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# High school students' interpretation when solving a MEA in the context of water scarcity

Dinorah Méndez-Huerta<sup>1</sup>, Verónica Vargas-Alejo<sup>1</sup> and Luis E. Montero-Moguel<sup>2</sup>

<sup>1</sup>Universidad de Guadalajara, Mexico; [dinorah.mendez7682@alumnos.udg.mx](mailto:dinorah.mendez7682@alumnos.udg.mx)

<sup>2</sup>The University of Texas at San Antonio, United States

*This research describes the results of the implementation of a Model Eliciting Activity related to the problem of water scarcity in the state of Puebla, Mexico. The activity was designed as part of a qualitative research. The activity was implemented with a group of ten high school students. The theoretical framework was based on Models and Modeling Perspective. The results indicate that the MEA allowed the students to develop two interpretations during the model construction process.*

*Keywords: Model-eliciting activity, interpretation, proportions.*

## Introduction

This investigation describes the results of the implementation of a Model Eliciting Activity [MEA] (situation close to the real life) associated with one of the problems that has currently become relevant around the world, water scarcity. According to the United Nations Educational Scientific and Cultural Organization (UNESCO, 2018), it is necessary to address the problem of clean water and sanitation, since among other issues it is known that “approximately 70% of all water withdrawn from rivers, lakes, and aquifers is used for irrigation” (par. 17) so we need a mathematical teaching tool to empower students' abilities to play a role in creating a more sustainable world (UNESCO, 2017). Based on the Models and Modeling perspective [MMP] (Lesh & Doerr, 2003), a MEA was designed – called ¡El Día Cero! [Day Zero! MEA] – as part of a qualitative research whose objective was to encourage students to learn mathematics while solving problems close to real life. The Day Zero! MEA was implemented with a group of high school students. The research question that guided this study was: what is the process that students followed to build the model when solving the Day Zero! MEA? The focus was to explore the interpretations of high school students while developing the model. In this proposal, we define interpretation as the personal conception, organisation, and/or expression of the context, problem situation, and questions presented in the MEA.

## Theoretical framework

This research is based on MMP (Brady & Lesh, 2021; Lesh, 2010). For the MMP, learning mathematics is a process of model development, which involves a series of cycles. In that process, student's ways of thinking are constantly expressed, tested, and revised through interactions with other students and the teacher when solving problems. A model is defined as the:

conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviours of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently.

A mathematical model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems. (Lesh & Doerr, 2003, p. 10)

To foster the development of models, the MMP proposes activities that involve problems close to real life, which are called MEAs (Makar et al., 2020). According to Lesh and Doerr (2003), problem solving may occur through multiple cycles of interpretation and refinement: “the mathematical interpretation of a problem is not trivial, solutions typically involve several modelling cycles in which descriptions, explanations, and predictions are gradually refined, revised or rejected –based on feedback from trial testing” (p. 18).


The students create several answers and analyse them to choose the ideal one according to the purposes that the problem requires. When students engage with MEAs, they are presented with complex problems that require multiple cycles of interpretation and refinement. Collaborative work and communication play a fundamental role in helping students expand their conceptual system (Zawojewski & Carmona, 2001). During the initial model construction process, students develop multiple initial interpretations of the MEA. The focus of this proposal was to explore the interpretations of the students while developing their model.

## Methods

The research approach is qualitative. The participants were 10 students aged 16-18 years old who were in their 5th semester of high school in Puebla, Mexico. The MEA (Figure 1) was designed based on the six principles described by Lesh et al. (2003). The context involved a dam located near the students’ town, so they could understand the situation posed. Linear function and proportion concepts were implicitly involved.

Sonia read the news on day zero! She is very worried about this situation because her father is a farmer and depends on the supply from the dam, she thinks that something similar to Monterrey could happen and she wants to know how the lake will behave with the constant exploitation of the water.

How will water from the Valsequillo dam decrease due to supply to Irrigation District 030 over the years? When will the dam stop supplying the field (Distrito de Riego 030)?



Help Sonia, write her a letter where you explain a procedure to find answers to her questions and that will serve to predict the period of supply that any lake or dam can give to any city or irrigation district in the face of constant exploitation.

**Figure 1: Model Eliciting Activity problem**

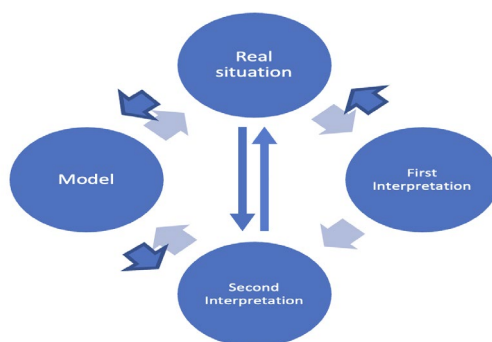
The activity was implemented in two zoom sessions with a duration of 60 minutes each. In the first session the students, organised in two teams, solved the “Day Zero! MEA”. In the second session the models built were discussed and the closing was done. The data collection included video recordings, the student’s letters, as well as the researcher’s logbook. The process of the evolution of students’ interpretations in solving the “Day Zero! MEA” was analysed.

To answer the research question, the researchers conducted qualitative research with a deductive strategy. “In deductive strategy, the researcher has some a priori orienting constructs and propositions to test or observe in the field. These analytic units are operationalized and then matched with a body

of field data” (Miles et al., 2014, p. 211). In the first cycle of analysis, the researchers used the following guiding preposition: the “descriptions, explanations, and constructions [...] ARE the most important components of responses [models]” (Lesh & Doerr, 2003, p. 3). This proposition was extracted from the MMP about the process that students develop during the construction of the model. Subsequently, two researchers developed an iterative process to analyse and code the descriptions, interpretations, and mathematical processes of the students. To ensure the validity, the third researcher reviewed the analyses of the first researchers. Finally, the researchers discussed the discrepancies based on the MMP until reaching a consensus. The next section reports the results of the analysis.

## Results

During the resolution process of “Day Zero! MEA”, the students exhibited two interpretations prior to the construction of the initial model (Figure 2). The first interpretation focused on making sense of the situation of water scarcity in the dam; then the students modified this first interpretation and created a new one addressed to describe, explain, and predict the period when the dam would run out of water. This second interpretation led them to create initial models. The letters built by the students clearly reflect how they systematised the ideas embodied in the two interpretations. The emphasis was mainly on answering the second question posed in the MEA. Through the process, the students expressed, revised, and refined their ways of thinking by interacting with their peers in the corresponding teams. The interpretations and their evolution are described below.



**Figure 2: Interpretation cycles to solve “Day Zero! MEA”. Adapted from Lesh and Doerr (2003, p. 18)**

### First interpretation

It was observed that when the students read the “Day Zero! MEA”, they thought that the dam supplied water to the community and, therefore, expressed that the shortage depended on the use of water by the population. This is seen in the following excerpts, where each team described and explained how they interpreted the situation.

- Team 1: The shortage depends on how many people there are; knowing this we can put together a plan.
- Team 2: According to the misuse that we give to that service... for example, we can know what is the amount of water that a person can use during a day.

The students were concerned about the water shortage phenomenon and even suggested preventive measures to regulate water consumption, as shown in the following excerpts.

- Team 1: We must have a regulated consumption and use water only for basic needs.

Team 2: Because of the bad use that is being made of the water, it is going to run out extremely fast.

## Second interpretation

The interaction between team members, and the teacher led the students to modify their first interpretation. The students realised that the dam irrigated the fields, and they began to “mathematize” (in terms of Lesh and Doerr, 2003) the situation. Mathematical descriptions and explanations associated with the lack of water supply emerged, as shown in the following excerpts.

Team 1: It will decrease due to irregular consumption by farmers or planters.

Team 2: It is not the same use that I give it as a farmer who is going to use it as a means of irrigation for his crops.

Based on this second interpretation, both teams tried to describe and explain the phenomenon. Team 1 attempted to answer the first question: How will the water in the Valsequillo dam decrease after supplying Distrito de Riego 030 over the years? They considered that the same amount of water was not spent to irrigate the different products produced in the region.

Team 1: How will water decrease? Well, it is based on knowing what quantities of water reach the farmers.

Teacher: How do you know how much water reaches the farmers?

Team 1: It depends. Some plants need more water than others, it also depends on the time it takes for the plants to grow.

In their speech, Team 1 used lettuce farming as an example of the water consumption that might be required for irrigation and identified the influence of three variables on the emptying of the dam: river, climate, and water consumption. The team generated three assumptions: 1) Atoyac River does not supply the dam, 2) Drought, 3) Only lettuce is planted.

Based on these assumptions and using proportions, the team suggested a procedure to determine the specific time in which the water in the dam would run out to answer the second question: When will the dam stop supplying the field (Distrito de Riego 030)? Team 1 explained step by step how to proceed, without including specific data or mathematical procedures.

Team 2, on the other hand, made the following assumption to answer the question: When will the dam stop supplying the field (Distrito de Riego 030)?

Team 2: There are 3 to 6 litres of water per square meter in a temperate climate, which is more or less the climate we have in this area of Puebla...if we multiply 10,000 square meters by an approximate 4 litres, we will know how much water is used to irrigate a hectare.

With this assumption, the team made operations without any order in the sheet to estimate the amount of 40 000 litres of water needed to irrigate one hectare. With this data and considering the total capacity of the dam (404 hm<sup>3</sup>), not the actual amount of water contained in the dam, they determined how many hectares the 404 hm<sup>3</sup> was sufficient for. They found that it was sufficient for 10 100 000 hectares.

Team 2: This would only be enough in the event that water would no longer be received from other sources... from the river and from rainfall, which are the most important sources that supply this dam.

Perhaps due to lack of time the students did not complete their procedures.

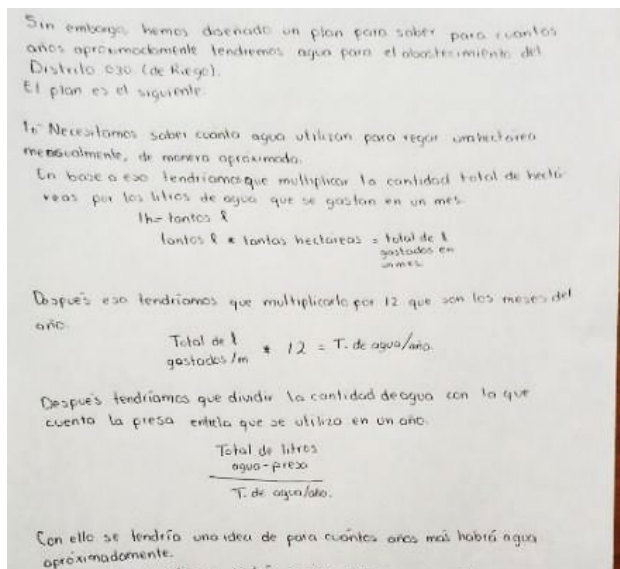
### Models

The new interactions between the teams' members with the MEA and teacher led them to refine their procedures and construct letters, in which they included their models. They improved their descriptions, and estimated the specific time during which the dam could supply the field.

#### Team 1 model

In response to the first question posed in the "Day Zero! MEA": How will the water in the Valsequillo dam decrease after supplying the Distrito de Riego 030 over the years? The team referred to the fact that it is not possible to know exactly the specific time when the dam will stop supplying the field. To answer the second question, the team proposed a general model (Figure 3), situated in terms of Lesh (2010), to calculate the specific time in which the dam will stop supplying water.

The team suggested multiply the amount of water per month needed to irrigate one hectare by the total number of hectares to be irrigated; the purpose was to obtain the total amount of water spent on all hectares per month. They multiplied this amount by 12, which are the months of the year. Finally, they used this amount as a divisor of the amount of water available in the dam.



However, we have designed a plan to know for approximately how many years we will have water to supply Distrito de Riego 030.

The plan is as follows:

We need to know how much water they use to irrigate one hectare monthly, approximately. With this we would have to multiply the total amount of hectares by the liters of water used in a month.

$$1h = \text{quantity of } l$$

$$\text{quantity of } l * \text{quantity of hectare} = \text{total of } l \text{ spent in a month}$$

Then, we would have to multiply by 12 which are the months of the year.

$$\text{Total of } l \text{ spent in a month} * 12 = T. \text{ of water/year}$$

Then we would have to divide the amount of water available in the dam by the amount of water used in a year.

$$\frac{\text{Total of liters water - dam}}{T. \text{ of water/year}}$$

With this we would have an idea of approximately how many years there will be water for.

Figure 3: Extract of the Team 1 model

#### Team 2 Model

The team proposed the amount of 40 000 litres of water to irrigate one hectare and multiply this amount by the number of hectares (20 619 h) supplied by the Valsequillo dam. The purpose was to know the amount of water they needed to irrigate the hectares that belong to Distrito de Riego 030 (Figure 4).

The team considered that farmers irrigate every day all the hectares and divided the amount of water in the dam by the amount of water needed by the 20,619 h of District 030. The students used 266.64 hm<sup>3</sup>, which is the actual water in the dam and not the 404 hm<sup>3</sup> used previously. They obtained that in 323 days the dam will run out of water under the initial conditions.



Es necesario tomar en cuenta la cantidad total de agua que contiene la presa, de acuerdo a los datos registrados son 404 hm<sup>3</sup>, que realizando la equivalencia serían 404 000 000 000 litros de agua.

Realizando una investigación se comprendió que la presa solo está llena en un 66% de su capacidad total. A lo cual equivale con 266.64 hm<sup>3</sup>, que esto sería igual a la cantidad de 266 640 000 000 litros de agua.

Después se analizó cuántos litros se llevaría aproximadamente por cada metro cuadrado de tierra de cultivo. Y haciendo un estimado de esto, se aproximó que son 4 litros de agua por metro cuadrado por riego.

Haciendo el cálculo y considerando que por hectárea son 10 000 m<sup>2</sup>, se utilizarían 40 000 litros de agua por hectárea por cada riego.

Tomando en cuenta que por día se realiza un riego y abasteciendo a 17 municipios irrigando una superficie agrícola de aproximadamente 20,619 hectáreas.

Para abastecer las 20,619 hectáreas es necesaria la cantidad de 824 760 000 litros de agua, tomando como referencia que es necesario 40 000 litros de agua por hectárea.

Considerando que todos los agricultores ocupan esa cantidad día con día, irá disminuyendo.

Tomando como referencia que los agricultores decidan regar al mismo tiempo.

Realizando una operación matemática, se dividirá la capacidad entera de la presa que es de 266 640 000 000 litros de agua entre la cantidad total de agua que se necesita para abastecer las hectáreas de cultivo de este distrito que es de 824 760 000 litros de agua. Que nos daría como resultado la cantidad de 323 días.

It is necessary to take into account the total amount of water contained in the dam, according to the data recorded there are 404 hm<sup>3</sup>, which would be equivalent to 404 000 000 000 liters of water.

An investigation revealed that the dam is only 66% full of its total capacity. This is equivalent to 266.64 hm<sup>3</sup>, which is equal to 266,640,000,000 liters of water.

Then it was analyzed how many liters would be carried approximately per square meter of cropland. And making an estimate of this, it was approximated that it is 4 liters of water per square meter per irrigation.

Making the calculation and considering that per hectare there are 10,000 m<sup>2</sup>, 40,000 liters of water per hectare would be used for each irrigation.

Taking into account that one irrigation is performed per day and supplying 17 municipalities irrigating an agricultural area of approximately 20,619 hectares.

To supply the 20,619 hectares, 824,760,000 liters of water are needed, taking as a reference that one hectare needs 40,000 liters of water.

Considering that all farmers occupy that amount day by day, it will decrease.

Taking as a reference that the farmers decide to irrigate at the same time.

Performing a mathematical operation, we will divide the entire capacity of the dam, which is 266,640,000,000 liters of water, by the total amount of water needed to supply the hectares of crops in this district, which is 824,760,000 liters of water. That would give us as a result the amount of 323 days.

**Figure 4: Extract of the Team 2 model**

## Discussion of results

We agree with Lesh and Doerr (2003), that solving a MEA may involve several interpretations. “Day Zero! MEA” provoked students to go through two interpretations (Table 1) to build the model (Figure 2). In the first interpretation, the teams associated the water shortage of the Valsequillo dam with the water supply for the population. They proposed strategies for the adequate use of water. Through the negotiation of ideas between the students and re-reading of the “Day Zero! MEA”, they realised that the dam is intended for irrigation of District 030. In the second interpretation, the teams made assumptions to describe the situation; they began to mathematize in terms of Lesh and Doerr (2003); for example, they established the amount of water needed to irrigate one hectare. Finally, the students systematised and refined their interpretations and explanations, they wrote the letter to present their models to solve the problem. Both teams concluded that the inadequate use of water, not only in irrigation, is of vital importance because we are exposed to the lack of this natural resource. The teams were worried about the situation and tried to give recommendations.

The students not only developed mathematical knowledge and skills but also generated proposals for the proper use of water, thereby enabling a more extended period of water availability for agricultural purposes. This is a crucial source of income for students’ families. According to UNESCO (2017), the students not only developed knowledge and skills but also learned “how to contribute to a sustainable transformation of society” (p. 19).

**Table 1: Teams interpretation when solving “Day Zero! MEA”**

Team	1 <sup>st</sup> Interpretation	2 <sup>nd</sup> Interpretation	Model
1	-The context was misunderstood.	-The team redirected their interpretation: The dam supplies the field. -The team tried to answer the two questions posed in the “Day Zero! MEA”.	Team 1 model: situated and reusable (in terms of Lesh, 2010). Answer to first question:

	The dam supplies water to the population, so preventive measures are required.	Using certain initial assumptions and proportions, a general procedure was constructed without numerical data to determine the specific time in which the Valsequillo dam would run out of water.	It is not possible to estimate how long the dam will stop supplying the field: “the water will decrease based on the exploitation given to the dam”. By means of a letter the team explained a general situated model to answer the second question. The model based on the refinement can be reused for different types of production.
2		-The team redirected their interpretation: The dam supplies the field. -The team tried to answer only the second question posed in the “Day Zero! MEA”. Using initial conditions (the amount of water needed to irrigate one square meter: 4 litres of water) a procedure was started to estimate when the Valsequillo dam would run out of water.	Team 2 model: situated. Answer to second question: In 323 days, the dam will stop supplying the field. By means of a letter they explained step by step the answer to the second question. The team built a situated model that depends on the context of the situation.

## Conclusions

According to the interpretations, whose evolution gave rise to the models that emerged, the following can be mentioned. The first interpretation allowed the teams to understand that the amount of water in the dam was decreasing and this depended on its use. However, the descriptions and explanations they gave were associated with water use by the population. Their interpretations were reoriented as they re-read the problem. This is consistent with Lesh and Doerr’s (2003) findings that first ideas are often crude. In the second interpretation, students explained the water shortage through certain assumptions or initial conditions. Students generated conjectures and tried to answer the second question posed in the task. Finally, team 1 managed to identify that there were two questions in the “Day Zero! MEA”, unlike team 2, only answered the second question. Team 2’s model is considered situated in terms of Lesh (2010); it means that it depends on the context of the situation. Team 1’s model is also situated, but in addition, it can be reused for different types of production.

The “Day Zero! MEA” was an activity that produced interest among the students, resulting in two interpretations that made it possible to answer the second question posed in the MEA, and where the constant modification of their thinking was observed. This contributes to the findings of Brady and Lesh (2021) according to which “in privileging realistic problem settings, the MMP aims to place students in the role of bricoleurs, making sense of settings by ‘making-do’—innovating by piecing together known mathematical structures to fit the bill” (p. 96). As to our future work, we plan to implement a Model Exploration Activity, supported by technology (SimCalc MathWorlds), to help students in further developing, enriching, and refining their comprehension of the linear function



concept included in the models built. This includes exploring its associated meanings, such as variation and covariation.

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