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► **To cite this version:**

Lóa Björk Jóelsdóttir, Pernille Bødtker Sunde, Paul Andrews. Age- and gender-related differences in Danish students' use of number-based strategies when solving multidigit addition tasks. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04416440

HAL Id: hal-04416440

<https://hal.science/hal-04416440>

Submitted on 25 Jan 2024

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Age- and gender-related differences in Danish students' use of number-based strategies when solving multidigit addition tasks

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In Denmark, both the national curriculum and school textbooks privilege number-based strategies above standard algorithms for the solution of arithmetical tasks. However, while strategy adaptivity and flexibility are widely accepted as key goals of mathematics education, internationally students turn to standard algorithms. In this paper, therefore, we investigated the influence of both gender and age on students' strategies for solving the addition tasks $386 + 214$ and $482 + 218$; tasks designed to elicit number-based strategies. The analyses confirmed the international trend, with an increasingly high use of the standard algorithm with grade level. However, where number-based strategies were used, they were no less accurate than standard algorithms. Importantly, the results also showed that girls used standard algorithms significantly more often than boys, who used number-based strategies significantly more often than girls.

Keywords: Arithmetic strategies, multidigit addition, adaptivity, flexibility, gender.

Introduction

The acquisition of strategy flexibility (having access to multiple strategies for a given task) and strategy adaptivity (being able to select the optimal strategy) are important goals of mathematics education (Verschaffel et al., 2009). In the specific context of arithmetic, research has highlighted the importance of number-based strategies, often the most efficient, over standard algorithms (Hickendorff et al., 2018; Sievert et al., 2019; Torbeyns et al., 2017). However, the same studies typically find that students reject such strategies in favour of standard algorithms, frequently in ways that are both context- and gender-determined. In this paper, in an under-researched context, we examine how age and gender influence Danish students' multidigit addition-related strategy choices.

Students' arithmetic-related strategy flexibility and adaptivity

Broadly speaking, while students' arithmetic-related strategy use becomes more adaptive with increasing age (Lemaire & Brun, 2017; Torbeyns et al., 2018), the overall picture is anything but simple. Studies undertaken in Flanders and the Netherlands have shown that while young children rarely volunteer strategy adaptivity, most have access to a range of arithmetic-related strategies and can, when asked to do so, switch between them (Torbeyns et al., 2018). By way of contrast, a German study found most grade three students having access to a range of strategies, that, in many cases, they were able to use adaptively (Heinze et al., 2009). However, when presented with tasks designed to elicit the use of number-based strategies, including shortcut strategies (strategies that involve the flexible adaptation of numbers and operations (Xu et al., 2017), students rarely exhibit task-related

adaptivity (Hickendorff, 2022; Hickendorff et al., 2018). For example, a study of Dutch and Flemish students, based on tasks designed to elicit number-based strategies, found, across grades three, four and five, most children consistently relying on one strategy, with only a minority of the high achieving or older students varying their strategies (Torbeys et al., 2017). Finally, in this section, the didactical introduction of standard algorithms not only constrains students' strategy repertoires but their preferred strategies (Heinze et al., 2009; Sievert et al., 2019; Torbeys et al., 2017). In other words, instructional practices, including the material presented in textbooks, tend to collude in the denial of opportunities for children to grow mathematically (Csikós, 2016; Heinze et al., 2009; Torbeys et al., 2017). That being said, little is known about the growth of students' strategy repertoires and strategy choice in curricular contexts, like Denmark, where the teaching of standard algorithms is discouraged.

Gender and students' arithmetic-related strategy use

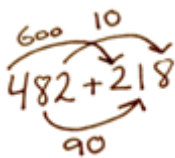
Internationally, the influence of gender on children's general arithmetical competence seems greatly influenced by national context (Mullis et al., 2020). That being said, even within the same national context research is ambivalent. For example, one study of grade six Dutch students found, with respect to multidigit arithmetic, that girls, who tend to use the standard algorithm, were more successful than boys, who tend to rely on unspecified mental calculations (Hickendorff, 2013). By way of contrast, large scale studies of number-related achievement found Dutch boys significantly outperforming Dutch girls (Mullis et al., 2020). Elsewhere, gender differences in strategy choice for single-digit arithmetic have been found from the early school years, with girls typically showing a preference for counting strategies and boys a preference for retrieval strategies (Bailey et al., 2012; Carr & Davis, 2001; Sunde et al., 2020). Similar differences, in respect of multidigit arithmetic, have been in the older grades, where Dutch studies have exposed boys' preferences for mental or short-cut methods and girls' preferences for written methods (Hickendorff, 2018).

This Study

The Danish national curriculum asserts that teachers should (our translation) "challenge and support individual students to develop their arithmetic strategies based on their number understanding rather than learning procedures for setting-up and calculating standard algorithms. The aim is not to practice standardised algorithms" (Ministry of Children and Education, 2019a, p.15). These objectives are reflected in Danish textbooks. For example, grade 2 students are encouraged to work with the open number line and 100 complements (Kristensen & Teglskov, 2018) or 'clever' strategies (Jensen et al., 2015). However, acknowledging that Danish grade 4 boys' number-related competence is significantly greater than that of girls (Mullis et al., 2020), there has been very little systematic evaluation of Danish students' arithmetic-related strategy use, as Danish national tests, which do not ask for written evidence of students' strategies, typically elicit only answers. That said, recent analyses of grade 9 students' answers indicate that many incorrect answers were due to a misapplication of a standard algorithm (Ministry of Children and Education, 2019b). In other words, the standard algorithm, even in a system that encourages number-based strategies, seems to dominate students' decision making and, for many, proves problematic. In light of such matters, the aim of this study is to examine, in relation to multidigit addition, which strategies Danish students choose to use at different stages of compulsory school, focusing explicitly on grades three, six and eight (ages 9, 12 and 14), and whether such choices are influenced by gender. We focus on multidigit addition

because students at all three grade levels should be able to solve three-digit addition tasks with high levels of accuracy and, importantly, feel confident in so doing, as this should improve the likelihood that they will be able to communicate their strategies. Furthermore, as shown in Table 1, tasks were designed to be amenable to a range of solution strategies.

Table 1: Possible solution strategies for the multidigit addition task 482 + 218

Examples of digit-based strategies for 482 + 218			
Standard algorithm		Calculation with digits	
$\begin{array}{r} 11 \\ 482 \\ + 218 \\ \hline 700 \end{array}$		$\begin{array}{r} 4 + 2 = 6 \\ 8 + 1 = 9 \\ 2 + 8 = 10 \\ 6 \ 9 \ \boxed{10} \\ \hline 700 \end{array}$	
Examples of number-based strategies for 482 + 218			
Split	Sequential (adding on)	Compensation	100 complements
$400 + 200 = 600$ $80 + 10 = 90$ $2 + 8 = 10$ $600 + 90 + 10 = 700$ OR (with drawing) 	$482 + 200 = 682$ $682 + 18 = 700$ OR $482 + 200 = 682$ $682 + 10 = 692$ $692 + 8 = 700$	$482 + 218$ $-2 \quad +2$ $480 + 220 = 700$	$82 + 18 = 100$ $400 + 200 = 600$ $100 + 600 = 700$

This paper, drawing on data from 20 schools in five demographically different municipalities in Jutland, Denmark, involved 2298 pupils in 37 grade 3, 39 grade 6 and 45 grade 8. This paper analyses data from the 2195 (96.4%) pupils for whom gender-related data were available. Prior to participation in late 2020, parents/carers received information on their child’s participation, their rights to withdraw their children from participation, and general data protection regulation (GDPR). The test from which the two items involved here were derived was designed to elicit shortcut or number-based strategies for addition, subtraction and multiplication tasks. In this paper, we focus on students’ solutions to two three-digit addition tasks, namely, 384 + 216 and 482 + 218, with Table 1 showing a range of possible solution strategies for the latter. To reduce the risk of copying, two equivalent tests were developed, so that students sitting in adjacent seats during the classroom-based but researcher-led implementation completed different tasks. Students were encouraged to scrutinise each task’s

numbers for clues as to how to proceed and then write in as complete a form as they could, the solution procedures they adopted. If solving ‘in their head’ they were asked to write or draw as best they could the process used for solving the task.

Analysis

Each student’s solution, from the perspectives of both strategy type and accuracy, was coded by the first author and at least one of several research assistants. Solution strategies were coded for the six strategies shown in Table 1, plus a further four categories of *counting*, *drawing or number line*, *no communicated method*, which included all tasks with an answer, but no strategy communicated, and *other*, which included strategies that were not possible to categorise in any other way. The results were then compared and contrasted for all disagreements. In the case of uncertainty, the opinion of a third expert was sought, and a resolution achieved. The results for the two tasks were compared for both strategy type and accuracy. A two-sample t-test showed no significant difference (95% significance level) between the use of standard algorithm ($t(2343) = -1.11, p = 0.268$) or split strategies ($t(2343) = -0.65, p = 0.518$) for the two tasks. The accuracy patterns of the two tasks, shown in Table 2, were also comparable ($t(2343) = -0.04, p = 0.9712$). Therefore, the results for the two tasks were combined to form one sample.

Results

Students’ strategy choice and accuracy for each task can be seen in Table 2. Despite the tasks being designed to elicit number-based strategies, in particular the 100 complements strategy, students’ use of the standard algorithm dominated their attempts, increasing from a low of 44% in grade 3 to 82% of tasks in grade 8. The most frequently used number-based strategy was the split strategy, with a frequency across the grades of 17%, 16% and 8% respectively. The remaining strategies were used rarely, although the 100 complements strategy, for which the tasks were explicitly designed, was the most commonly occurring, with frequencies of 8%, 7% and 5% respectively.

It could be argued that the split strategy is unique among the number-based strategies in that it does not require the taking account of the specific characteristics of the numbers. That is, the split strategy, along with the standard algorithm, can be construed as number-independent, which means that the strategy is not adapted to the number characteristics. The success or otherwise of the other number-based, or number-dependent, strategies - sequential, compensation or 100 complements - is highly dependent on the specific characteristics of the numbers. If we compare the total use of number-independent - standard algorithm, split and digit-based strategies - with the total use of number dependent strategies - compensation, sequential and 100 complements, the ratios are 65:16 in grade 3, 86:10 in grade 6 and 88:8 in grade 8, showing that students’ use of number independent strategies is not only very high but increases across the grades, while their use of the easier, number dependent, strategies is not only low but falls across the grades. However, it is worth noting, with respect to accuracy, that the number-independent and number-dependent strategies have comparably high accuracy levels, indicating that students’ strategy choice may be more influenced by beliefs about what is expected than the facility number-dependent strategies offer.

Table 2: Strategy totals for the two tasks by accuracy by grade

	Grade 3 (N=721)		Grade 6 (N=687)		Grade 8 (N=786)	
	Solved tasks (%)	Accuracy (%)	Solved tasks (%)	Accuracy (%)	Solved tasks (%)	Accuracy (%)
Total (all methods)	94	71	99.7	92	100	96
Standard algorithm	44	79	69	92	82	97
Calculation with digits	4	60	1	75	1	80
Split	17	80	16	97	5	95
Sequential (adding on)	3	95	2	100	2	100
Compensation	5	91	1	88	1	88
100 complements	8	82	7	96	5	100
Drawing or number line	3	52	-	-	-	-
Counting	1	80	-	-	-	-
No communicated method	11	60	3	81	2	93
Other	3	65	1	100	1	83

Results from an earlier study involving students' solutions to multidigit addition, subtraction and multiplication, highlighted substantial gender differences with respect to use of standard algorithm and shortcut strategies (Hickendorff, 2018). However, it is not known how these differences play out in relation to specific tasks with specific number characteristics, in this case addition tasks designed to encourage the use of the 100 complements strategy. To this end, the figures of Table 3 show the results split by gender. Due to data pertaining to gender not being made available until after initial coding and results had been calculated, the strategy information for 104 students in Table 3 was categorised in the *other* category. This resulted in an artificially large number of solutions in the *other* category and the omission of the *calculation with digits* strategy compared to data in Table 3. However, even with a reduced sample, the results in Table 3 offer a clear sense of gender-related differences in students' strategy use. Irrespective of grade, girls' use of the standard algorithm far

exceeded that of the boys. In this respect, the ratio of girls' use to boys' use of the standard algorithm was 49:34 in grade 3, 84:55 in grade 6, and a more equitable 88:76 in grade 8. If, as above, we collapse the frequencies of the number-dependent strategies into one group, the ratio of boys' use to girls' use of such strategies was found to be 18:7 in grade 3, 16:3 in grade 6 and 11:3 in grade 8. Taken together, these results show that girls are far more likely than boys to use number-independent strategies and boys far more likely than girls to use number-dependent strategies. A two-sample t-test show significant difference for standard algorithm $t(2193) = -9.56, p < 0.001$ and for split method (as number-independent strategi, $t(2193) = 2.67, p = 0.01$.

Table 3: Gender differences in strategy use by percentage of solved tasks

	Grade 3		Grade 6		Grade 8	
	Boys	Girls	Boys	Girls	Boys	Girls
Standard algorithm (%)	34	49	55	84	76	88
Split (%)	14	13	18	11	6	3
Sequential (adding on) (%)	2	1	3	0	2	0
Compensation (%)	5	1	2	0	1	0
100 Complements (%)	11	5	11	3	8	3
Drawing or number line (%)	4	2	0	0	0	0
No communicated method (%)	15	15	6	0	3	2
Other (%)	17	13	5	2	3	2

Discussion

In this paper, our goal was to examine the influence of age and gender on the strategies used by Danish students when solving multidigit addition tasks designed to elicit a range of number-based strategies. From the perspective of the former, the results clearly show not only that such strategies are rarely used but that their use diminishes with age. Indeed, from a high of 25% in grade 3, the proportion of tasks solved by means of number-based strategies fell to 8% by grade 8. By way of contrast, the use of the standard algorithm, which accounted for barely two-fifths of all tasks in grade 3, had almost doubled by grade 8. This fall in the use of number-based strategies, which resonates with studies from elsewhere (Heinze et al., 2009; Sievert et al., 2019 Torbeyns et al., 2017) is disappointing in a curriculum context that privileges informal over formal strategies, and indicates that either instructional practices may be complicit in denying children opportunities (Csíkos, 2016; Heinze et al., 2009; Torbeyns et al., 2017) or, warranting further research, parents may have intervened at home.

From the perspectives of gender, the results show, irrespective of grade, substantial differences, resonant of other studies (Hickendorff, 2018) in how boys and girls approach multidigit addition tasks. Girls, almost exclusively, turn to the standard algorithm, while boys, albeit with a strong standard algorithmic tendency, are more likely to attempt number-based strategies. The general failure of students, especially girls, to demonstrate the cognitive flexibility and adaptivity involved in choosing and executing number-based strategies is problematic, particularly as such competences support later learning (Sievert et al., 2019; Torbeyns et al., 2017), and should be of major concern to the Danish curricular authorities, teachers, teacher educators and parents.

Overall, this study has raised several questions in need of further research. Why, in a system that privileges number-based strategies, do so many students resort to standard algorithms? Is it simply a consequence of instructional practices that suppress such strategies, or are other issues, like the role of parents, at play? Why, in particular, do girls take refuge in the procedural certainty of the standard algorithm or, alternatively, why do so few girls take strategic risks when solving multidigit addition tasks? These questions do not yield simple answers, but they are questions worth asking.

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