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# An educational experience on modelling for in-service mathematics teachers from the Panamanian school system

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*We report the preliminary results of an educational experience on modelling for in-service secondary education mathematics teachers from the Panamanian school system. The context is the Diploma Course in Mathematics Education Applied to Secondary Education, taught by the University of Panama, which includes a submodule on modelling. Using the modelling cycle taught in the submodule, we analysed the solving procedures to three problems posed to the teachers, in which we made evident their tendency to consider the problem as solved when they obtained mathematical results from the mathematical model, without the need to interpret or validate these results in the real situation context. With these preliminary results, we intend to provide a first overview on the teaching and learning of modelling in the Panamanian context for further teacher professional development.*

*Keywords: Diploma course, in-service teachers, mathematical modelling, professional development.*

## Introduction

One of the competencies that allows individuals to be able to link their mathematical knowledge to the needs and demands of the 21st century is mathematical modelling (Maass et al., 2022). In this sense, it is necessary to prepare teachers with the adequate competencies for modelling and, in this way, educate students in skills to develop this process (Blum & Borromeo Ferri, 2009). Due to this relevance, several studies have reported different educational experiences on the teaching of modelling for both pre-service and in-service teachers. For example, Borromeo Ferri (2018) and Greefrath et al. (2022) stress the importance of promoting modelling competencies in teachers as key elements for working with this type of problems, through the solving and creation of new problems.

In the case of our study, we report on a pioneering educational experience with in-service mathematics teachers from Panama. The school education system in Panama consists of General Primary Education (students aged 4–15) and Secondary Education (students aged 15–18) levels. In this context, the University of Panama, in collaboration with foreign universities, implemented two diploma courses for in-service teachers of both educational levels in 2022: «Didactic Strategies for the Teaching of Mathematics» (EDEM, its acronym in Spanish) for General Primary Education, and «Mathematics Education Applied to Secondary Education» (EMAS) for Secondary Education. The objective of both diploma courses is to broaden the mathematics teachers' pedagogical skills. This research focuses on the EMAS Diploma Course, where modelling was addressed.

The relevance of our study lies in two facts. On the one hand, we address a topic that has been little explored in the Panamanian context, such as the in-service mathematics teachers' education on modelling. Even though the national curricular documents of Panama do not include a systematic work with modelling for the mathematics teaching, the EMAS Diploma Course included this process

as a relevant topic to be taught. On the other hand, derived from the latter, the University of Panama (which taught these diploma courses) is considered a benchmark in teaching and research at the Central American level (Morales-Maure, 2019), so this educational experience can be replicated in other countries of the region due to sociocultural similarities.

Under these circumstances, our study intends to provide a first overview of a pioneering educational experience on modelling for in-service mathematics teachers from the Panamanian school system, guided by the question: What are the solving procedures to modelling problems of the in-service mathematics teachers involved in the EMAS Diploma Course? To this end, we use a modelling cycle to analyse the solving procedures of three problems posed to the teachers during a submodule of this diploma course, and we conclude by reflecting on these preliminary results.

### Mathematical modelling

Mathematical modelling briefly describes the translation of a real problem into mathematics and the translation of the results back into reality (Pollak, 2007). In the specialised literature, different cycles have been proposed to analyse the modelling process (Borromeo Ferri, 2006) and different perspectives on its implementation have emerged (Preciado et al., 2023). This study adopts the «modelling cycle from a cognitive perspective» (see Figure 1), proposed by Borromeo Ferri (2018), which is framed within the «realistic perspective» of working with modelling (Abassian et al., 2020).

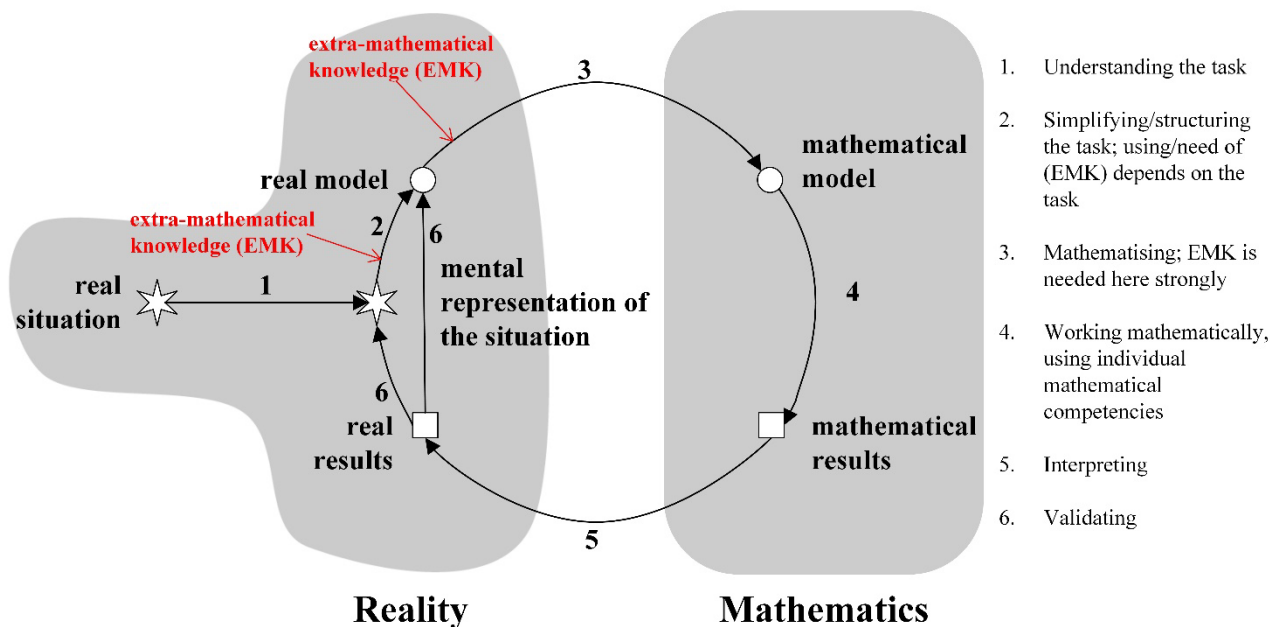


Figure 1: Modelling cycle from a cognitive perspective (Borromeo Ferri, 2018, p. 15)

The choice of this cycle is justified by (a) our previous experience in theoretical work (see Ledezma et al., 2022) and (b) its use is part of the EMAS Diploma Course. To explain how the cycle in Figure 1 works, we use the *Sugarloaf Problem* as an example, since we presented it in the diploma course.

*Sugarloaf Problem:* The Sugarloaf cableway tasked approximately three minutes for its ride from the valley station to the peak of the Sugarloaf Mountain in Rio de Janeiro. It runs with a speed of

30 km/h and covers a height difference of approximately 180 m. The chief engineer, Giuseppe Pelligrini, would very much prefer to walk – as he did earlier on, when he was a mountaineer, and first ran from the valley station across the vast plain to the mountain and then climbed it in 12 minutes. How big is the distance, approximately, that Giuseppe had to run from the valley station to the foot of the mountain? (Source: Adapted from Blum & Leiß, 2007, p. 224)

The *real situation* is given by the wording of the *Sugarloaf Problem*, in the form of a written text. By analysing the problem, we can form a *mental representation of the situation*, which involves, for example, establishing relationships between holidays and tourist sites, in order to understand what is required to solve the problem. We must simplify and structure this *mental representation* to obtain a *real model* that represents the *real situation* posed; in this case, we can simplify the mountain and the cable as segments, and the cableway as a point, and then structure these simplifications in a drawing. The *mathematical model* considers the mathematical objects that allow explaining the *real situation* posed; in this case, we can apply the Pythagorean theorem. By working with this *mathematical model*, we can obtain *mathematical results*, which we must interpret in the context of the *real situation* to obtain *real results*; in this case, we approximately obtain 1.49 kilometres. Now, does this answer make sense in the context of the *Sugarloaf Problem*? One way to validate these *real results* would be to use a map application to measure distances at the real location of the Sugarloaf in Brazil.

## **Methods**

In this study, we followed a qualitative research methodology from an interpretative paradigm (Cohen et al., 2018), and in this section, we describe its methodological aspects.

### **Research context**

We conducted this research in the context of the EMAS Diploma Course, taught by the University of Panama during May–October 2022, with a total duration of 320 hours. The objective of this diploma course is to contribute to the continuous teacher professional development of Panamanian secondary education mathematics teachers, which includes the design, implementation, assessment, and improvement of mathematical teaching and learning processes. In the EMAS Diploma Course there were 113 participating teachers from different Panamanian regions, who were grouped into the four virtual rooms of the platform designed by the University of Panama. This programme consisted of six modules taught in hybrid mode: (a) Introduction to Mathematics Education; (b, c, d) Didactics of Mathematics I, II, and III; (e) Social, familiar, and educational contexts; and (f) Reflection on own practice. At the end of this programme, the participating teachers received a certificate for completing the course. The EDEM and EMAS Diploma Courses are pioneering educational experiences in the Panamanian context, which are not only supported by a government research project awarded by the University of Panama, but also by scholars from foreign universities in Spain and Hispanic America.

### **Submodule on mathematical modelling and modelling problems**

Within the «Introduction to Mathematics Education» module there was a submodule on «Mathematical modelling», which consisted of two sessions. In the first session (lasting 90 minutes), the presenter (the first author) began explaining to the participating teachers what is meant by mathematical modelling, what characterises this type of problems, what work strategies with this

process are suggested to follow in the classroom, and how the solving of a modelling problem can be analysed using the cycle from a cognitive perspective (explained in the previous section). After the presentation, the participating teachers had to individually solve (due to the hybrid format of the diploma course) the following modelling problems posed by the presenter:

*Bales of Straw Problem:* At the end of the summer one can see a lot of bales of straw. Bales of straw on the picture are piled up in this way that in the bottom line are five, in the next four, then three, then two and on the top one ball. Try to find out, how high this mountain of bales of straw is. (Source: Adapted from Borromeo Ferri, 2018, p. 14)

*Meanders Problem:* On the Yamal Peninsula in north-western Siberia, a series of active and abandoned meanders in highly sinuous rivers can be seen from the air. In light colour are the most recent sediments deposited in the convex parts of the meanders. What is the approximate length of the river with sediments? (Source: Authors' archive)

*Boston Light Problem:* In the bay of Massachusetts, a lighthouse called "Boston Light" was built in 1716, measuring 31 metres (102 feet) in height. Its beacon was meant to warn ships that they were approaching the coast. How far away, approximately, was a ship when it saw the lighthouse for the first time? (Source: Adapted from Borromeo Ferri, 2018, p. 106)

Although these modelling problems were started to be solved during the first session of the submodule, due to time constraints, the participating teachers were allowed to solve them as homework for the second session. To this end, the participating teachers had to upload their solving procedures and answers to the virtual platform designed for the EMAS Diploma Course. In the second session (lasting 30 minutes), the same presenter gave collective feedback on the solutions for the three problems, explaining one possible solution for each problem.

### **Analysis of the participating teachers' productions**

Since the authors of this proposal are scholars involved in the EMAS Diploma Course, we had full access to the virtual platform where the participating teachers uploaded their solutions and answers for the three problems posed as written productions. Our study focuses on the analysis of these productions using the modelling cycle considered as a theoretical reference. In this way, we can have a first overview of the preliminary results of this pioneering educational experience in the Panamanian context.

### **Preliminary results**

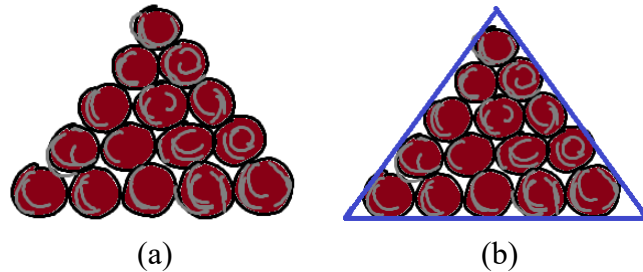
In this section, we present the preliminary results of this study, by illustrating some representative productions of the participating teachers (labelled from P<sub>1</sub> to P<sub>5</sub>) for the problems posed.

#### **Bales of straw problem**

Since there is no measurement of the diameter of a bale of straw, we use 1 bale as a standard reference. So, the mountain of bales of straw will be 5 bales high. (P<sub>1</sub> production)

The P<sub>1</sub> production shows a fairly obvious answer to the problem but does not make evident a complete understanding of the task or the extra-mathematical aspects of its wording. This production does not include any solution or representation of the situation.

The shape of the mountain is an equilateral triangle [see Figure 2]. With the sides of 5 bales, that is,  $l = 5$  bales, and with the formula of an equilateral triangle [sic] we have:  $h = \left(\frac{\sqrt{3}}{2}\right)l$ ;  $h \cong 4.3330127019$  bales. (P<sub>2</sub> production, translated by the authors)

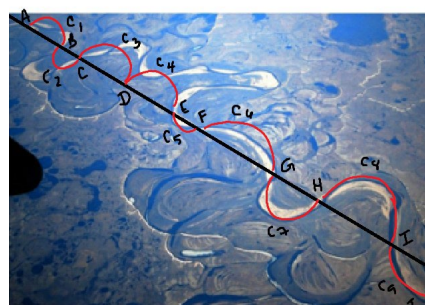


**Figure 2: Representations included in P<sub>2</sub> production (authors' archive)**

In contrast to the P<sub>1</sub> production, the P<sub>2</sub> production makes evident a reflection on the wording of the task. The solver set up a *real model* of the situation (Figure 2a) and simplified it as an equilateral triangle (Figure 2b). The *mathematical model* for the problem was the formula for calculating the height of an equilateral triangle ( $h = \left(\frac{\sqrt{3}}{2}\right)l$ ). However, the solution ended with the *mathematical results* that the *mathematical model* produced, without their interpretation in the context of the *real situation*.

### Meanders problem

A mathematical expression can be established, based on the arcs of circumferences, which allows us to obtain the length of the river. That is, for each of them, the measurement of the angle is 180° and we use the expression:  $L = \frac{2\pi r\theta}{360^\circ}$ . Thus, by applying it to each semicircle of the previous figure [Figure 3] we have:  $L_{AB} = \pi r_1$ ;  $L_{BC} = \pi r_2$ ;  $L_{CD} = \pi r_3$ ;  $L_{DE} = \pi r_4$ ;  $L_{EF} = \pi r_5$ ;  $L_{FG} = \pi r_6$ ;  $L_{GH} = \pi r_7$ ;  $L_{HI} = \pi r_8$ ;  $L_{IJ} = \pi r_9$ . Therefore, the length of the river is given by:  $L = \pi(r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7 + r_8 + r_9)$ . (P<sub>3</sub> production, translated by the authors)



**Figure 3: Representation included in P<sub>3</sub> production (authors' archive)**

In the P<sub>3</sub> production, the solver began by setting up a *real model* of the situation (Figure 3), in which the meanders of the river were simplified as semicircles around a line segment that represents the linear length of the river. The *mathematical model* for the problem was the sum of perimeters of each semicircle. However, as in the P<sub>2</sub> production, the solution ended with the *mathematical results* that the *mathematical model* produced, where real values were not assigned to the radii of the semicircles.

## Boston light problem

In the following scheme we have [Figure 4]. By using basic trigonometry [*sic*] we have:  $\tan \theta = \frac{31 \text{ m}}{x} \Rightarrow \frac{31 \text{ m}}{\tan \theta}$ . This would give us an idea on how distant a ship is, based on the elevation angle. (P4 production, translated by the authors)

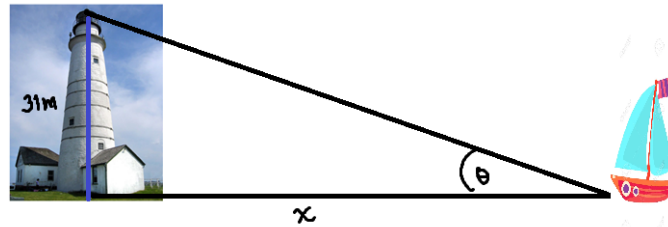


Figure 4: Representation included in P<sub>4</sub> production (authors' archive)

In the P<sub>4</sub> production, the solver began by setting up a *real model* of the situation (Figure 4), in which the distance between the lighthouse and the ship was simplified, forming a right angle. Unlike P<sub>2</sub> and P<sub>3</sub> productions, the solution of P<sub>4</sub> of the problem ended with the construction of a *mathematical model* (tangent trigonometric ratio) from which no *mathematical results* were obtained.

The only data provided by the problem is the height of the lighthouse, which is 31 metres [Figure 5]. We investigated what the range of the lighthouse is and, according to Wikipedia ([https://en.wikipedia.org/wiki/Boston\\_Light](https://en.wikipedia.org/wiki/Boston_Light)), we have:  $x = \sqrt{(50000)^2 - (31)^2} = 5000$  metres or 5 km [*sic*]. So, approximately, a ship can be 5 km away when it sees the light of the lighthouse for the first time. (P<sub>5</sub> production, translated by the authors)

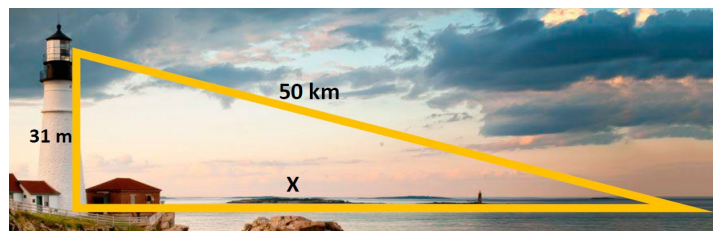


Figure 5: Representation included in P<sub>5</sub> production (authors' archive)

During the submodule, the presenter commented that, since a modelling problem does not always provide all the information necessary for its solving, we could resort to other sources to find or complement data, or even to validate our *real results*. In this sense, the P<sub>5</sub> production presents a particular case, in which the solver used Wikipedia as a source to obtain real data from the Boston Light. However, the solution with the Pythagorean theorem was not correct.

## Final reflections

In this paper, we presented the preliminary results of a pioneering educational experience on modelling for in-service secondary education mathematics teachers from the Panamanian school system. To do this, we used a modelling cycle to analyse the participating teachers' solution procedures to three problems posed during a submodule of the EMAS Diploma Course. In this section, we reflect on two aspects: firstly, on the preliminary results of this first implementation and, secondly, on the modifications and improvements for the submodule on modelling.

From the analysed participating teachers' productions, we could make evident that none of them developed a whole modelling cycle. Given this situation, we identified three types of solving procedures from the perspective of the cycle in Figure 1. The first type corresponds to the P<sub>2</sub> and P<sub>3</sub> productions, where the solvers performed the *real model* → *mathematical model* → *mathematical results* phases. In these two productions, obtaining *mathematical results* from the *mathematical model* was considered as sufficient to solve the problems, without the need to interpret them as *real results*, much less validate them in the context of the *real situation* posed; in other words, the problem was worked on mathematically to give it an answer. The second type corresponds to P<sub>4</sub> production, where the solver performed the *real model* → *mathematical model* phases. In this production, formulating a *mathematical model* was considered as sufficient to solve the problem, without any mathematical work on it; in other words, the problem was mathematised to give it an answer. Finally, the third type corresponds to P<sub>1</sub> and P<sub>5</sub> productions, where the solvers did not make evident a complete understanding of the task, making accommodative simplifications and incorrect calculations.

As mentioned at the beginning, working with modelling for mathematics teaching is not part of the national curricular documents of Panama; therefore, these teachers do not have enough experience regarding strategies for solving this type of problems in their educational practice. As evidenced by these preliminary results, the teachers did not pass all the phases of the modelling cycle and stopped when they considered that the problem was mathematically solved. The first two types of solution procedures are consistent with some of the findings of Ledezma et al. (2022), in which the prospective secondary education mathematics teachers were not interested in returning to the «real world» to interpret or validate the results obtained from the *mathematical model* used, but focused their attention on the «mathematisation» and consequent «mathematical work» with the problems posed.

These preliminary results provide a first overview on the teaching and learning of modelling in the EMAS Diploma Course and for us, as scholars involved in this context, allow us to ask the following question: What should be improved/modified in the submodule on modelling for future implementations? Based on previous research on this type of educational experiences (see Borromeo Ferri, 2018) and our preliminary results, we can conclude that, firstly, two sessions are not sufficient for the teachers to acquire modelling competencies by themselves and furthermore to think about how they could teach/implement this process in their educational practice; secondly, that lack of sessions leads us to reformulate the didactic aspects of the submodule by starting from the teachers' previous knowledge and competencies, dividing the modelling process into sets of the phases that make up the cycle and then propose tasks that work on specific modelling sub-competencies (see Maaß, 2010), and solving and creating new modelling problems; and thirdly, designing a modelling lesson that promotes reflection on their own practice after its implementation. Finally, we stress that this study is seminal in the line of research on mathematical modelling in the Panamanian context for further professional development for teachers.

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## References

- Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning*, 12(1), 53–65. <https://doi.org/10.1080/19477503.2019.1595360>
- Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, 1(1), 45–58.
- Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222–231). Horwood. <https://doi.org/10.1533/9780857099419.5.221>
- Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *Zentralblatt für Didaktik der Mathematik*, 38(2), 86–95. <https://doi.org/10.1007/bf02655883>
- Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Springer. <https://doi.org/10.1007/978-3-319-68072-9>
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (8<sup>th</sup> ed.). Routledge.
- Greefrath, G., Siller, H.-S., Klock, H., & Wess, R. (2022). Pre-service secondary teachers' pedagogical content knowledge for the teaching of mathematical modelling. *Educational Studies in Mathematics*, 109(2), 383–407. <https://doi.org/10.1007/s10649-021-10038-z>
- Ledezma, C., Font, V., & Sala, G. (2022). Analysing the mathematical activity in a modelling process from the cognitive and onto-semiotic perspectives. *Mathematics Education Research Journal*. Advanced online publication. <https://doi.org/10.1007/s13394-022-00411-3>
- Maaß, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik*, 31(2), 285–311. <https://doi.org/10.1007/s13138-010-0010-2>
- Maass, K., Artigue, M., Burkhardt, H., Doorman, M., English, L. D., Geiger, V., Krainer, K., Potari, D., & Schoenfeld, A. (2022). Mathematical modelling – A key to citizenship education. In N. Buchholtz, B. Schwarz, & K. Vorhölter (Eds.), *Initiationen mathematikdidaktischer Forschung: Festschrift zum 70. Geburtstag von Gabriele Kaiser* (pp. 31–50). Springer. [https://doi.org/10.1007/978-3-658-36766-4\\_2](https://doi.org/10.1007/978-3-658-36766-4_2)
- Morales-Maure, L. (2019). *Competencia de análisis e intervención didáctica del docente de primaria en Panamá* [Primary Education Teacher's Didactic Analysis and Intervention Competency in Panama] (Doctoral dissertation, University of Barcelona). Dipòsit Digital de la Universitat de Barcelona. <https://hdl.handle.net/2445/151343>
- Pollak, H. (2007). Mathematical modelling – A conversation with Henry Pollak. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 109–120). Springer. [https://doi.org/10.1007/978-0-387-29822-1\\_9](https://doi.org/10.1007/978-0-387-29822-1_9)
- Preciado, A. P., Peña, F., Ortiz, Y. A., & Solares, A. (2023). Diversity of perspectives on mathematical modelling: A review of the international landscape. In G. Greefrath, S. Carreira, & G. A. Stillman (Eds.), *Advancing and Consolidating Mathematical Modelling: Research from ICME-14* (pp. 43–57). Springer. [https://doi.org/10.1007/978-3-031-27115-1\\_3](https://doi.org/10.1007/978-3-031-27115-1_3)