project's future and its perspectives.

Theoretical considerations

The trainer's questions have given rise to an initial research question: what are the training issues involved in teaching and learning mathematical modelling in primary school? In the context of the “Rallye Maths”, the problems proposed are based on a real context. However, starting from situations rooted in reality requires first of all that the problem be made accessible through a mathematical treatment.

It must be considered that a question of everyday life is rarely directly a mathematical question. It becomes one through a process of mathematization or mathematical modelling that simplifies and interprets reality. It is important to make this stage visible, the choices that are made and the way in which they condition the apprehension of reality, by actively involving the students. (Preface by Artigue in Masselin (2020, p. 13), translated by the author)

This necessity to mathematise the situation leads me to work within the theoretical framework of the Realistic Mathematics Education (RME). I consider the distinction introduced by Treffers and taken up by Freudenthal (1991) between horizontal mathematization which “leads from the world of life to the world of symbols” and vertical mathematization, as work within the mathematical system itself.

Treffers, in his thesis of 1978, distinguished horizontal and vertical mathematising [...]: Horizontal mathematising, which makes a problem field accessible to mathematical treatment (mathematical in the narrow formal sense) versus vertical mathematising, which effects the more or less sophisticated mathematical processing. (Freudenthal, 1991, p. 40)

I followed the idea of Israel (1996), who defines a mathematical model as “a piece of mathematics applied to a piece of reality” and specifies that “a single model not only describes different real situations, but this same piece of reality can also be represented by different models” (p. 11). By crossing this point of view with the RME, horizontal mathematization seems relevant to explore educational issues related to the research question at stake. It led me to define different forms of horizontal mathematization that seem relevant to characterise this type of work, namely: choosing a piece of reality to question in order to answer the problem; identify and choose the relevant aspects of the piece of reality (context elements, attributes); relating together the chosen aspects to construct a mathematical model. In addition, I added a form of horizontal mathematization: quantification which refers to the association of some aspects of reality to quantities (essentially consisting in measuring). As detailed in Yvain-Prébiski (2021), based on these definitions and on elements of an initial epistemological study of researchers' modelling practices, I have developed an adaptation of the modelling cycle proposed by Blum and Leiß (2007), highlighting the dialectical relationships between the horizontal and vertical aspects of mathematization, in Figure 1.

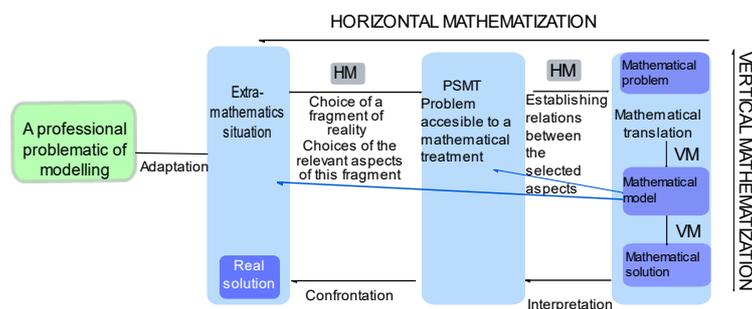


Figure 1: Modelling cycle considering the horizontal and vertical aspect of mathematization

To highlight the interconnection between horizontal and vertical mathematization in a mathematical modelling activity, I have added double arrows from the mathematical model to "the real situation" and to "the mathematically tractable problem". These arrows show that the choice of the starting model can be a known mathematical model which allows the researcher (or the student researcher) to envisage vertical mathematization work with a view to enlightening him or her on the problem, even if it means refining or rejecting the chosen model by reconsidering the choices made (Yvain-Prébiski, 2021). Horizontal mathematization is constitutive of the activity of modelling from an extra-mathematical situation. Within the context of this project, my main research question is the following: what are the effects of the consideration of this mathematization (or lack thereof) by the different actors involved in the "Rallye Maths" device, on the need to actively involve the pupils in the work enabling them to move from a situation anchored in reality (proposed in video form), to a mathematical problem? To initiate the study of this issue, I first set up a pre-experiment. The aim is first to identify whether or not horizontal mathematization is considered by the teachers involved in the "Rallye Maths". Based on the results of previous research presented at CERME11 (Yvain-Prébiski & Chesnais, 2019), I hypothesise that not only is this mathematization not considered, but also that this creates difficulties for the teacher in supporting the pupil's modelling work. In the following paragraph I explain the methodology of this pre-experiment.

Pre-testing: method

I have observed and then analysed a part of training session concerning the analysis of their pupils' modelling approaches by primary school teachers involved in the "Rallye Maths" (in May 2022). The problem proposed in the classes (4th and 5th grade) was the following one: https://educabagnols.net/storage/videos/2018-2019/niveau1_etape4.mp4

The next text summarises the video problem "The Big Wheel".

The Big Wheel

Carole is in the big wheel that turns. The camera films the city around (Lyon in France). Her phone rings.

Carole: Allo

Jean: Yes Carole, it's Jean. I'm waiting for you here, where are you?

Carole: I 'm in the big wheel.

Jean: But you're still on the big wheel?

Carole: I 've seen the summit at least twice now.

Jean: You did two laps, I think it's five laps in total, right?

Carole: I think so, I've already done two laps.

Jean: So, you have three laps to go... How long do you think it will take to get to the end?

Carole: I don't know, about... in about 5 minutes.

Jean: I think you need 15 minutes.

Throughout the conversation, you can see the big wheel spinning. At the end of the conversation, it spins again, and the video stops without the Big Wheel having made a complete lap.

Brief a priori analysis of the “Big wheel”

As explained previously, there is no question asked in the video. Two questions could most likely be chosen: Who is right? How much time is left? To answer those interrogations, two more questions arise: How many full rotations are left to complete? What is the duration of one full rotation? We can suppose that Carole completed two full rotations and there are three more left. As the video does not show the wheel doing a full rotation, we cannot measure the time it takes. We can for example measure the time needed for half a rotation, or even a quarter of a rotation, and then come back to a full rotation. The simplifying assumptions underlying this procedure are that the wheel speed is constant, and the wheel does not stop. For instance: we can track any cab for a quarter of the wheel’s rotation, the stopwatch will display 85 seconds, then we calculate $3 \times 85 = 255\text{s}$. There is 4min15sec remaining then, so Carole is right. To make this problem accessible to mathematic treatment, we need to make a choice on which parameters or context elements should be considered, which falls under the scope of horizontal mathematization. We can anticipate that – the piece of reality « Carole’s cab » could be studied by the pupils (where is Carole during the video, precisely), that – the piece of reality « the wheel » brings the pupils to question themselves on the rotational speed of the wheel, and on possible stops of the wheel. We can consider that pupils make choices either from considerations that are taken from reality (for example, a full rotation takes 3 minutes because I’ve already been on a wheel like this), either without justification (e.g., a full rotation takes 5 minutes) or from a mathematical treatment (for example, $15 : 3 = 5$, a full rotation takes 5 minutes)

Method

The data collection consists of audio recordings of this training time. The district educational trainer also gave me audio recordings of ten classes that addressed the Big Wheel problem (classes of some teachers attending the training). At first, I have analysed audio recordings of ten classes. As part of my theoretical reflections, I noted the questions pupils asked themselves during the resolution and the choices of answers they gave. I have classified these choices into three categories: choices based on reality (RC), a priori choices (PC) and choices based on mathematical work (MC). (These analyses are still ongoing, for example, see table 1)

Table 1 Examples example of analysis of pupils' work

	RC	PC	MC
How much time is a full rotation?	<ul style="list-style-type: none"> - We should go to Lyon. - teacher, I, I have already been on that wheel in Lyon, so I know that a full rotation takes around 1 min 30 to complete - I think that the only solution to find answers is to go there and ask how much time it takes for the wheel to rotate fully 	<ul style="list-style-type: none"> -1 full rotation is 5 minutes. -1 equals maybe 5 minutes 	<ul style="list-style-type: none"> - We know that 1/2 a rotation is 30s so a full rotation is 1 minute. - We should set a watch to 0 and calculate because the watch tells the number of seconds well.

How much time are three full rotations	-I counted the seconds in my head for 1/3 of a rotation, between each third there is a stop. Each stop could be up to a minute, to wait for the people to go on the wheel.	-Three full rotations take about 5 minutes to complete. The wheel's speed is 10 km/h	-We calculated the time it took her to get off the wheel, but we need a calculator because I cannot divide 5 by 3.
In which cab is she?		-We knew where she almost was (showing with hands), but we did not precisely know where... and I think that it would not help us anyway	-She should show us in which cab she was. César – we could use a red arrow
Can the wheel stop turning?	-We do not know whether there is going to be a problem with the wheel or not... It could stop... We do not know why... because it could face issues... and then it would take more time so we cannot really calculate the time needed for a full rotation		

These initial analyses show that not all the pupils necessarily deal with this situation mathematically by proposing, for example, "to go to Lyon and measure the time it takes for the Big Wheel to make a turn". And that in the same class, for the same question, the pupils make different choices. This highlights that in order to deal mathematically with the video-based problem, a first work of horizontal mathematization is required: choices and simplifying hypotheses must be made, such as - the wheel does not stop, - we do not need to know where Carole's cabin is precisely. Then, I plan to analyse the audio recordings of the training time. During this session, the trainer asked the teachers to analyse the pupils' approaches in the ten classes I analysed above. My aim is to get some initial feedback on how the teachers have tackled the different questions of the pupils that I have identified above. To conduct my first qualitative analyses of the exchanges between teachers during this phase, I have defined the following six indicators, based on the forms of horizontal mathematization presented in the *theoretical considerations* section : Discussion on the choice of the fragment(s) of reality (Indicator 1), discussion on the choice of identifying relevant magnitudes and/or their values (Indicator 2), Discussion of the relationships between the identified magnitudes (Indicator 3), Discussion of the need for numerical data (Indicator 4), discussion on the mathematical work, in the vertical mathematization sense (Indicator 5) and discussion not linked to a work of mathematization, e.g. classroom organization issues (Indicator 6). I plan to use these indicators to categorise the nature of exchanges between teachers studying pupils' work.

Pre-testing: Findings

I have not finished these analyses, but I already have some evidence that the teachers did not anticipate that the pupils would not deal directly with the problem mathematically and that this non-anticipation presented some difficulties for them, mainly in the following cases:

When a pupil remains “connected” to the real world”, which means produces an answer relying on arguments attached to reality, and thus does not engage herself or himself in a mathematical process.

“There is a pupil that makes a connection between the situation exposed in the problem, and her/his life experience since she/he said, at some point, that she/he has been on this large wheel. Thus, hearing her/his sentence, I was wondering, to what extent, in the situations in which there is a real attachment to reality, to their life experience, would this slow down or even stop their understanding of the situation, and also of the resolution? They may indeed not be able to deviate from this!”

When different choices of context elements or quantities are offered by the pupils, leading to many different mathematical treatments.

“So, at first, there are a lot of questions that have been raised without having an answer. There have been questions, and answers, and when it is the teamwork time, some posters replied to a question, and other posters replied to other questions.”

“Now that we noticed such a thing, I think it becomes more complex because, if some pupils raise different questions, then it is the teacher’s role to prepare some hints. It enlightens the idea that the teacher must properly prepare his class, prepare his problem so think of all the potential procedures which could be followed.”

When pupils do not authorize themselves to make choices

“The fact that they cannot precisely determine which cab is Carole’s one, it embarrassed them and some of the pupils wanted to know in which cab she was. The video serves them for the understanding, but they also miss an element, and knowing they were missing something, the problem seemed difficult to them.”

During this time of exchange, with the trainer's help, the teachers realised the need to get the pupils to move from the video to a mathematical problem (e.g., "the fact that they could not determine precisely the cabin where Carole is, it really bothered them and there are several children who want to know where she is"). The main point that emerged was the importance of not reducing the modelling activity to the translation of a statement into mathematical language. Indeed, the exchanges focused on the need to specify to the pupils the choices (of contextual elements, of quantities) on the basis of which the situation will be treated mathematically and even to show that, depending on the choices made, the resolution may be different (for example, if it is established that the wheel does not have a regular speed). It can also be noted that taking instrumental measurements from the given situation, such as using a stopwatch, opened up discussions on the fact that this can lead the teacher to deal with the difficulties inherent in a possible change of didactic contract. They expressed their training needs in terms of modelling teaching and learning in primary school based on so-called *everyday situations* in the French curriculum. This has led the trainer to redefine his training content to better answer these requirements.

Horizontal mathematization in training

This year, in preparation for his training, he has decided on the following course of work: *Stage 1* (same as previous training sessions): He asked teachers (42 volunteer participants) to reflect on the problem of the Big Wheel and to state the question they address. *Stage 2* (new): He asked teachers to answer the following three statements with yes, no or I don't know: Carole did two laps on the Big Wheel, she is in the cab at the top of the Big Wheel and to get off, Carole jumps off the Big Wheel. *Stage 3* (new): He presented Yvain-Prébiski’s work on considering the dialectical relationship

between the vertical and horizontal aspects in a modelling activity (Yvain-Prébiski, 2021). In an interview, I asked the trainer to explain to me why he had developed his training in this way. Here is his response:

“I have realised that when modelling extra mathematical situations, choices must be made and that it is therefore important to train teachers to manage such a phase with pupils. During the previous training, I realised that teachers are not aware that many implicit choices are connected to their mathematical resolution, what you call the horizontal mathematization work. that's why I designed stage 2, so that they realise that there are choices to be made. and then I planned to explain the role of horizontal mathematization by presenting elements of your work. That's why!”

In the stage 1, the three questions expressed by teachers are who is right? How long is a lap? Who is closer to the truth? To address these questions, they mainly timed the time of half a lap and doubled this time, and then tripled the result (implicit choice that three laps were left to be made). The Figure 2 shows the results obtained in stage 2. (The participants were divided into 14 groups of 3)

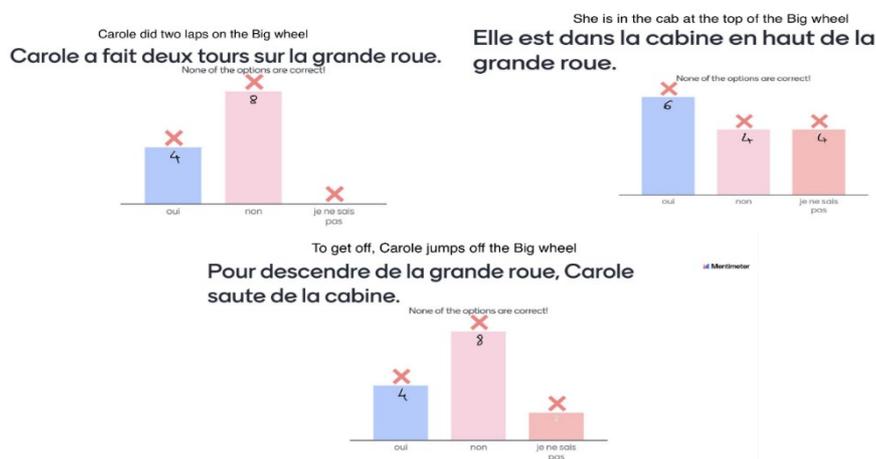


Figure 2: Teachers ‘productions-Stage 2

This highlights, for example, that even if the majority had made the implicit choice that there were three rounds left, only 4 groups answered YES to the statement "Carol did two laps on the Big Wheel». Does this mean that teachers realise that choices have to be made before dealing with the problem mathematically? Or does it mean that they are aware that choices have to be made but they do not consider it necessary to make them explicit? The forthcoming analyses of the audio recordings of this training course (this training took place at the end of January 2023) should provide us with information and some answers to our research question regarding the effects of the consideration of the horizontal mathematization by the different actors involved in the "Rallye Maths" device.

The project's future and perspectives

I have shown why and how horizontal mathematization has been introduced into the contents of a training device. The first analyses highlight that most teachers do not make explicit their choices related to their mathematical treatments thus generating difficulties in supporting the pupils' modelling work. I plan to analyse the exchanges between teachers during the January training to better identify the extent to which teachers have identified the challenges of horizontal mathematization in a modelling activity. Then, I intend to follow these teachers in the implementation of the “Rallye Maths” in their classes. My aim is to study how content related to horizontal

mathematization in education can support the teaching and learning of modelling in primary school, as I had tried to show in secondary education at CERME 11 (Yvain-Prébiski & Chesnais, 2019). This research contributes to a recent broader collaborative project, consisting of a group of associated educational institutions, called LéA MaPcv (Mathematization of concrete Problems from videos) which aims to identify training issues to promote the teaching and learning of modelling in primary education.

Acknowledgement

I would like to thank Jean Discours, the trainer who designed this device and collected all the data.

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