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# Problem-posing training and its impact on the quality of the posed problems

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*In this study, a year-long intervention on problem posing involving primary school students aiming to investigate its impact on both the students' problem-posing abilities and the quality of the posed problems is presented. The intervention helped the participating students to be acquainted with a variety of problem-posing strategies. After the intervention, the students' choices gradually shifted from less to more powerful strategies based on the students' ability to see and use the underlying structure of the problems. This ability to use structure by initially identifying, and then negating or adding attributes, in conjunction with the openness of the produced problems in terms of affording multiple solutions or multiple problem-solving paths, add to the quality of the posed problems.*

*Keywords: Problem posing, structure, training, intervention.*

## Introduction

Cai et al. (2015) synthesizing the current state of the art related to problem posing, discuss the work that has been done in the field as well as questions that remain unanswered and that should attract the interest of the research community. In one of the questions, they are wondering whether students and teachers can be effectively trained to pose high-quality problems. They also acknowledge that the notion of strategy constitute a key element in the way the participants are trained to pose problems. So, in this chapter they invite the community to explore further the kind of strategies that are more productive for problem posing and which determine the 'quality' of the produced problems. There are a few studies that provide some evidence on the topic. It seems, for example, that students improve the level of their problem posing when their training involves the combination of exploration and problem solving with problem posing (Koichu & Kontorovich, 2013). Moreover, emphasis should be given to the development of the students' ability to reflect on the mathematical structure of the task (Patsiala & Papadopoulos, 2022), and use their knowledge, skills, concepts, and relationships from their previous mathematical experiences, to create one or more new mathematical problems (Baumanns & Rott, 2019). The underlying structure refers to the mathematical relationships between the entities and quantities within the given problem. In this setting, we attempt to respond to the question made by Cai et al. (2015) focusing on primary school students to investigate the quality of the posed problems in terms of seeking and using the structure of the given problems after participating in a problem-posing intervention that lasted a whole school year. Thus, our research question is: What are the effects of a problem-posing intervention on the quality of the problems elementary school students pose, in terms of seeking and using structure?

## Theoretical background

Two of the ways research literature on problem posing can be categorized are (i) research on problem posing as a goal of mathematics instruction and (ii) research that treats problem posing as a goal, meaning that treats problem posing as a phenomenon that is itself the object of study (Cai & Leikin,

2020). Indeed, for the first category, the study of Koichu (2020) with pre-service mathematics teachers gives evidence that problem posing in mathematics teacher education can be promoted as an implicit goal. In the same spirit, and as an example of involving students, Chen et al. (2015) implemented a training program for Chinese students to develop their problem-posing and problem-solving abilities. It seems that the training program improved the students' problem-posing abilities. The researchers found that the participants were able to pose more original problems, but not more complex or diverse problems. In her study with 5<sup>th</sup> graders, English (1997) organized a problem-posing training program aiming at developing the students' ability to recognize and use the problem's structure, and her findings were very encouraging.

The second category of research literature mentioned above reflects the researcher's perspective. In our paper, this is connected to the emphasis on the quality of the problems posed by students. English (2020) reviewing several articles notes that quality is connected, among others, to the openness of the task (open at task start, open at task end, or both), its mathematical demand and challenge, and the opportunity for multiple solutions. Cankoy (2014) mentions that the mathematical structure preferred by the students in posing problems can be considered a quality indicator. Therefore, we suggest that this quality is expressed through the chosen problem-posing strategies that make use of the given problem's structure.

This connection between problem solving/problem posing and the structure of the problem has been highlighted by several researchers (English & Watson, 2015; Kwek, 2015). English and Watson (2015) examined the role of problem posing in developing students' statistical literacy. They claim that the significance of seeing and using the problems' structure was seen in the students' responses. Kwek (2015) tried to identify patterns in students' mathematical learning and thinking during classroom-based problem-posing tasks. The findings revealed that the students' ability to identify the mathematical structure of the problem was a crucial cognitive factor.

To approach the quality of the produced problems, Patsiala & Papadopoulos (2022) developed an instrument that captures the degree of students' awareness of the structure of the given problem, based on the problem-posing strategy used. The connection of this instrument to the quality of the posed problems lies in the acknowledgment that the more the posers use advanced problem-posing strategies (e.g., types A and B) the more they have developed their ability to seek and use structure. This finally results in productive problem posing (Cai et al., 2015).

The instrument includes all the problem-posing strategies found in the literature, which are organized into three groups A, B, and C (Table 1). The grouping is indicative of how powerful the strategy from the mathematical point of view is, from the most (A) to the less (C) powerful. This use of more advanced problem-posing strategies indicate that the students posing the problem have grasped the underlying structure, which adds to the issue of the quality of the posed problems. Therefore, during and after the training period, the progressive shift of the students' choices from C to A type of strategies or the combination of them is considered as a shift towards posing problems of higher quality.

**Table 1: Groups of problem-posing strategies**

The answer is a method (A)	“Tailless” word problems (B)
What-if-not (A)	Change the question/Form a question (B)
What-if-yes (A)	Change numbers (B)
Change the context (A)	Reversing known and unknown information (C)
“Frontless” problems (B)	Change numbers (C)
Missing middle problems (B)	No evidence

All four strategies labeled as A category indicate a deep structural understanding. In the “answer is a method” strategy the new problem requires a description of how the problem would be solved if there weren’t numbers. In the “what-if-not” the poser negates one of the attributes of the given problem asking “What if this attribute was not so?”. Employment of the “what-if-yes” strategy means that the poser adds new information which has an impact on the problem’s solution. Finally, in the “Change the context” strategy the poser modifies or changes completely the task environment. The B-category includes five strategies. In “Frontless” problems only the question of the problem is kept. In “Missing middle problems” the original question is kept but some numbers, necessary for solving the problem, are left. In “Tailless” problems the question at the end of the problem is omitted. The “Change the question/Form a question” concerns changing the existing question or posing from scratch a question that fits the given data. The strategy “Change numbers” refers to the change of some or all the numerical data of the given problem and appears in both B and C categories because according to the nature of change, the result might be similar to or more complex than the given one, as it will be demonstrated in the Results section. Finally, in “Reversing known and unknown information” the given and the goal are swapped.

## Methods of the study

This study is part of a broader study investigating how accumulated experience in problem posing might have an impact on the development of seeking and using structure. In this study, the participants were forty grade-5 students from two classes (20 students per class) from a private school in Greece. Students from both classes had the same background and followed the same Mathematics curriculum. The first (Group A) had no prior experience with problem-posing whereas the second (Group B) had attended a problem-posing intervention that took place in parallel to the normal teaching of mathematics during a whole school year. This intervention consisted of iterative cycles including four steps per cycle, one step per week (one month per cycle). Problem-solving activity (step 1), problem posing at the classroom level (step 2), reflection on some of the posed problems selected because of the strategy used (step 3), and individual problem posing (step 4) (Papadopoulos & Patsiala, in press).

For the purpose of this study, we presented the same word problem in both groups and asked the students to pose as many problems as possible they considered challenging for their classmates.

The statement of the given problem was: *“I want to buy a book that costs 8,50 €. If I save 0,50 € every day, in how many days will I be able to buy the book?”*

The specific problem was chosen for two reasons: (i) its content is limited to decimal numbers, a topic already taught in earlier grades and revisited at the beginning of grade five, and (ii) it affords a

variety of different problem-posing strategies to generate new problems. The study was initiated by a whole-class discussion in both groups to make sure the students understood the problem which was then solved in the classroom. Then the participants in both groups were asked to individually produce problems inspired by the given one. Their produced problems constitute our data that were collected and categorized on the basis of content analysis, according to the problem-posing strategies used. The analysis was done independently by each author and then their results were compared to clarify categories and resolve possible differences.

## Results - Discussion

In this section, we present the analysis of the students' use of problem-posing strategies as they were identified in their posed problems. The analysis illustrated that the students without experience in problem posing (group A) were limited mainly in category-C strategies, almost without any instance of combining more than one strategy for posing new problems. On the contrary, the students who attended the training program exhibited an ability to produce many problems, using all three categories of strategies, exhibiting thus a solid structure sense, combining very often more than one strategy. At the same time, there was a certain number of problems that were irrelevant to the given one, or their statement did not make any sense. These results are presented in a summarized way in Table 2.

**Table 2: Distribution of the generated problems per group and category**

Problems	Category A	Category B	Category C	Combination of Strategies	No evidence	Sum
Group A	0	0	10	1	24	35
Group B	10	19	54	28	59	170

Below we discuss each category using representative examples from the students' scripts. It is important to note that we were not able to collect evidence from every strategy from Table 1.

### Category C

This category includes two strategies: "*Change numbers*" and "*Reverse known and unknown information*" and it seems that they were the most popular strategies (10 and 54 instances for groups A and B respectively). The solution to the initial problem is 17 days. By "reversing known and unknown" a student from Group A posed the following problem (Problem 1): "*Every day I save 50 cents. I will be able to buy my favorite book in 17 days. How much does the book cost?*". So, the number of days that was unknown in the initial problem became known, and the cost of the book became the goal. The next problem is an example of employing the "*Change numbers*" strategy and was created by a student from Group B (Problem 2): "*I want to buy a book that costs 19 €. If I save 25 cents every day, how many days will I need to buy the book?*". The new problem is of the same complexity and structure as the original one and thus its solution is in the same spirit as the solution to the given problem. The dominance of these two strategies might be attributed to the fact that they allow students to "play safe" by mainly changing the numbers, mimicking thus what they recognize as word problems in their textbooks. Given the lack of experience on problem posing and especially for Group A this is something reasonable, mentioned in the literature as a "surface reformulation

technique” (Grundmeier, 2015). The students do not change the structure of the problem but just one of their surface characteristics.

### **Category B**

Problems that reflect strategies of this category were identified only in students from Group B. An example of the employment of the “*Missing middle problem*” strategy (Problem 3): “*I want to buy a book. If every day I save 50 cents every day, in how many will I be able to buy the book? What information do you need to answer the question?*”. In this problem, the required answer is a number (i.e., the cost of the book) which is not given, but this is not the answer to the question the problem poses (i.e., the number of days). According to the choice made for the cost of the book, a different solution might exist. Interestingly there might be suggested numbers that would provoke meaningful discussions. What if the cost of the book is not a multiple of 0.50?

Another strategy employed by the students was the “*Change the question*” one (Problem 4): “*I want to buy a book that costs 8,50 €. If I save 50 cents every day, will I be able to buy it in 2 weeks?*”. In this problem, the question has changed and now the solution to the problem can be got either by calculating the required days or by calculating the total amount of savings in two weeks and comparing it with the cost of the book. To form such new questions presupposes that the posers are able to focus on the structure of the problem and see what they can figure out from the information at hand.

An example of applying the “*Tailless problem*” strategy is (Problem 5): “*I want to buy a book that costs 8,50 €. If I save 50 cents every day, what can you ask about this problem?*”. In this case, the poser intentionally leaves the problem without question inviting the potential solver to come up with a question that fits the mathematical situation described in the problem. To be able to use mathematics well, the student must possess the ability to figure out what questions fit the mathematical situation.

### **Category A**

Again it was only the students from Group B who exhibited awareness and use of these problem-posing strategies. The posers’ ability to create problems using category A strategies is indicative of a deeper understanding of the problems’ structure. The first example concerns a problem created by applying the “*What-if-not*” strategy. The produced problem was (Problem 6): “*I want to buy a book that costs 8,50 €. What if the amount I save per day was not 0,50€ and instead, I save twice the money per day and I do not save money for the first 5 days? How many days do I need to buy the book?*”. In this example, the student chose two attributes to negate. The first was the amount of money saved per day which becomes double than in the original problem. The second was the time schedule. Instead of saving daily starting from the first day, now the whole process starts after the first five days. This process means that the student first made a list of the problem’s attributes and chose two of them. Then, he manipulated them through the process of asking “what-if-not” and suggesting alternatives. Finally, he posed the new problem inspired by the alternatives. In this way, the process of problem posing becomes more systematic and structured (Brown & Walters, 1983).

The second example in this section is a problem produced by employing the “*What-if-yes*” strategy (Problem 7): “*I want to buy a book that costs 8,50 € and a notebook that costs 4,20 €. If I save 50*

*cents every day and the book is offered at a discount of 38%, how many days do I need to buy both?''*. The employment of the *“what-if-yes”* strategy meant that new information (i.e., the notebook and the discount) has been added to the problem which has an impact on its solution. Both strategies add to the quality of the problem since they give students the space to think about how details contribute to what is being thought. So, instead of jumping to calculations or giving up, they continue thinking and they realize that problems have attributes, and these attributes can be challenged.

### **Combination of different strategies**

In many cases, the students combined more than one strategy belonging either to the same or different categories. Only one student in Group A managed to combine strategies to form a new problem (Problem 8): *“My parents want to buy a car that costs 5.000 €. They earn 100 € each week. How much money will they earn in a year? Will there be enough money to buy the car?''*. The problem seems to be of the same structure as the given problem although the object of interest is different (car instead of a book). The first thing to say here is that obviously, the student used the *“change numbers”* strategy. However, carefully reading the statement of the problem we see that the word *“save”* has been substituted by the word *“earn”*. We can calculate the total amount they earn in a year (first question), but to answer whether there will be enough money to buy the car it is necessary to know how much they save regularly. So, the problem now became a *“missing middle”* problem.

On the contrary, in Group B we had a plethora of new problems (in total 28 problems) that combine strategies. One example of their problems is (Problem 9): *“I want to buy 2 books that cost 17€ each. If I save 50 cents every day, what can you ask about this problem?''*. Obviously, the student used the *“change numbers”* strategy as the number of books and the cost of each book have been changed compared to the corresponding numbers in the initial problem. Moreover, the student omitted the question of the given problem. Instead, invites the potential solver to think of questions that fit the given data, exhibiting thus the use of the *“tailless problems strategy”*. Some of these questions might be *“trivial”*, or similar to the ones that are omitted. But others might be a challenge for the poser by demanding some figuring out. In this way, the use of this strategy serves the diversity in students' thinking.

Considering the numerical data in Table 2 and the discussed examples in this section it can be said that Category C strategies dominated the students' preferences. Indeed, for Group A the number of the posed problems was small and all but one were based on the *“reverse known and unknown information”* and *“change numbers”* strategies (category C). However, while students from Group A were limited to category C strategies, students from Group B who were recipients of the training project on problem posing were able to exhibit the ability to use more advanced problem posing strategies that demand deeper structure sense. So, although category C strategies still dominate, it is evident that the use of more powerful strategies (categories B and A) as well as some combinations of them have also emerged. The problems produced by the students with the training experience afford multiple solutions (Problem 3), they have one solution that can be solved in more than one way (Problem 4), and they are open at end (Problems 5 and 9), which are, according to English (2020) quality indicators for the posed problems. At the same time, these Group B students were able to

make use of the structure of the tasks by initially identifying, and then negating or adding attributes (Problems 6 and 7), which also is considered a quality indicator (Cankoy, 2014).

## Conclusions

This study aimed to investigate the impact of a problem-posing intervention on both the students' development of their problem posing abilities and the quality of the problems they posed in terms of seeking and using structure. To do that, we compared the problems posed by two different groups. Group A had no prior experience in problem posing while Group B participated in a year-long intervention. When students are not given the opportunity to explore the limits of various mathematical situations, they are limited to the types of problems they already know (Crespo & Sinclair, 2008). Group A reasonably used mainly Category C strategies to pose new problems. Group B had the opportunity to explore many different problem-posing situations and become familiar with a variety of problem-posing strategies. The findings of our study strongly indicate that there was a difference between the two groups. This difference was expected (although there is a significant difference in the number of the posed problems). The problems posed by the students from Group B are distributed across all the three categories, but there is a considerable accumulation in categories B and A and even combination of multiple strategies in the same problem. This shift towards cognitively more demanding strategies is indicative of a rather deeper understanding of the notion of structure. Moving from the modification of surface characteristics of a problem to recognizing and changing the structure of the problem, is an important step that students could not have achieved without the intervention. Moreover, these findings rather strengthen the claim that students can be effectively trained to pose high-quality problems (Cai et al., 2015) when they receive proper training. In a future study, a greater number of participants or more sessions with each group could provide us with even stronger indications.

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