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# Theory-related pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools - empirical analysis of the promotion of pre-service teachers

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*Teaching simulations and mathematical modelling with digital tools requires theory-related pedagogical content knowledge. This type of knowledge consists of theories about the corresponding potentials and possible uses of digital tools and – as meta-knowledge about processing steps – also modelling cycles. These can be used, for example, in the processing of more complex reality-related problems. The article conceptualises this dimension of pedagogical content knowledge and describes a course for pre-service mathematics teachers at the University of Münster and the University of Würzburg, designed to promote this dimension of knowledge, amongst others. Using an intervention study in a pretest-posttest design, it is quantitatively ( $N = 146$ ) shown that this knowledge can be effectively developed within the treatment.*

*Keywords: Mathematical modelling, simulations, digital tools, pedagogical content knowledge, teacher education*

## Introduction

Since digital tools have become part of mathematical learning, mathematics education research has been analysing and evaluating the multitude of possible uses of those tools in modelling and simulation activities. Well-known digital tools are dynamic geometry softwares, spreadsheets, function plotters and computer algebra systems. Their use can be helpful at many points during the working process (Geiger, 2011; Greefrath, 2011), for example when experimenting, visualising, calculating or controlling. Specifically, digital tools can be used to process larger amounts of data, to create more complex models and to outsource more complex calculations. They also enable “a new kind of ‘experimental’ work” (Doerr et al., 2017, p. 78) for example, through the use of simulations and the possibility of including multiple models in the solution process. In a *GeoGebra* application, for example, the number of people in a traffic jam can be determined by deliberately varying sliders (Gerber et al., 2022). Digital tools thus facilitate the integration of authentic real-life references into teaching practice.

These uses show the great potential of digital tools for teaching in reality-related lessons (Niss et al., 2007). Doerr et al. (2017) concretise this: digital tools help to “empower the mathematical processes involved in modelling activities” (p. 79). For example, multiple digital representations can improve students’ abilities to solve reality-related problems (Geiger, 2011), although this cannot be attributed to the use of a digital tool alone (Greefrath et al., 2018).

In addition to the potential for teaching mathematical modelling, however, the use of digital tools also poses challenges. Demanding translation processes to and from the digital tool are necessary for simulations and mathematical modelling (Siller & Greefrath, 2010). For example, in order to

outsource a complex calculation to a digital tool, the underlying mathematical model must first be translated into the syntax of the digital tool. After the calculation, the digital result must then be translated back into the world of mathematics. In the field of probability calculation, for instance, numerical values are issued digitally during the simulation of stochastic processes. A decision must then be made as to whether the values can be interpreted as probabilities or relative frequencies using mathematical terms. Subsequently, the values have to be validated in terms of correctness in the real world.

To meet these challenges and to use digital tools profitably in teaching simulations and mathematical modelling, specific pedagogical content knowledge is required. One dimension of this pedagogical content knowledge is the theory-related dimension, which addresses the basic content of the use of digital tools in reality-related teaching, as examined in the following. After explaining this knowledge dimension, we investigate whether it can be adequately promoted at an early stage of teacher education, i.e. through a university course for pre-service mathematics teachers.

## **Theoretical Background**

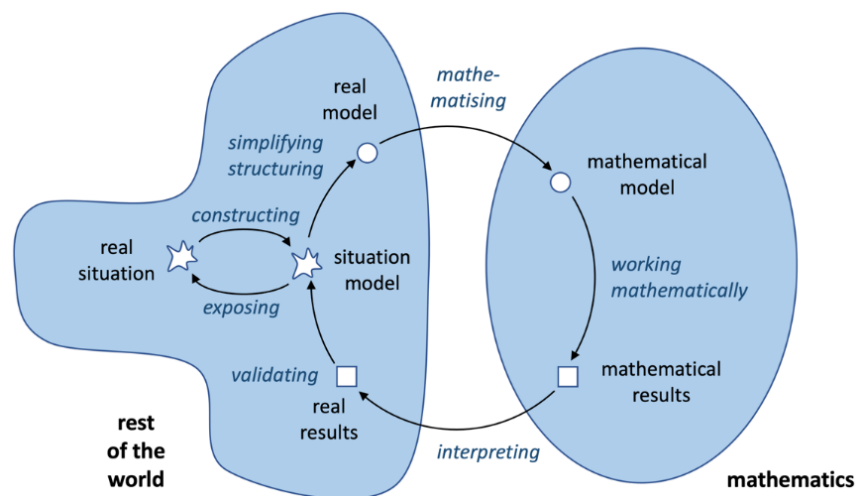
### **Theory-related pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools**

In order to describe and investigate pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools, reference was made to corresponding knowledge dimensions from modelling research (Borromeo Ferri, 2018; Greefrath et al., 2022). These following dimensions were concretised and empirically tested for the use of digital tools and simulations: (i) the *theory-related dimension*, (ii) the *task dimension*, (iii) the *diagnosis dimension* and (iv) the *intervention dimension* (Gerber et al., 2022). This domain-specific pedagogical content knowledge represents a cognitive aspect of a structural model of teachers' professional competence.

The *theory-related dimension* of pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools, which the paper focuses on, includes theoretical knowledge about the basics of pedagogical content and declarative meta-knowledge in this domain-specific area of teaching. This dimension includes aims, perspectives and potentials of the use of digital tools (see *Introduction*) in reality-related teaching. More specifically, knowledge about terms such as 'simulations' or how multiple representations can complement each other during the working process for a more holistic picture of the real situation (such as graphical and numerical representations) is one of the integral parts of this dimension.

### **Modelling cycles as part of the theory-related dimension of professional knowledge**

Declarative meta-knowledge also includes modelling cycles that take the use of digital tools into account. A modelling cycle represents central steps in the processing of a reality-related problem (Figure 1) and can serve as a tool for teachers to identify and analyse modelling processes (Vos & Frejd, 2022). Thus, it is important for teachers to have knowledge of modelling cycles in order to have a theoretical foundation for diagnosis or intervention (Blum & Leiß, 2007). As a basis for the following considerations, Figure 1 shows a seven-step modelling cycle by Blum and Leiß.



**Figure 1: Modelling cycle (following Blum & Leiß, 2007, p. 225)**

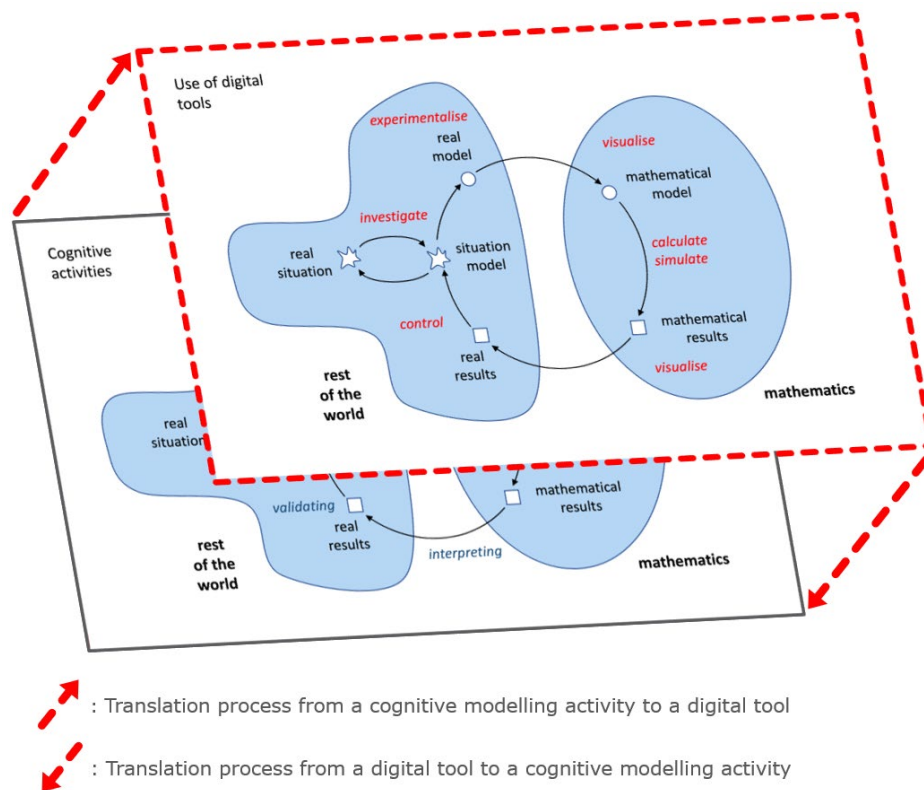
When using digital tools in simulations and mathematical modelling, knowledge about versatile uses of digital tools on the one hand (Greefrath, 2011) and the necessary translation processes to and from the digital tool on the other hand (Siller & Greefrath, 2010) are required. Both can be depicted in modelling cycles that consider digital tools. Digital support for simulation and modelling processes can therefore be looked at from two different perspectives: (firstly) from the perspective of the multiple possibilities of use, and (secondly) from the perspective of the translation processes.

These perspectives will now be linked by highlighting translation processes for the use of digital tools *at each point* of the modelling cycle (Figure 1) to support cognitive modelling activities. Following Vos and Frejd (2022), this will be illustrated in a multi-level representation. This is illustrated in Figure 2. The back layer shows the modelling cycle from Figure 1 as the structure of modelling processes. The front layer shows Greefrath's (2011) modelling cycle which adds possible uses of digital tools that are intended to support cognitive processing steps.

On the one hand, the arrow pointing from the back layer to the front layer describes the necessary meta-knowledge about potentials of digital tools in the corresponding modelling situation in which the digital tool is to be used. This includes the knowledge about why it makes sense to use the tool in this phase or what support can be expected from it. At the same time, it also describes the knowledge about the digital tool's syntax that is required to be able to use its functions.

On the other hand, the arrow pointing from the front layer to the back layer expresses the knowledge to use digitally generated output, which is now available in concrete terms, to support a cognitive modelling activity and to retrieve it to the right place in the working process. For this purpose, it is also necessary to translate the digital output back into the context of the respective processing step. The digital tool thus provides concrete support in this modelling phase.

Figure 2 hence describes that the use of digital tools is possible at all points during the working process (front layer: red italics). Furthermore, it becomes clear that cognitive modelling activities (back layer or Figure 1: blue italics) can only be supported by the use of digital tools if necessary translation processes (red arrows) are carried out (Frenken et al., 2021).



**Figure 2: Modelling cycle – use of digital tools**

### Promoting the theory-related dimension of pedagogical content knowledge

Because of the importance of the theory-related dimension of pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools, the promotion of this dimension is already beneficial in the first phase of teacher education (on the importance of pre-service teacher training, among others, see for example Baumert & Kunter (2013)). Therefore, we have designed a mathematics education course for pre-service mathematics teachers at the University of Münster and the University of Würzburg on simulations and mathematical modelling with digital tools. The conception of this course is based on Greefrath et al. (2022) and is structured in three phases: In the *preparation phase*, theoretical basics for simulations and mathematical modelling with digital tools are presented including pedagogical content knowledge of the theory-related dimension. For this purpose, modelling cycles and how they consider digital tools are analysed, among others. In this phase, simulation and modelling tasks are also processed with different functions of digital tools and working steps are evaluated with a special focus on the use of digital tools. In the *practical phase*, theoretically acquired knowledge is applied to concrete situations with students or fellow pre-service teachers in practical exercises: Self-developed tasks are presented, and modelling cycles are used to diagnose modelling processes. The *reflection phase* serves to recapitulate and analyse the results and observations achieved in the preceding phases, for example on one's own competence development. Therefore, the course does not only address contents of the theory-related dimension, which this article focuses on, but also, for example, the area-specific task knowledge as well as the knowledge of diagnosing and intervening in students' difficulties.

## Research question

Concerning the course conception, the question arises regarding the effectiveness of the course for the promotion of theory-related pedagogical content knowledge of pre-service mathematics teachers. Therefore, we pose the following research question: *To what extent can the theory-related pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools of pre-service mathematics teacher be developed by a university mathematics education course which is structured into the three phases mentioned above?*

## Methods

Our course is now evaluated in a quasi-experimental intervention study. For this purpose, data from 146 pre-service mathematics teachers at the University of Münster and the University of Würzburg were collected in the winter semester 2021/22 and the summer semester 2022 and cumulatively analysed. The data were collected using a self-developed test instrument on teachers' professional competences in simulation and mathematical modelling (Gerber et al., 2022). For this purpose, the pre-service teachers took an identical test at two measurement points approximately three months apart: at the beginning of the semester (pretest) and at the end of the semester (posttest). Between the measurement points, the pre-service teachers in the experimental group took part in the specially designed mathematics education course (intervention). During the same period, the pre-service teachers in the control group did not attend any mathematics education course or attended another course (lecture, tutorial or seminar) that did not offer any corresponding support for simulations and mathematical modelling with digital tools. A detailed sample description can be found in Table 1:

**Table 1: sample description**

	Number	Gender m/f/d/n. s.	Age		Semester		A-levels grade	
			M	SD	M	SD	M	SD
<b>EC</b>	80	28/51/0/1	22.24	3.84	6.25	3.00	1.78	0.49
<b>CG</b>	66	28/38/0/0	23.58	3.18	8.45	3.24	1.90	0.51
<b>Total</b>	146	56/89/0/1	22.84	3.60	7.25	3.29	1.84	0.50

The use of digital tools ...	
... requires a standardized approach to mathematical modelling.	<input type="checkbox"/>
... in mathematical modelling is only possible in calculation.	<input type="checkbox"/>
... makes it possible to work on mathematical models with complex function terms.	<input type="checkbox"/>
... is not helpful in understanding the factual context.	<input type="checkbox"/>

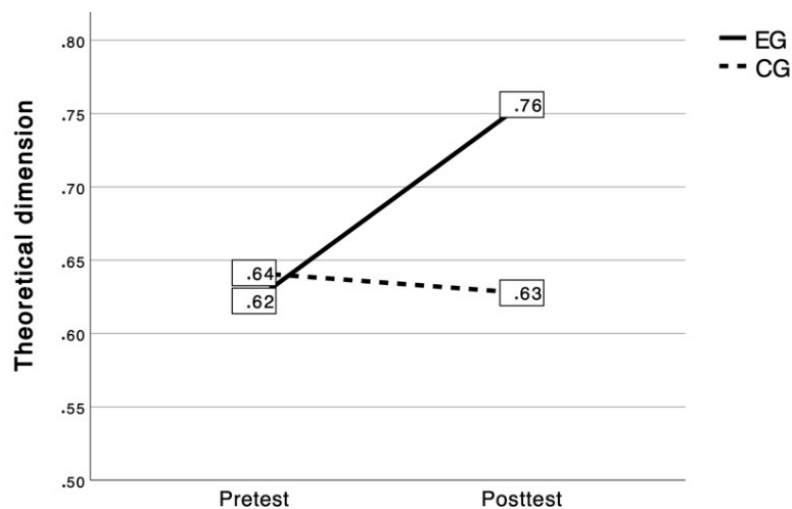
**Figure 3: Example item of the theory-related dimension (Gerber et al., 2022, p. 4)**

The theory-related scale of the test instrument, which was used to measure the knowledge dimension, comprises nine items in a multiple-choice answer format. They each consist of a beginning of a sentence and four sentence completions, one of which must be selected as correct. The items are

evaluated dichotomously (0 = wrong answer selected; 1 = correct answer selected). As an example, Figure 3 shows an item of the theory-related scale. The item refers to the potential of digital tools in simulations and mathematical modelling. Other items focus, for example, on the steps of the modelling cycles mentioned above or the interaction of different digital tools. This content is developed (preparation phase) and deepened in the treatment.

## Results

Figure 4 shows a line chart in which the arithmetic mean values of the proportions of points achieved in the pretest and posttest are shown for the experimental and the control group respectively (possible minimum: 0 = no item answered correctly, possible maximum: 1 = all items answered correctly).



**Figure 4: Development of the theory-related pedagogical content knowledge**

In the pretest, the theory-related pedagogical content knowledge of simulations and mathematical modelling with digital tools is  $M_{Pre,EG} = .62$  ( $SD = .19$ ) in the experimental group; in the control group it is  $M_{Pre,CG} = .64$  ( $SD = .17$ ). At this point, no significant difference can be found between the two groups,  $t(144) = -.649$ ,  $p = .517$ . In the posttest, knowledge in the experimental group is  $M_{Post,EG} = .76$  ( $SD = .16$ ) and in the control group  $M_{Post,CG} = .63$  ( $SD = .17$ ). In order to assess the development of knowledge, the difference between the two measurement points is considered below. It can be seen that the development of domain-specific pedagogical content knowledge differs significantly between the groups,  $t(144) = 4.238$ ,  $p = .001$ ,  $d = 0.71$ . While knowledge in the experimental group significantly increases and does so with a mean effect size according to Cohen (1988),  $t(79) = 5.954$ ,  $p = .001$ ,  $d = 0.67$ , knowledge in the control group actually decreases compared to the pretest. However, the change in the control group is not significant,  $t(65) = -0.506$ ,  $p = .614$ .

## Discussion

Pedagogical content knowledge represents an important component of the professional competence of teachers (Baumert & Kunter, 2013). In particular, this also applies to the pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools (Gerber et al., 2022). In addition to an understanding of terms and knowledge of the potential of digital tools, this also includes knowledge of modelling cycles as a basis for the analysis of modelling processes. Based on the results of our study, we showed that pre-service teachers in the experimental group were able

to significantly increase their theory-related pedagogical content knowledge while no corresponding significant knowledge development could be demonstrated for pre-service teachers in the control group during a comparable period. Thus, the intervention can be considered effective. This suggests that the course design with its three-part structure (preparation/practical/reflection phase) was chosen wisely. Courses with a comparable conception regarding content and organisation thus put the prospect of promoting the theory-related knowledge of pre-service mathematics teachers forward.

At the same time, it has not yet been determined which elements of the course design were (particularly) responsible for the individual knowledge gains. Due to the design of our study, it could not be determined whether it was rather the practical phase or the pure teaching of theoretical content that favoured the gains. Therefore, we can consider the intervention *as a whole* to be effective but we are not able to describe individual elements as particularly effective.

Although it was possible to identify the theory-related knowledge dimension (Gerber et al., 2022) and to theoretically substantiate its significance for other knowledge dimensions regarding the content, this significance has not yet been empirically examined or quantified. An example of a content-related theoretical justification is that knowledge about modelling cycles helps when analysing working processes. This in turn provides the basis for diagnosing students' difficulties. Measurable correlations in the promotion of diagnostic activities and the gain in theory-related knowledge dimensions – i.e. effects in both directions – are therefore possible. In follow-up studies, this correlation between the theory-related dimension and further knowledge dimensions could be empirically investigated.

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