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# Teaching mathematical modelling along with didactic resources

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*In this paper, we shall report a teaching experiment of mathematical modelling at university level. In our didactic engineering, we deal with two didactic resources: the seven-step schema of mathematical modelling cycle and the questions and answers map. We implemented an activity of creating modelling logs in the form of a questions and answers list into the existing mathematical modelling course for the first-year university students and conducted analyses on the modelling logs and students' reflection comments collected. Based on the result of the analysis, we discuss the characteristic of students' mathematical modelling activities and the potential of using the above two didactic resources in teaching mathematical modelling.*

*Keywords: Mathematical modelling cycle, questions and answers map, tertiary education.*

## **Introduction: On the duality of the teaching of mathematical modelling**

In this paper, we shall report the result of our didactic engineering and experimentation of the teaching of mathematical modelling at university level. When teaching mathematical modelling, we are supposed to organise two different kinds of didactic situations implicitly or explicitly. The first kind of situations, needless to say, is didactic situation *for mathematics*. In the ordinary courses of mathematical modelling, students encounter mathematical knowledge involved in modelling process. The second kind is (meta-)didactic situation *for didactics* (or epistemology), so to speak<sup>1</sup>. In such courses, the students should also acquire the *knowledge about mathematical modelling*, parts of which, for example, can consist of the concepts of model, system, object, reality, and mathematics. Usually, the second kind of situations is implicit and supplemental. In some cases, that may be intentionally avoided as the anti-didactic move to off-topics. Nevertheless, we consider that knowing such a didactic aspect of modelling is quite important for mastering mathematical modelling. In fact, some textbooks of mathematical modelling have the (short) chapter for explaining the nature of mathematical modelling (e.g., Mooney & Swift, 1999). In our view, what is crucial is the appropriate collaboration of mathematics and didactics. Here is the umbrella question of ours: *What is the effect and potential of teaching mathematical modelling together with relevant didactic resources?* This paper is the first report of the teaching experiment for answering this question.

## **Theoretical background**

In our didactic engineering, we deal with two didactic resources, which we believe to be useful for students' understanding of the nature and function of mathematical modelling. The first is the seven-

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<sup>1</sup> In this paper, the adjective *didactic* basically means “about teaching, studying, and learning something like mathematics”. In this sense, didactic phenomena are at meta-level for mathematical ones. Note that *didactic resources* include theoretical tools for analysing didactic situations: e.g., the notions of *metacognition* and *mathematisation*.

*step schema of modelling cycle* (Blum & Leiß, 2007) and the second is the *questions and answers map* (or simply Q-A map) in the anthropological theory of the didactic (ATD) (cf. Winsløw, Matheron, & Mercier, 2013). The seven-step schema is a cyclic system consisting of *six elements*: a) real situation; b) situation model; c) real model; d) mathematical model; e) mathematical results; and f) real results. The elements a, b, c, and f belong to real world, while the elements d and e are included into mathematical world. And then, there are *seven steps* of transitions between the elements: 1) understanding [ $a \rightarrow b$ ]; 2) simplifying/structuring [ $b \rightarrow c$ ]; 3) mathematising [ $c \rightarrow d$ ]; 4) working mathematically [ $d \rightarrow e$ ]; 5) interpreting [ $e \rightarrow f$ ]; 6) validating [ $f \rightarrow b$ ]; and 7) exposing [ $b \rightarrow a$ ]. We call this schema as Blum’s model metonymically in the following. The modelling cycle is often used as an analytic tool to describe students’ modelling process (e.g., Greefrath et al., 2022), and then occasionally combined with some supplemental frameworks for clarifying different aspects of modelling process (cf. Vos & Frejd, 2022). In this study, we focus on the *dimension of problem-posing* in the process with the help of Q-A map. The Q-A map is a directed graph with questions and answers as vertices. Here is a quite simple example:  $Q_1 \rightarrow Q_2 \rightarrow A_2 \rightarrow Q_3 \rightarrow A_3$ . The Q-A map has been introduced to describe inquiry process within the framework of the ATD, and often applied to investigate mathematical modelling (see also, Barquero & Jessen, 2020). In this paper, the above two didactic resources have twofold functions. The cyclic schema and the Q-A map are didactic stakes in the modelling course of ours, thereby being assigned with the parts of the contents to some extent, while at the same time, they are also used as our own tools to analyse the course, as usual.

Let us add here a short comment about our design principles for the teaching of didactic resources. Basically, we take care of that they are *never* too emphasised more than modelling activity itself. We call this the *humility principle*, thereby avoiding the *metacognitive shift* which is the “*sliding*” of didactic stakes from mathematical knowledge to be taught to didactic tools for studying and teaching it (Brousseau, 1997). Following this principle, the cyclic schema and the Q-A map are introduced with some simplification—we call it the *minimalist principle*. The details will be described later.

## **The design of the course and the instruction to students**

The reported course of mathematical modelling was organised based on the idea of integrating students’ documentation of Q-A maps as “modelling logs” explained later into the existing course within the framework of “*Modelling as Contents*”. It is a 2-credit, 1-semester course, meeting for 90 minutes each week for 15 weeks. Students are invited to three mathematical modelling tasks during the semester. Students are divided into groups of 3 to 4 per group and work in groups on mathematical modelling tasks. Groups are rearranged for each task. The outline of the course is as follows:

- Week 1: Introduction including descriptions of mathematical modelling.
- Week 2: Guidance for digital tools.
- Week 3-6: Modelling Task 1 (Deer and wild boar reduction targets in Japan).
- Week 7-11: Modelling Task 2 (The practicality of a new cancer test using AI).
- Week 12-15: Modelling Task 3 (Changes of the market share of smartphone operating systems).

### **Modelling tasks**

The modelling tasks used in the course were chosen under two criteria: that each of them should be a real or authentic problem in our societies, and that real data on the problem should be easily accessible by students themselves. In this paper, we focus only on the first modelling task, in which students are

asked to read an editorial of a Japanese newspaper (only the summary shown below) and to address the question (1) and (2) below (Figure 1).

Wild animals cause serious damage in many parts of Japan. Agricultural damage reaches around 20 billion yen annually. As of FY2011, the number of deer and wild boars in Japan was estimated at 3,250,000 and 880,000, respectively. If nothing is done, the number of deer will increase at a rate of about 20% per year. The Ministry of the Environment and the Ministry of Agriculture, Forestry and Fisheries have set a goal of halving the populations of deer and wild boar by FY2023, and local governments across Japan are stepping up their catching activities. [Summarised from a part of “Damage by Wildlife: How to Reconnect with People” (2015) by the authors.]

- (1) At the time of the article, what specific measures could be taken to achieve the goal within the deadline? Make a plan including numerical targets.
- (2) Examine the information available as of 2022 and discuss the future prospect in the light of the plan established in (1).

### Figure 1: Modelling Task 1: Deer and wild boar reduction target in Japan

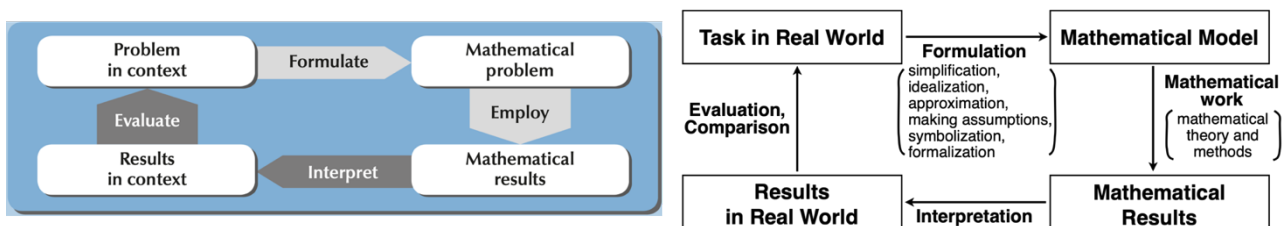
Students are asked to address the two questions in this order. They can use any information, including the internet and books. They are encouraged to use computers or other devices for calculation and graphing, and any digital device such as laptops, smartphones, scientific calculators, etc. can be used in the classroom. Students work on the task for three weeks (Week 3-5) and write reports on it in groups. They discuss the task in Week 3-4, creating their modelling logs. The report writing activity begins in Week 5. If they continue to discuss the task, they will continue to produce their modelling logs in Week 5. Reports are due within a week after the Week 5 class. The Week 6 class is devoted to the reflection upon the modelling activities by sharing the reports of all groups with the whole class.

This task can be approached using numerical sequence models with recurrence relation  $a_{n+1} = pa_n - q$  or  $a_{n+1} = p(a_n - q)$  with the natural rate of increase  $p$  and the annual catch  $q$  as parameters, the geometric progression model  $a_{n+1} = p(1 - r)a_n$  with the same  $p$  and the proportion of the annual catch  $r$ , or linear approximation models of population trends based on the historical data. These pieces of mathematical knowledge are included in the Japanese high school curriculum, and in fact, this is the reason why we chose this task as the first task. This task is intended to focus on *mathematical modelling* activities rather than *purely mathematical* activities. The natural rate of increase  $p$  can be set as 1.2 according to the editorial, but the accuracy and variability of this figure itself can be debated. Students can access the real data published each year by the relevant ministries on the estimated population of deer and wild boar, the number of these animals captured, and so on.

### Instruction of the concept of mathematical modelling

The concept of mathematical modelling was introduced in Week 1 by showing the two figures of mathematical modelling cycle with some examples. Although Blum’s model will be used in the analysis to be conducted later, we deliberately present students with the models different from it, according to the minimalist principle. In Figure 2, the one on the left is the model described in the PISA 2015 assessment framework (OECD, 2017, p. 68, Figure 4.1), which shares the basic idea of the cyclic process of mathematical modelling with Blum’s model. The right one is a modified version of Miwa’s triangle model (Miwa, 1983), which is well-known and accessible to Japanese educational

communities. The modification was made by the lecturer (the first author) to fit it to PISA’s quadrilateral model. Using these two models, we intended to make students understand the function and structure of mathematical modelling easily but without oversimplifying them and containing too many components. In Week 1, the lecturer informed that the step represented as “Formulate” and “Formulation” in Figure 2 was called “mathematisation,” and then emphasised the following didactic facts about modelling: simplification, approximation, and making assumptions are needed in the mathematisation; mathematical modelling is a cyclic process to create and improve a certain model repeatedly; there is no unique “right” solution and there are many possible interpretations and results; in this course, the modelling process towards results is more important than the results themselves.



**Figure 2: Two models of mathematical modelling cycles introduced to students**

### **Instruction of the modelling log in the form of a list of questions and answers**

Each group was asked to create their own logs of the modelling activities of each week in the form of a sequential list of questions and answers generated in the process, which we call *Q-A logs* in short in the following. A guidance on how to create Q-A logs was given to the students in Week 3 when the first modelling task started. The lecturer first showed an example of Q-A logs by using an easy modelling task. Then the procedure of creating Q-A logs for the first modelling activity was shown to the students as follows: write the two questions (1) and (2) given in the above modelling task as  $Q_0$  and  $Q_{0-2}$ , respectively; numbering the questions arose in modelling activities in time order, write down as  $Q_1, Q_2, \dots$ ; when questions occur in parallel at the same time, write  $Q_{2-1}, Q_{2-2}$ , and so on; for an answer  $A$  to a question  $Q$ , write  $A$  with the same subscript as  $Q$  (e.g.,  $A_1$  for  $Q_1$ ); if there is more than one answer for a question  $Q$ , then add sub-numbers (e.g.,  $A_{1-1}, A_{1-2}, \dots$  for  $Q_1$ ); there can be a question  $Q$  without any answer  $A$ , and it is possible that some question remains unanswered or generates another question; every answer  $A$  cannot exist without some question  $Q$ , and any activity is regarded as an action for producing a certain answer  $A$  to a determined question  $Q$ .

Indeed, Q-A logs are not exactly Q-A maps, but Q-A logs possess the structural information of Q-A maps implicitly with the help of the subscript numbers. Actually, teachers or researchers can draw a Q-A map from a Q-A log if needed. The reason for introducing Q-A logs instead of Q-A maps following the minimalist principle is that we wanted students to focus more on their own questions and did not want them to work hard on the drawing of Q-A maps.

### **The lecturer’s interaction with the students**

During the modelling activities, the lecturer sometimes interacted with the groups to give them comments. The lecturer also read their Q-A logs after the class and gave feedback to the whole class in the following week, picking out important questions from the Q-A logs and commenting on them by relating them to the steps in the modelling cycle.

## Data collection and analysis

The course described above was conducted in the autumn semester of the 2022 academic year at a Japanese university. The lecturer of it was the first author. The participants of it were first-year students in engineering, agriculture, computer science, psychology, etc. There were 56 participants in the class, 39 of whom were engineering students. The participants were divided into groups of four and 14 groups were formed. For most of them, it was their first time to experience mathematical modelling in the course. During the modelling activity in the Week 3-5, the Q-A logs of 14 groups were collected in digital format (PDF or Word) through the learning management system Moodle. All the groups submitted their logs in Week 3 and 4. No additional logs were submitted in Week 5.

The Q-A logs were analysed in order to examine students' modelling activities with the use of Blum's model as an analytic tool. The analysis was carried out as follows. First, errors in numbering and descriptions without the labels of  $Q$  or  $A$  in the data were carefully examined and corrected. In the cases of the missing of the labels, we added appropriate numbering with labels in case they could be recognised as  $Q$ s and/or  $A$ s. For each pair  $(Q; A)$  of  $Q$  and  $A$  that could contain more than one answer, i.e.  $(Q_1; A_{1-1}, A_{1-2}, \dots)$ , we identified the corresponding steps from among the seven steps in Blum's model. We assigned more than one step to  $(Q; A)$  if it involves several steps. We then located all the pairs  $(Q; A)$  of each group onto the diagram of Blum's model, together with arrows representing the process corresponding to the pairs (e.g., Figure 3 in the next section). In the case that pairs with consecutive numbering corresponded to the same steps, we mapped them to share a single arrow.

In addition, we collected students' reflection comments during Week 3-6 and their final comments after Week 15. The data were collected using Moodle and qualitatively analysed to examine students' use of modelling terminologies and their thinking about creating Q-A logs. For the former aim, we counted the number of students who used modelling terminologies in their comments collected during Week 3-6. For the latter, we conducted a qualitative coding by using the descriptive coding method on the data obtained in the final comments in which students described their thinking about Q-A logs.

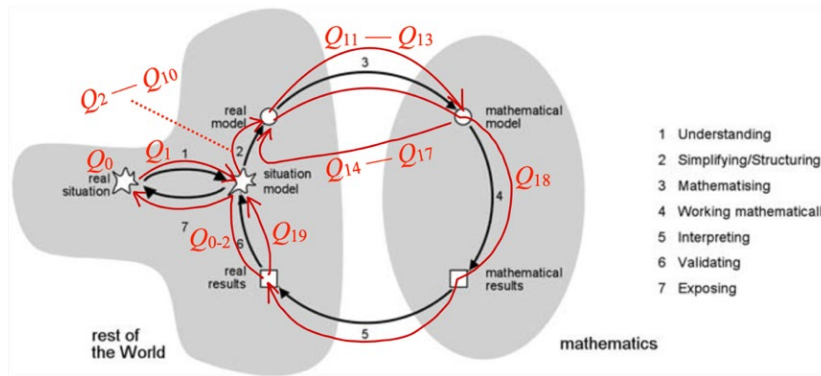
## Results and discussions

### The Q-A log of one of the groups and the visualisation of the modelling path

Below we show a part of the Q-A log created by one of the groups and the result of the visualisation of the modelling path of the group (Figure 3). The questions and answers below were originally written in Japanese and translated into English by the authors. As shown in Figure 3, the group had only one round modelling cycle but had the activity going back to the situation model from the mathematical model ( $Q_{14}$ - $Q_{17}$ ) to adjust their real model by considering some assumptions.

- $Q_1$ : How can we reduce the deer and wild boar population by half by FY2023?  
→ $A_1$ : Reduce the populations by capturing.
- $Q_2$ : How many trappers are needed? → $A_2$ : [...]
- ...
- $Q_{11}$ : How do we make a mathematical model? → $A_{11}$ : [...]
- $Q_{12}$ : What equation do we create?  
→ $A_{12-1}$ : Create a recurrence relation (on the population).  
→ $A_{12-2}$ :  $a_n = a_{n-1} \times 1.2 - \alpha$  ( $a_n$  is the population in year  $n$  (final value),  $\alpha$  is the number of decreases per year.)
- $Q_{13}$ : What do we set as constants and variables?

- $A_{13}$ : Fix the rate of increase at 1.2, vary the number of decreases  $\alpha$ .
- ...
- $Q_{17}$ : How do we assume the number of hunters? → $A_{17}$ : [...]
- $Q_{18}$ : What is the expected value of  $\alpha$   
→ $A_{18}$ : Using the recurrence relation in  $A_{12-2}$ , [...] approximately 690,000 deer.
- $Q_{19}$ : How many animals will be caught by each hunter?



**Figure 3: An example of visualisation of modelling paths overlapped on Blum's model**

### Summary of the results of all the groups

From the result of visualisation for all the groups, we observed the following: 13 groups had one or two rounds of the mathematical modelling cycle; 11 groups had activity going back and forth between the situation/real model and the mathematical model. Figure 3 shows that the group was most active in Step 2, but this was a general characteristic shared by the other groups. Table 1 summarises the frequencies of every step of all groups, which are totally 213 steps corresponding to the pairs ( $Q$ ;  $A$ ). This survey includes  $Q_0$ ,  $Q_{0-2}$  and the two questions identified from descriptions without the labels of  $Q$  or  $A$  in the preparation of the analysis. Each frequency in Table 1 is represented as a percentage, which shows that Step 2 accounts for about half of the total. As this was the first experience of mathematical modelling for most of the students, this result was to be expected.

The step in Blum's model	1	2	3	4	5	6	7
The frequency of the step	4.7	49.3	22.1	11.3	6.6	5.2	0.9

**Table 1: The frequency of each step in Blum's model corresponding to the pairs ( $Q$ ;  $A$ )**

Moreover, Step 2 and Step 3 together account for more than 70% of the total, suggesting that the students' activities were concentrated on creating mathematical models from situation models. Compared to Step 2-3, the proportion of Step 4 is very small. One of the reasons for this is probably that the mathematical work was not so difficult in this task. Although whether the Q-A logs are a faithful reflection on students' actual modelling activities or not needs to be verified in the future study, it was indeed observed in the classroom that the students did a lot of trial and error in Step 2 and 3. This suggests that the result shown in Table 1 reflects to some extent the reality of the modelling activities. There are two points to note here. The one is that the degree of summarisation, or granularity so to speak, of  $Q$  and  $A$  varied widely between groups. Regarding the number of questions generated in groups, there was one group in the range of 0 to 5, five groups in the range of 6 to 10, four groups in the range of 11 to 15, and four groups in the range of 16 to 20. The mean value

was 12.0 and the median was 11.5, but the minimum was 3 and the maximum was 19. The other is that many groups did not include the record of  $Q_{0-2}$  in their Q-A logs; only six groups did. This may be due to the fact that the work on  $Q_{0-2}$  took place at the same time as the writing of the report. These two points indicate a need for improvement in the instruction and the method of producing Q-A logs.

### **Can Q-A logs help students' modelling process?**

There were very few descriptions in the Q-A logs which used the modelling cycle terminology introduced in Figure 2. For example, the word "assume" was found in  $Q_{17}$  of the above example but that was quite rare, and the Q-A logs were basically written in the context of the given task. Regarding the usefulness of the Q-A logs, 50 out of the 56 students responded positively in their final reflection comments: easy to record (4 students); easy to understand the thought process (11 students); easy to recall previous class work (16 students); helpful for thinking and discussing (14 students); and so on. These data imply that, in this case, Q-A logs do not contribute much to the students' recognition of the modelling cycle explicitly nor directly, but they do play, implicitly and indirectly, a role to help students clarify and recall the thought process in their modelling activities. Of course, we are supposed to be careful about students' tendency of making "good" answers for teachers.

### **What is the potential of introducing of the modelling cycle?**

Although the modelling terminology was rarely found in the Q-A logs, it was found in the reflection comments of many students. The followings are some of such comments taken from students' reflection during Week 3-5, which were translated and italic by the authors:

"From the point of view of hunting ability, we tried to confirm the *validity* of the mathematical model but were unable to do so."

"I think we only have two rounds of *cycles* to create the mathematical model we learned in the first week."

The number of students who used the modelling terminology in their reflections is 20 (35.7%) during Week 3-5 and 37 (66.1%) if the summary reflection in Week 6 is included. As shown by this result, many of the students reflected on their own modelling activities with reference to the modelling cycle framework. From this observation, we can say that teaching the modelling cycle have the potential to help students monitor and control their own modelling activities from a higher perspective.

### **Final remarks: On assessment of mathematical modelling**

This paper revealed a possibility introducing didactic resources into the teaching of mathematical modelling. Especially, about the Q-A map, this paper is probably one of the first teaching trials which includes students' self-analysis of their own mathematical inquiry with it, beyond the context of teacher education (cf. Barquero & Bosch, 2022). We mainly discussed the benefit to students, but there also exists one to lecturers. The students' documents within the terminology of modelling cycle and the format of Q-A log are rich data-sources for assessment. As we have already said in the introduction, the teaching of mathematical modelling is a twofold reality to a certain extent, that is, the teaching of mathematics and didactics. This leads us to the difficulty to assess students' modelling activities. On the one hand, ordinary mathematics examination, obviously, is not relevant, because it ignores the didactic or epistemological aspect of students' modelling capacity. But, on the other hand, modern assessment methodologies, e.g., performance-based assessment, seem to tend to leave from



mathematics. We hope that the didactic organisation reported in this paper is a promising approach towards developing more relevant assessments of mathematical modelling.

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