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Analysis of a mathematical modelling activity performed in the industry: The design of controllers used in durability tests and its didactical potential

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This research, framed in the Anthropological Theory of Didactics (ATD), is focused on the analysis of a mathematical modelling activity performed by engineers in a project integrator company. This activity is based on the control design of a durability test applied to windshield wipers motors. It is intended to show the mathematical models, how engineers use and adapt them, and how time is an important factor that forces them to decide certain routes in their making and specific ways to justify them. This analysis will allow, at a second stage, to transpose this professional mathematical modelling activity into a school activity to create a didactic proposal for training future engineers.

Keywords: Engineering education, mathematical modelling, control theory, project integrator company.

Introduction

The research focused on the mathematics used in workplaces has shown that engineers use it to solve problems by adapting well-known mathematical models (Bissell & Dillon, 2000). They use mathematics as a description and analysis language (Mustoe & Lawson, 2002). They get support from several information technologies (Kent & Noss, 2002), and they acquire techno-mathematical literacies (van der Wal et al., 2017). Likewise, it has been shown that the epistemology associated with mathematics used in the workplace has its own characteristics (Castela & Romo-Vazquez, 2023), which are determined by multiple aspects that can evolve over time. Thereby, it is necessary to analyse current engineering companies that allow us to understand the role of mathematical modelling in them. According to this, a projects integrator company from Mexico, which develop specific projects to attend to the necessities of automotive companies, has been analysed. This company employs highly trained engineers to guarantee the efficiency and perfection of a high number of automated processes, which suggest the use of control theory and hence the use of mathematical models (Romo-Vazquez, 2009). This led to questioning: which mathematical modelling activities are developed to automatise and control processes in these kinds of companies? To give an answer, a praxeology of mathematical modelling based on the control design developed for a torque test applied in motors used for windshield wipers has been analysed.

Some elements from the Anthropological Theory of Didactics

In the Anthropological Theory of Didactics (ATD), initially proposed by Chevallard (1999), any human activity could be analysed by an epistemological unique model called *praxeology*, from an institutional perspective. A praxeology $[T, \tau, \theta, \Theta]$ has four components: Type of task T , what must be done; the technique τ , the way the task could be done; the technology θ , the discourse that justifies the technique; and the theory Θ , which is a more general discourse that justifies the technology. Social stable organizations where human activities are developed, and which also provide resources to their

subjects to perform these activities are called *institutions*. Thus, Romo-Vazquez (2009) has identified three types of institutions, considering their relation to knowledge in the training of engineers; these are *scientific* or *producing institutions*, where knowledge or praxeologies are generated (e.g., engineering sciences); *teaching institutions*, which transmit praxeologies (e.g., engineering courses), and the *using institutions*, where praxeologies are used to solve practical problems (e.g., industry). Within institutions, *subjects* develop new praxeologies and/or adopt them from other institutions. If the latter is the case, the praxeology is transposed, considering the necessary conditions to reproduce this in the receiving institution. Especially when a praxeology is taken from any institution to be taught, the process followed is known as a *didactic transposition* (Chevallard, 1991). Different institutions and phases are involved when carrying out a didactic transposition. According to Gascón & Nicolás (2022), this can be schematized as shown in Figure 1.

As mentioned by Bosch et al. (2021, p. 142) “The process of selecting, adapting, organizing and

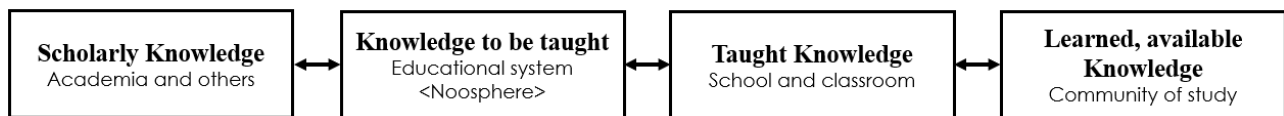


Figure 1: Didactic Transposition Process

declaring the *knowledge to be taught*, beginning from scholarly knowledge and ending with teaching materials proposed in a given course, is called *External didactic transposition (EDT)*”. Making an external transposition of industrial knowledge in order to bring it into the classroom is associated with a unit of analysis involving three institutions: the *industry* (instead of academia), the *educational system* (syllabus, math department) and the *school and classroom* (mathematics and engineering courses). A key step is to elucidate the industrial knowledge corresponding to a mathematical modelling praxeology used in industry to determine the EDT’s viability. That is, to determine the relevance of bringing it into the classroom. Castela & Romo-Vazquez (2023) have proposed six epistemic tasks to analyse mathematical modelling praxeologies used in the industry to recognise the way mathematics techniques produced or adapted there were justified under industry regulations, which could be different across disciplines. The epistemic type of tasks are classified into six categories: *describing* the technique, *explaining* its efficiency, *validating* it, which means guaranteeing that the expected result will be obtained when using this technique; *facilitating* its use, that is to consider the knowledge users should have to apply the technique efficiently; *appraising* the efficiency that its use presents in comparison with other techniques available; and *motivating* its stages to achieve objectives, this latter demands to explicit the goals that are expected to reach while using the technique and its effects and the consequences that will appear, although it does not require analytical explaining why the technique will allow reaching these objectives (Romo-Vazquez, 2009). The following sections will show the analysis of a praxeology of one of the many activities of engineers in the industry to identify the mathematical knowledge at stake.

Methodology

In this study, the unit of analysis comprises three institutions: a projects integrator company W (industry), Mexican engineering training (educational system) and mathematics and engineering courses (school). In this communication, we present the analysis of the company W. We focus on a

specific project that allowed us to analyse a mathematical modelling praxeology called: *Durability test applied to windshield wipers motors*. To make it intelligible, elements of the control theory are presented before the analysis.

The study of projects integrator company W

Project integrator companies, specialised in engineering, develop projects to meet the unique requirements of different industries such as automotive, aeronautic, food, manufacturing, pharmaceutical. Most of their activities are based on the design of product tests, the automation and control of processes, the manufacturing of electric and mechanic equipment, as well as software development. Consequently, their engineers are trained to solve multifaceted challenges. These companies provide specialised, cost-effective services and promote the economic development.

To analyse activities performed by engineers, we studied the project integrator company W, established in 2010 and which mainly attends to demands from the automotive industry¹. Following an initial approach, several types of tasks whose performance demand a deep understanding of electric motors performance, the design of controllers for velocity, position and acceleration, fundamental control theory principles and the mathematical models associated to them were identified. During a second approach, an automotive industry's demand was identified: the design of a durability test bench for motors used in windshield wipers. This test bench would cover a range of assessments, including a specialized torque test. To analyse how this demand was addressed, six interviews were conducted with Luis, a mechatronic engineer who was deeply involved in the project and who also provided us with several documents about client requirements, the proposed design, the equipment, and the programs used. Interviews were carried out by the first author, who is also an engineer, which allowed her to understand the project's development and decide to focus on the design of the controller, as this activity usually requires mathematical modelling. Once the reconstruction of the technique followed by the engineer's team was done, Luis evaluated it and confirmed its validity via this analysis.

Durability test applied to windshield wipers motors

The torque test was requested to company W considering the following scenario: the motor used in windshield wipers performs a swaying movement at a constant slow, medium, or fast velocity. It is considered that the motor will be exposed to natural loads like rain or snow and that these loads will apply a counterforce to the motor's movement. Due to the swaying movement, when the motor changes its direction, it needs enough torque to overcome the inertia associated with the direction change and the additional load applied by the natural load. All above occurs while the motor is exposed to temperature ranges from -40°C to 80°C. A multidisciplinary team, formed by five engineers with specialised profiles in mechanics, electronics, programming, documentation management and ensemble, was selected to develop this test. According to Luis' descriptions, the technique followed by the team has been reconstructed, which has the following two stages:

¹ Due to confidentiality, the real name of the company is not provided.

Stage 1. The characteristics and requirements the client asked for were analysed, recognising the type of tests to be developed and the information about the motor required for this. The necessary instrumentation was identified: sensors and actuators to perform tests and to collect data, as well as the type of architecture. To simulate the natural loads on the motor the team proposed to use a brake coupled to the motor. The resulting torque of the interaction between the motor and the brake would be measured by using a torque meter (sensor of torque), as shown in Figure 2.

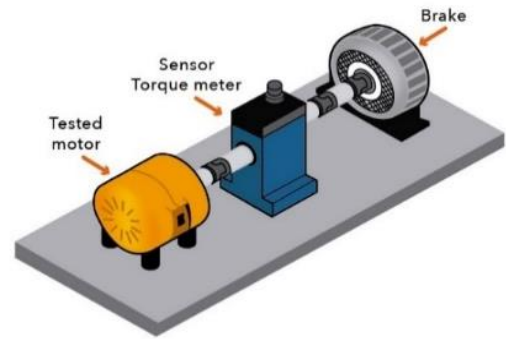


Figure 2: Test's connections

Stage 2. The mechanic, electronic and software design was done in order to implement the mechanic and electronic structure. Once the team was certain there were no failures, the implementation test was done. This means that the first version of the software was run for programming the motor's tests. During the programming phase, the team noticed that the motor's torque varied when the velocity changed. That is, if the motor's velocity increased, the torque generated by the interaction between the motor and the break decreased. Therefore, the break should be able to recognise this change in torque and adapt to keep the pre-set torque constant (the one which represents the natural load applied to the motor). To achieve this self-regulation, a controller was implemented. Once the programming was done, the complete system was assembled, and the final performance tests were done. To illustrate a part of this process, the praxeology of the controller's design is presented.

Mathematical modelling in designing a controller

In control theory, designing a controller is based on modifying the behaviour of a process or physical element, which is called *plant*, to reach a certain goal. This modification is achieved by using a controller that mathematically manipulates the *transfer function*, i.e., a mathematical model that characterises the input-output relationship of a linear time-invariant system, which is described by n^{th} order differential equations whose coefficients are real constants: $\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \dots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$. Where $y(t)$ is the output, and $r(t)$ is the input of the system. The transfer function $G(s)$ is described by the ratio $\frac{Y(s)}{R(s)}$ obtained by calculating the Laplace transform of both sides of the differential equation (zero initial conditions), i.e., $G(s) = \frac{Y(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$ see (Kuo, 1996).

Closed-loop control systems have several signals such as the input signal $r(t)$, which is the desired value; the output signal $y(t)$, which is the real value of the system response to the input signal; the controller signal $c(t)$ and the error signal $e(t)$, this latter corresponds to the difference between the desired value and the real value, i.e., $e(t) = r(t) - y(t)$ (see Figure 3). What is

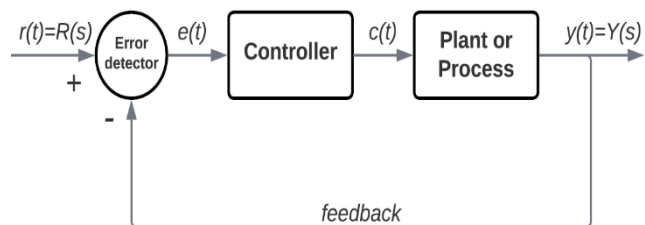


Figure 3: Block diagram of a closed-loop system

attempted to be done is that the controller $c(t)$ modifies the system's behavior for $y(t)$ to be equal to $r(t)$. Controllers modify the error signal by applying a combination of equations known as the *proportional action P*, where $c(t) = K_P e(t)$; the *integral action I*, where $c(t) = K_I \int e(t)dt$; and the *derivative action D*, where $c(t) = K_D \frac{de(t)}{dt}$. The constants K_P, K_I, K_D are known as *gains* and the process to assign values to them is called *tuning*. Classic controllers are based on a combination of these actions, for example, the Proportional-Integral-Derivative controller (PID), where $c(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$. The choice of the controller will depend on the desired design features.

Analysis of a praxeology based on the control design of a torque test

In the analysis of this praxeology, where the *type of task* consists in designing a brake's controller for durability tests applied to motors used in windshield wipers, this controller must ensure that the brake's behaviour, initially oscillatory, will be stabilised in a time set by the automotive company t_f . The *techniques*, developed by the engineer's team, were identified, and they will be described by means of the subtasks 1, 2, and 3 (Chaachoua et al., 2019), which constitute three strategies, more or less related, elements preserved from the preceding and modifications that originate the subsequent. The *technologies* associated with each of them were also identified. All the above is made explicit through the epistemic tasks of *motivating*, *appraising*, and *validating*.

Subtask 1. Choosing the controller. To select the type of controller, the digital tools that the company made available to the engineers were considered. The team streamlined this task by selecting a PID controller pre-approved by the company. Making use of this tool implied less invested time in the designing phase. At the same time, this decision also implied to set values to proportional, integral, and derivative gains. This latter *motivated* to obtain the system's transfer function; thus, it was necessary to develop the differential equation that characterised the system.

Subtask 2. Identifying the plant. To obtain the differential equation that characterised the plant, it was necessary to identify the system to be controlled. This activity required describing the set of sensors, actuators, and signals involved and how they would interact with each other. The input signal was proposed as the torque that the brake should keep constant when the motor was being tested, while the output signal would be the one obtained from the read value by the torquemeter, and which simultaneously would serve as feedback. In a first attempt to obtain the transfer function, the plant was described as the interaction between the brake and the motor. Hence, the equation should consider the behaviour of both elements. *Appraising* the characteristics of this potential model, the team reflected on the low adaptability that this presented. This means that several motor models would be tested in the same test bench, and if the plant were modelled based on the brake-motor interaction, the transfer function parameters would change with each new motor to be tested. Hence, the gains set to the controller would only work with the motor for which it was modelled. This latter *motivated* the team to modify the approach, and a new plant was considered. The new plant should be based on a fixed element that simultaneously was an actuator; the brake fulfilled both characteristics. With this change, the differential equation should characterize the brake's behaviour. However, during the modelling process, the team *appraised* the possible difficulty in obtaining the model and the time

restrictions of the project, which *motivated* them to adjust their approach and proceed without the transfer function.

Subtask 3. Tuning the controller. The theoretical methods used to tune a controller were dismissed without the transfer function. Nevertheless, the team identified that it was possible to experiment with the physical system. This *motivated* them to attempt an empirical tuning method. A set value for the gains proportional K_p , integral K_I and derivative K_D were proposed as $K_p = K_I = K_D = 1$. Subsequently, they considered that derivative action would not significantly influence the behaviour of the signal to be controlled and to propose a high value for this action could even make the control more difficult, hence, they fixed $K_D = 0.5$. For proportional and integral gains, several combinations were performed manually; however, it was ineffective due to the vast number of potential combinations along the time restrictions they were subjected to, which *motivated* them to change the strategy. They developed a program in LabVIEW² for testing combinations automatically, this executed combinations of the parameters while the motor was performing. To implement this strategy, fixing two of the three parameters was necessary, while the third would vary. Initially, and according to the precedent experience, $K_D = 0.5$ and $K_I = 1$ were fixed. Thus, the interval of values to determinate would be for K_p . Considering that integral action would reduce the oscillations presented in the signal when K_p was modified, K_I would be the last value to vary. Thus, the program iterated for K_p starting from $K_p=1$, adding 0.05 to this value in each iteration. A spreadsheet associated with the program saved the K_p value used, the time when the controlled signal reached the input signal t_r and the time in which the signal presented no oscillations t_e .

To exemplify the type of graphic the team analysed, we graphed the behaviour of a standard system. Figure 4 illustrates the oscillatory response of the system, showing a peak in the oscillations, which indicates the maximum value that oscillations reached. The sum of the times t_r and t_e , resulting in time t , represents the time the system's signal equals the input signal.

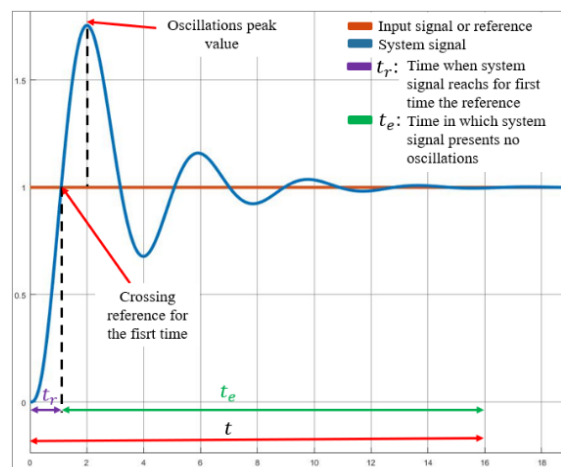


Figure 4: Times observed in signal

The program was run a second time, subtracting 0.5 to K_p in each iteration, starting from $K_p=1$. Once a set of values was available, the team used the commands of the spreadsheets to delimit the interval of the possible numeric values for K_p considering the ones whose sum of times t_r and t_e was close to the time required t_f for the client. The engineers concluded that t_e could be reduced when K_I was modified while t_r would not be reduced more; thus, priority was given to the K_p values where $t_r < t_e$. According to these criteria, a set of fifteen possible values for K_p was obtained. Each one of this fifteen values served as a set point for iterating K_I . A new set of combinations was obtained from which values where $t_r + t_e > t_f$ were excluded. Combinations where the peak value of oscillations

² Graphic programming environment used in engineering to develop software.

was out of the error range allowed for the client, were also excluded. Finally, the combination of values obtained for the gains of the controller was fine tuned manually. The tuning was *validated* when the values of K_P , K_I , y K_D managed the system for it to follow the input signal. At the same time, the value of the error of the controlled signal was within a range of three percent from the reference value.

According to the classical control theory, subtask 1 should have consisted of obtaining and analysing the characteristics of the system's transfer function to determine whether the system can be controlled and the appropriate controller type. In this case, it can be observed that the engineer's team initially proposed the type of controller based on the assumption that the system's transfer function was known (which defines the behaviour of the brake coupled to the motor). They accessed it empirically by running the motor. Likewise, tuning methods used in control theory are based on the study of the functions of the numerator and denominator that form the transfer function and the modification in the behaviour that each action K_P , K_I , K_D , would do on these functions. This means that tuning by theoretical methods, based upon the transfer function, allows direct control of variables, times of stabilization, and even the error ranges, subtasks done here by trial-and-error methods but supported by theoretical and practical knowledge.

Conclusion

The epistemic tasks, motivating, appraising, and validating, allowed us to show how the three strategies considered by the engineers' team were happening. Furthermore, we demonstrate how theoretical and empirical knowledge was combined to obtain an optimal solution for the automotive company. The decision to tune the control while the motor was running is associated with a parameter calculation job supported by a spreadsheet. It may seem simple, but it is supported by a high number of theoretical and empirical knowledge, just like engineers' creativity. This analysis allowed us to show this, at least partially, by considering as reference mathematical and control theory elements related to the theoretical modelling of plant control. Also, this has revealed factors that are important in the day-to-day decision-making of engineers in industry, for example, time, which we tend to overlook but may be interesting to make students aware. Determining the tasks and techniques involved in this industrial praxeology make it possible to assess which will be necessary to maintain and which will have to be adapted while performing the didactic transposition. In addition, the analysis of curricula and university courses directly related to this industrial activity (e.g. courses in differential equations or control theory) will show how this praxeology can be best brought into the university. Performing this didactic transposition allows the creation of links between the mathematics used by engineers in their workplace and the mathematics taught in the training of future engineers. This may improve future engineers' performance in modelling and solving control-related problems at their future workplaces.

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