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Digital self-assessment in learning multiplication

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Digital technologies have the potential to support the assessment of students' mathematical understanding, but they are rarely used to support students in self-assessment. For this reason, the BASE project is developing and researching a digital self-assessment tool (BASE tool) that focuses on self-assessment of basic arithmetic knowledge. This paper describes a case study investigating how a dynamic and interactive sample solution in the BASE tool affects a student's multiplicative understanding and activation of metacognitive activities. It turns out that the student develops a multiplicative understanding that is strongly related to the sample solution, even if it is only one of two possible solutions. Regarding metacognitive activities, the sample solution causes the student to doubt his mathematically correct answer on the one hand and to correctly evaluate his self-assessment on the other.

Keywords: Feedback, Educational Assessment, Technology, Multiplication, Metacognition.

Introduction

As Black and Wiliam (2009) described, formative assessment is widely recognised as a crucial aspect of effective teaching in the classroom. With the rise of digital technologies, opportunities for digital assessment are also increasing. Referring to previous research, Drijvers identifies several arguments favouring digital assessments. For example, he sees potential for digital assessment in rich interactive tasks or automatic feedback (Drijvers, 2018). However, many digital formative assessment environments lack student-centred practices like self-assessment, which can significantly enhance students' mathematical understanding and metacognition (Ruchniewicz & Barzel, 2019). Furthermore, as digital self-assessment environments are limited, we know little about how dynamic and interactive sample solutions provided in such environments are interpreted and used by students in their self-assessment process and how the sample solutions affect students' learning.

The BASE project addresses this need for development and research. It aims to design and investigate a digital self-assessment tool (BASE tool) that helps students to self-assess their basic arithmetic knowledge.

Potentials of digital technologies for formative self-assessment

“Self-assessment is a process of formative assessment during which students reflect on the quality of their work, judge the degree to which it reflects explicitly stated goals or criteria, and revise their work accordingly” (Andrade & Valcheva, 2009, p. 13).

Self-assessments promote metacognitive skills, such as self-observation or self-reflection. Encouraging students to engage metacognitively with their learning through self-assessments makes them more involved with the mathematical contents (Andrade, 1996).

Digital technologies provide various potentials to enhance self-assessment processes (Ruchniewicz & Barzel, 2019). The following three potentials merit particular consideration (2018, pp. 47-48):

- (1) Rich items: Digital learning environments can include dynamic elements, for example, videos or animations and interactive items.
- (2) Adaption: Digital learning environments enable the automatic assignment of assessment items to students' learning levels, allowing real-time adjustment of the difficulty of subsequent exercises based on responses to earlier tasks.
- (3) Automatic feedback: Digital learning environments can provide automatic feedback, for instance, on the correctness of the task solution or self-assessment.

For example, Ruchniewicz and Barzel (2019) developed the SAFE tool, a digital self-assessment tool in the area of functional thinking that uses dynamic sample solutions to support students' self-assessments. In the SAFE tool, students complete a digital assessment task, receive a dynamic sample solution and subsequently assess their solution with the help of a task-specific checklist "to guide the student's reflection on his/her own solution" (Ruchniewicz & Barzel, 2019, p. 50). However, in addition to the potential to support students' mathematical understanding and metacognition, the research on the SAFE tool revealed that students would like to get feedback on the correctness of their self-assessment and task performance (Ruchniewicz & Barzel, 2019).

The BASE tool

The BASE tool is a self-learning tool that targets fourth- and fifth-grade students transitioning from primary to secondary school. The tool enables students to practice their basic arithmetic skills from primary school individually and self-directedly. In the following, we will briefly elaborate on the design of the BASE tool using a multiplication task as an example. A detailed description of the design principles and structure of the BASE tool can be found in Graewert et al. (2023).

There are various assessment tasks in the BASE tool, each focusing on basic arithmetic knowledge, e.g., representing numbers in the place value chart or column addition. In the task used in the present study, the students had to represent the calculation 6×4 by jumps on a given number line. As multiplication is a binary commutative combination, the solution of the task can be done by calculating 6×4 or 4×6 . Thus, for a correct solution, it is possible to draw six jumps of length four or four jumps of length six.

After the students have submitted their solution (Figure 1, left, "My Solution"), they have to assess themselves using a task-specific checklist (Figure 1, left, "Self-Assessment"). To support their self-assessment, the students receive a dynamic and interactive sample solution (Figure 1, left, "Possible Solution"). To avoid cognitive overload of the students, only one of the two possible solutions is presented. Since the standard interpretation in Germany is that the first number indicates the number of jumps and the second number the length of the individual jumps, it was decided to present the corresponding sample solution. In the sample solution, the students can play the jumps by clicking on the "Play button" so that the jumps appear one after the other on the number line. Additionally, they can select visual hints that highlight and explain features of the sample solution (e.g., selecting "length of each jump").

After self-assessing their solution, students receive automatic feedback on the correctness of their self-assessment (Figure 1, right). Based on the correctness of the task solution and the self-assessment, the BASE tool adaptively assigns subsequent exercises to the students.

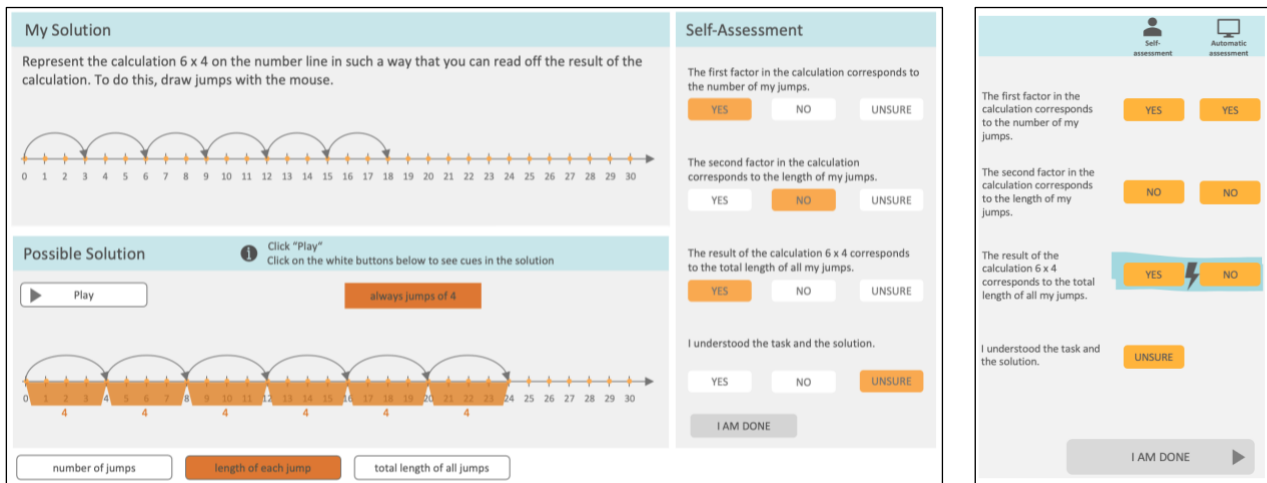


Figure 1: Self-assessment of a submitted task (left) and feedback on the self-assessment (right)

Research question and methodology

In the present paper, we focus on the following research questions:

- 1) *To what extent are the sample solution and its dynamic and interactive elements used?*
- 2) *How does using the sample solution affect a student's multiplicative understanding and metacognitive activities in a self-assessment process?*
- 3) *What can be inferred from 1) and 2) for the further development of the BASE tool?*

Data collection

As part of the study, we collected video recordings from 12 students attending the fifth grade of a German upper secondary school, who were thus about ten years old. The students were selected according to whether they and their parents agreed to participate in the survey. Other selection criteria, such as mathematics performance, did not play a role in selecting the participants. The students were supposed to independently practice their primary school knowledge of multiplication on the number line with the help of the abovementioned task. After completing the task, the students self-assessed their solutions. To capture the students' understanding and metacognitive activities, they were asked by an interviewer to think aloud and explain their actions and thoughts (Leighton, 2017). Lorenz's case was chosen for in-depth analysis because it shows a typical case that occurred several times.

Data analysis

The data analysis is based on Rezat's theoretical framework (2021, p. 1436), incorporating the theory of instrumental genesis (Rabardel, 2002) and the concept of scheme (Vergnaud, 2009) (Figure 2). According to Vergnaud, a scheme "*is the invariant organization of activity for a certain class of situations*" (2009, p. 88, emphasis original). It comprises various components, where operational invariants represent the implicit knowledge in schemes (Rezat, 2021). These operational invariants consist of *concepts-in-action* and *theorems-in-action*. Concepts-in-action are mathematical objects considered relevant or less relevant (e.g., factor, number of jumps), thus playing a crucial role in

identifying and selecting information. Theorems-in-action are mathematical statements that students believe to be true but may be mathematically false (Vergnaud, 2009).

Instrumental genesis describes a twofold process in which a subject transforms an artefact into a helpful instrument through its actions. The discovery and use of artefact properties and the assignment of functions to it are referred to as *instrumentalisation*. The use of artefacts shapes utilisation schemes (= schemes linked to the utilisation of an artefact), and the development and acquisition of these schemes is called *instrumentation* (Rabardel, 2002). As Drijvers and Trouche (2008, p. 28) noted, the instrumental genesis process can be time-consuming and laborious. Since we will focus on the processing of a single task in the following case analysis, we will only consider a small part of instrumental genesis. Furthermore, given that the student is using the BASE tool for the first time, it can be assumed that the use of the tool is rather explorative, and the development of schemes is preliminary.

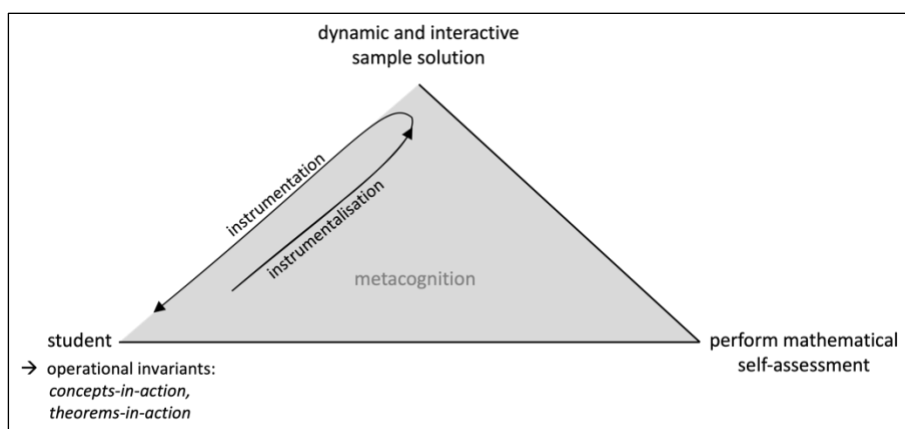


Figure 2: Theoretical framework based on Rezat (2021, p. 1436)

In the following, concepts-in-action will be highlighted with double lines $||\dots||$, theorems-in-action with greater-than or less-than symbols $>\dots<$ and metacognitive activities with asterisks $*\dots*$.

Results: The case of Lorenz

When solving the task, Lorenz initially draws a jump from zero to five but corrects his solution to five jumps of length six starting at zero. When the interviewer asks for an explanation, Lorenz explicates his solution and notices that he has drawn one jump too many. So Lorenz removes the last jump, resulting in four jumps of length six:

- 4 Interviewer: Why does it have to be removed?
- 5 Lorenz: Because that must be four (*points to the 4 in the task*). That (must be?) times four, so four jumps, I guess.
- 6 Interviewer: Mm, ok.
- 7 Lorenz: Yes, and I have just drawn the jumps till six.

Lorenz submits his solution with four jumps of length six and is then directed to the self-assessment screen:

- 11 Lorenz: (*Reads the hint in the sample solution and clicks on PLAY*)
Six jumps of length four are played on the number line.

- 12 Lorenz: Ah. I had to make four, not six. (I have to draw something here starting from four?) (*points to the jumps in the sample solution one after the other*) One, two, three, four, five, six. Six jumps, then. (*Reads through the first checklist statement and clicks NO*) I did NO because instead of doing six jumps (*points to the 6 in the task and then to his solution*), I did four jumps and instead of up to four (*points first to the 4 in the task and then to the 4 on the number line in his solution*) I did up to six (*points to the 6 on the number line in his solution*) because I thought that is how you have to do it.
- 13 Interviewer: Mm.
- 14 Lorenz: The second factor in the calculation corresponds to the length - also NO because, as I already said, I have also mixed it up there (*points to his solution*). The result of the calculation corresponds to the...(*points to the last jump in the sample solution and selects YES*).
- 15 Interviewer: Why YES?
- 16 Lorenz: Because I just quickly calculated four – six times four, and if that is the total length of all my jumps and the solution also came up with 24 (*points to the 24 on the number line in the sample solution*), then I calculated that. So my answer here was true (*points to the third checklist statement, which he answered YES*). (*Reads through the fourth checklist statement and answers YES*). My solution was wrong (*points to his solution*), but with the solution (*points to the sample solution*), I understood it (*clicks on I AM DONE*).

Instrumentalisation

Lorenz instrumentalises the sample solution to identify supposed errors in his solution. He concludes that his solution needs to be corrected because he mixed up the number and the length of the jumps compared to the sample solution (l. 12). However, Lorenz uses the “play function” of the sample solution only once.

Instrumentation

After correcting his solution twice, Lorenz submits a final solution of four jumps of length six. Referring to the four in the task, he states, “that must be four...so four jumps, I guess” (l. 5). Thus, the first theorem-in-action can be explicated as:

(TA 1) *>If I want to determine the number of jumps, then I must consider the second factor and draw the corresponding number of jumps<*

This theorem-in-action is guided by the concepts-in-action *||number of jumps||* and *||second factor||*. Concerning the length of the jumps, Lorenz points out that he has “just drawn the jumps till six” (l. 7), which results in the second theorem-in-action:

(TA 2) *>If I want to determine the jump length, then I must consider the first factor and draw jumps of the corresponding length<*

For this theorem-in-action, the concepts-in-action *||jump length||* and *||first factor||* are relevant. Playing and looking at the sample solution leads Lorenz then to **evaluate his solution as incorrect**. He says, “Ah. I had to make four, not six” (l. 12). Later, it becomes apparent that he meant “six, not four” since he continues saying, “One, two, three, four, five, six. Six jumps, then” (l. 12). Viewing the sample solution and the evaluation of his solution as incorrect lead to a change in the first theorem-

in-action (TA 1). This is because instead of relating the concept-in-action *||number of jumps||* to that of the *||second factor||*, Lorenz relates it to the *||first factor||*. The adapted theorem-in-action can be formulated as follows:

(TA 1.2) *>If I want to determine the number of jumps, then I must consider the first factor and draw the corresponding number of jumps<*

This theorem-in-action is also evident when Lorenz explains why he chose NO for the first checklist statement: “I did NO because instead of doing six, I did four jumps” (l. 12). Additionally, he states: “Instead of up to four I did up to six” (l. 12), referring to the four and six on the number line in his solution. Here, the theorem-in-action changes, which refers to the jump length (TA 2). It now can be expressed as follows:

(TA 2.2) *>If I want to determine the jump length, then I must consider the second factor and draw jumps of the corresponding length<*

Lorenz now associates the concept-in-action *||jump length||* with the *||second factor||* instead of the *||first factor||*. The repeated use of “instead of” indicates that in Lorenz’s case, answering NO to the first checklist statement is accompanied by an **evaluation of his own solution as incorrect**. This is also the case with the second checklist statement, which Lorenz answers with NO. He justifies his answer with “I have also mixed it up there” (l. 14). However, Lorenz answers the third checklist statement with YES. Lorenz explains: “I just quickly calculated four – six times four, and if that is the total length of all my jumps and the solution also came up with 24, then I calculated that”. Here, he refers to the sample solution and points to the endpoint of the last jump, that is, to 24 on the number line (l. 15, 16). It can be seen that Lorenz equals the *||total length of all jumps||* with the *||last jumps’ endpoint||* and equates these concepts-in-action with the *||product of the first and second factors||*, leading to the following theorem-in-action:

(TA 3) *>If I want to check the total length of all jumps, then I must calculate the product of the first and second factors and compare it to the endpoint of the last jump<*

Based on this theorem-in-action, Lorenz comes to **evaluate his answer to the third checklist statement as correct**: “So my answer here was true” (l. 16). After reading the fourth checklist statement, Lorenz summarily **evaluates his solution as incorrect** but notes that the sample solution helped him to understand the task: “My solution was wrong, but with the solution, I understood it” (l. 16).

Conclusion and outlook

In our study, we investigated to what extent a student uses the sample solution and its dynamic and interactive elements and how using the sample solution affects the student’s multiplicative understanding and metacognitive activities in a self-assessment process.

Use of the dynamic and interactive sample solution

Concerning using the dynamic and interactive sample solution (*instrumentalisation*), we find that Lorenz only plays the jumps once. This could be because the dynamic sample solution has no added value for Lorenz compared to a static sample solution. The interactive visual cues in the sample

solution were not used by Lorenz at all. A possible explanation could be that Lorenz was working with the BASE tool for the first time and was unfamiliar with its functionalities.

Impact of the sample solution on multiplicative understanding and metacognitive activities

Concerning the impact of the sample solution on Lorenz's multiplicative understanding (*instrumentation*), it turns out that the sample solution contributed to developing theorems-in-action, which are closely connected to the sample solution. When solving the task, Lorenz associated the number of jumps with the second factor (TA 1) and the length of the jumps with the first factor (TA 2). Looking at the sample solution, where the number of jumps is associated with the first and the length of the jumps with the second factor, resulting in a corresponding adaptation of Lorenz's theorems-in-action (TA 1.2 and TA 2.2). The sample solution did not support that Lorenz recognised multiplication as a binary commutative combination.

The sample solution affected not only Lorenz's multiplicative understanding but also his metacognitive activities. Looking at the sample solution led Lorenz to negatively evaluate his mathematically correct solution. Similar results were observed in a study conducted by Safadi and Yerushalmi (2013), in which students interpreted the sample solution as an "ultimate template and simply considered their deviation from it as mistakes" (p. 205). However, the sample solution also helped Lorenz to correctly evaluate his answer (YES) to the third checklist statement ("The result of the calculation 6×4 corresponds to the total length of all my jumps"). It becomes apparent that the sample solution can encourage not only the evaluation of one's own solution but also one's self-assessment.

Inferences for the further development of the BASE tool

Building on the results mentioned above, we have further developed the BASE tool as described in the following.

To support students in recognising multiplication as a binary commutative combination, we have reformulated the checklist statements (Figure 1, left, "Self-Assessment"). Instead of relating the checklist statements to the properties of the sample solution, the statements are formulated to apply to *both possible solutions*. For example, we reformulated the first statement: "One of the two factors in the calculation corresponds to the number of my jumps". In addition, we have included *feedback on the task level*, which means that the students also get feedback on whether their solution to the task was correct or incorrect. Given that Lorenz considered his mathematically correct solution to be incorrect, it would have been interesting to see how the feedback that his solution was correct would have affected his multiplicative understanding and metacognition. Regarding the dynamic and interactive sample solution, minor adjustments have been made concerning the design and arrangement of the buttons for selecting the interactive visual hints, which we hope to support that students use them more.

In summary, the results of this study illustrate the complexity of developing self-assessment environments that rely on interactive and dynamic sample solutions. Next, we will conduct a study with the revised tool and a larger group of students. We hope this research will contribute to a deeper

understanding of how artefacts affect students' mathematical understanding and metacognition in self-assessment processes.

References

- Andrade, H. (1996). *Student self-assessment: At the intersection of metacognition and authentic assessment*. Harvard University.
- Andrade, H., & Valtcheva, A. (2009). Promoting learning and achievement through self-assessment. *Theory Into Practice*, 48(1), 12–19. <https://doi.org/10.1080/00405840802577544>
- Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Measurement*, 21(1), 5–31. <https://doi.org/10.1007/s11092-008-9068-5>
- Drijvers, P. (2018). Digital assessment of mathematics: Opportunities, issues and criteria. *Mesure et évaluation en éducation*, 41(1), 41–66. <https://doi.org/10.7202/1055896ar>
- Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In M. K. Heid & G. W. Blume (Eds.), *Research on Technology and the Teaching and Learning of Mathematics: Vol. 2. Cases and Perspectives* (pp. 363–392). Information Age Publishing.
- Graewert, L., Thurm, D., Hußmann, S., & Barzel, B. (2023). Digital formative self-assessment [Manuscript submitted for publication]. *Proceedings of the 16th International Conference on Technology in Mathematics Teaching (ICTMT 16)*.
- Leighton, J. P. (2017). *Using think-aloud interviews and cognitive labs in educational research*. Oxford University Press. <https://doi.org/10.1093/acprof:oso/9780199372904.001.0001>
- Rabardel, P. (2002). *People and technology: a cognitive approach to contemporary instruments*. Université Paris.
- Rezat, S. (2021). How automated feedback from a digital mathematics textbook affects primary students' conceptual development: two case studies. *ZDM – Mathematics Education*, 53(1), 1433–1455. <https://doi.org/10.1007/s11858-021-01263-0>
- Ruchniewicz, H., & Barzel, B. (2019). Technology supporting student self-assessment in the field of functions – A design-based research study. In G. Aldon & J. Trgalova (Eds.), *Technology in Mathematics Teaching - Selected Papers of the 13th ICTMT Conference* (pp. 49–74). Springer. https://doi.org/10.1007/978-3-030-19741-4_3
- Safadi, R., & Yerushalmi, E. (2013). Students' self-diagnosis using worked-out examples. *SciRes*, 4(3), 205–216. <https://doi.org/10.4236/ce.2013.43031>
- Vergnaud, G. (2009). The theory of conceptual fields. *Human Development*, 52(2), 83–94. <https://doi.org/10.1159/000202727>