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Make it real: Students' mathematical modelling realized with 3D printing

Simone Jablonski, Tim Läufer and Matthias Ludwig

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Mathematical modelling addresses numerous demands of modern mathematics education: The learning of mathematics is directly linked to reality and students learn what they can use mathematics for. We focus on the link between architecture and geometry and observe the modelling steps of students in recreating Frankfurt's fair tower – a 257 meters' tall skyscraper. Hereby, the use of 3D print technology is taken up as a modern approach for the realization and production of the students' mathematical modelling. In an explorative, qualitative study with eleven secondary school students, we analyze the modelling activities and give a deeper insight into the relevance of individual selected modelling steps. The results show that particularly the simplifying and structuring and validation steps can be observed, partly in the use of the provided digital tool.

Keywords: Mathematical modelling, digital tools, 3D printing, 3DMP, validation.

Mathematical modelling

In the context of modern approaches to mathematics teaching and learning, mathematical modelling provides numerous possibilities. Because of its relation to reality, modelling distinguishes itself from the classical inner-mathematical problem and is thus intended to clarify the relevance of mathematics in everyday life (Blum & Leiß, 2007). Research activities have focused on the modelling process of students, e.g., describing the modelling process as a cycle with different steps in which students attempt to understand, simplify and structure, mathematize, work mathematically, interpret, validate and present a given problem from reality to solve the modelling problem adequately (Blum & Leiß, 2007). In addition, research focuses on the link between theory and practice, searching for innovative opportunities to support the design and implementation of modelling in mathematics classes (e.g. Barquero et al., 2022). Hereby, Greefrath et al. (2018) describe digital tools as a possibility to realize real-world modelling problems, e.g. by dynamic geometry software, calculation software or tools that provide feedback on a given solution. For example, El Bedewy et al. (2022) describe the potential of architecture as an integration of 3D mathematical modelling with GeoGebra in mathematics classes. In the scope of this article, we take up the focus on architecture and extend the focus to modelling activities that are linked to the use of 3D print technology.

3D print technology

With the term 3DMP (3D modelling and printing; e.g. Anđić et al., 2022), researchers currently examine the potential of students creating a 3D model using digital technology and printing it. Its potential for mathematical modelling is highlighted since “3D printing has made the realization of mathematical models easier than ever” (Asempapa & Love, 2020, p. 87). Hereby, 3DMP adds a new facet to mathematical modelling: The students create their mathematical models as a digital, to scale replica of a real-world object with modern 3D modelling software – a process that has been observed to enhance the geometric understanding of students (Lieban & Lavicza, 2019). A 3D printer then

prints this digital replica by executing a set of commands that a special software (“slicing”-software) generates based on a given 3D model. The results are physical 3D models that are still to scale and embodies the mathematical model that the students used to create (cf. Figure 1). Through that, it serves as a “work piece” (Dilling et al., 2021) which can be the basis of the students’ interpretation, validation, and presentation.

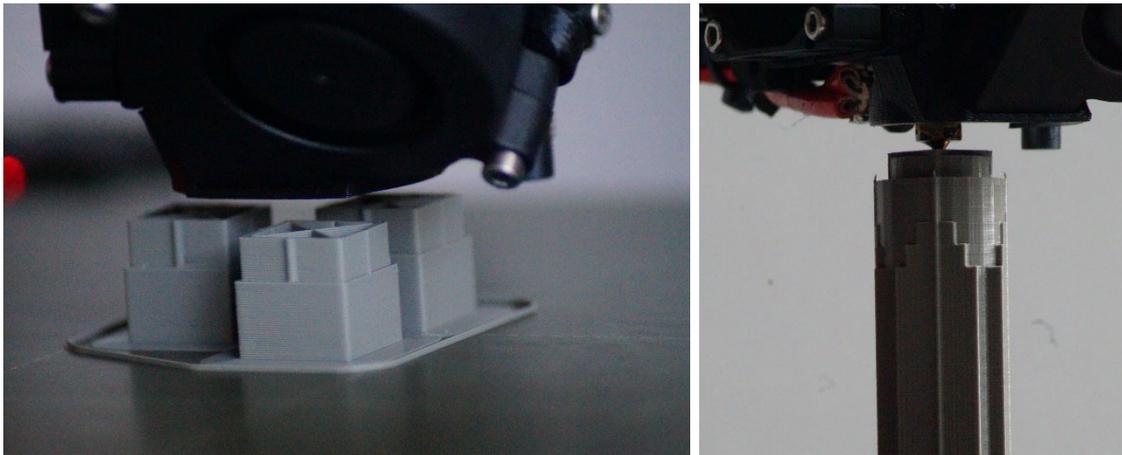


Figure 1: The 3D printing process of a skyscraper. This type of 3D printer prints additively in layers (here: 0.3 mm)

Research question

Based on the effort to use digital tools profitably and innovatively for mathematical modelling, on the one hand, and current research activities in the field of 3DMP on the other hand, it is the aim of the paper to empirically observe and analyze the modelling steps of students when creating a 3D printing model. In the article’s example, the focus is on the architecture of Frankfurt’s fair tower, a 257 meters’ tall skyscraper.

The theoretical basis for the connection of 3D print technology and mathematical modelling is highlighted in Figure 2. In the center, the general 3DMP process, as defined by Anđić et al. (2022), is presented in a simplified version as introduced by Läufer and Ludwig (2023). Hereby, the main steps are highlighted, excluding more detailed choices, e.g., on the technology used. In order to connect the process with the modelling steps of the modelling cycle (Blum & Leiss, 2007), the 3DMP process is presented together with these steps on the left. On the right side, Figure 2 shows the modelling process that the students are theoretically intended to carry out in the example of this article. As it can be seen here, the students’ main activities are assumed to happen in taking notes and modelling the fair tower (in terms of simplifying and structuring), the 3D modelling with a digital tool (in terms of mathematizing and working mathematically) and the validation of the 3D models (in terms of validating and interpreting). Between all three branches in Figure 2, similar colors underline these potential connections. With these considerations being solely from a theoretical point of view, we take a closer look at selected modelling steps during the 3DMP process from an empirical perspective and formulate the research question: *What characterizes selected modelling steps when students create a 3D print model of Frankfurt’s fair tower?*

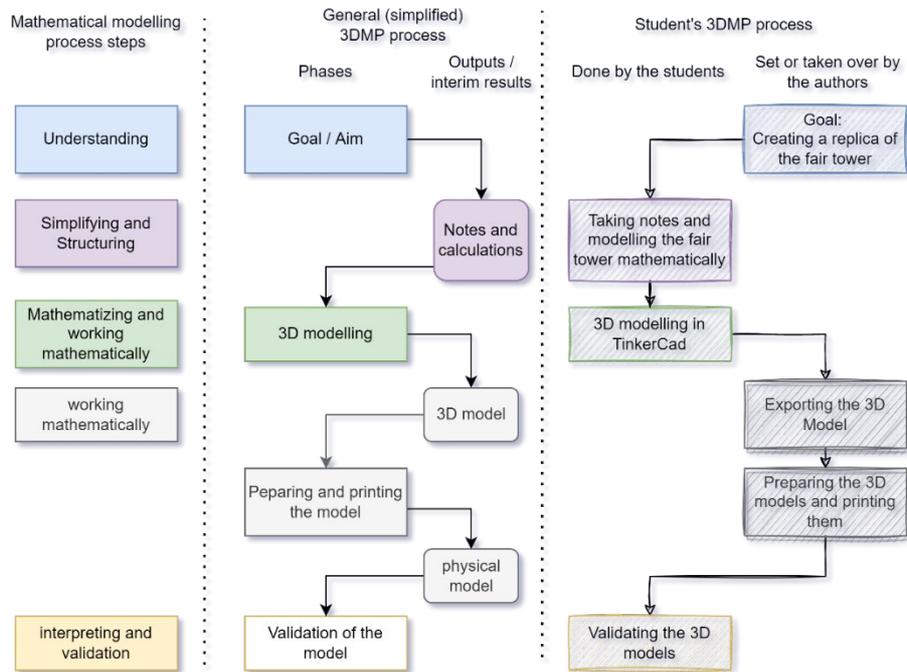


Figure 2: The simplified 3DMP process (centre), the students’ 3DMP process (right), and the related mathematical modelling steps (left)

Method

The research question is answered by means of a qualitative, explorative case study. In the study, eleven seventh graders (approx. 13 years old) participating in the enrichment programme “Junge Mathe-Adler Frankfurt” (cf. Jablonski & Ludwig, 2022) were involved. The students were divided into five groups of two or three, which are named A, B, C, D and E in the following. The modelling task was integrated into three 90-minute sessions that took place at Goethe University, starting in November 2022 and ending in January 2023. For the first session, a three-dimensional map of the quarter of Frankfurt’s fair and exhibition center was printed from free OpenStreetMap data (see Figure 3 left).

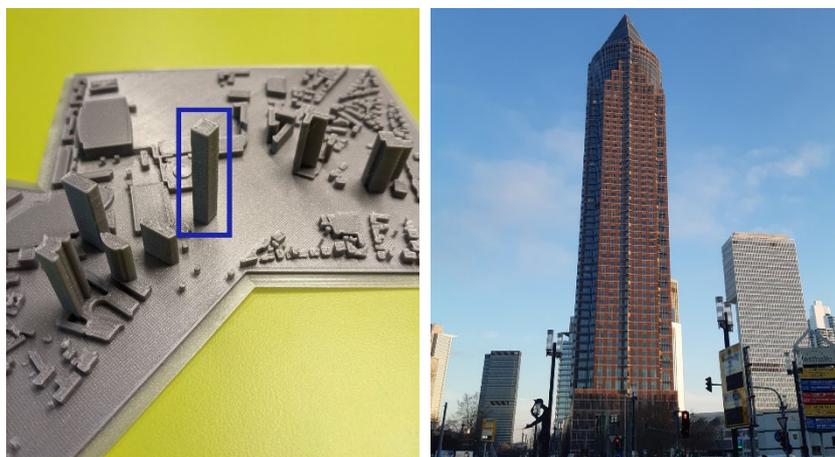


Figure 3: Representation of the fair tower in a 3D printed map (data from OpenStreetMap, left) and in a photograph (right)

After a short discussion, the students recognized the area and, in particular, the simplified version of the fair tower. The map data only shows a cuboid instead of the tower's original shape (see Figure 3 right) what the students described as insufficient.

Using this observation and rating as a motivation to recreate the originally 257 meters' tall fair tower as precisely as possible, we took the groups to a location outdoors from which the students could analyze the shape of the real tower in more detail. Standing in front of the object, the students should make a sketch involving all relevant geometric solids. Hereby, we provided the tower's total height and side lengths. Other lengths had to be estimated. In a next step, the students were asked to scale their estimated data so that we could print the tower in an appropriate size.

In the second session, the students got an introduction to TinkerCad by the authors. With this digital tool, the students were able to rebuild the fair tower with different geometric solids. TinkerCad is a free, web-based application that allows the creation of 3D models. There, geometric shapes can be placed on a grid, moved around and resized. One major function is that shapes that touch each other can be "grouped", resulting in a fusion as a single shape. In Figure 4 (left), the students of group C, for example, grouped all shapes used to create the model. A second function is that a shape can either be a solid or a hole. Grouping a solid shape with an overlapping hole shape subtracts the hole from the solid, allowing for more complex shapes to be created. The different colors in both Figure 4 left and right were chosen by the students. Before the introduction in session two, the students were not familiar with the programme.

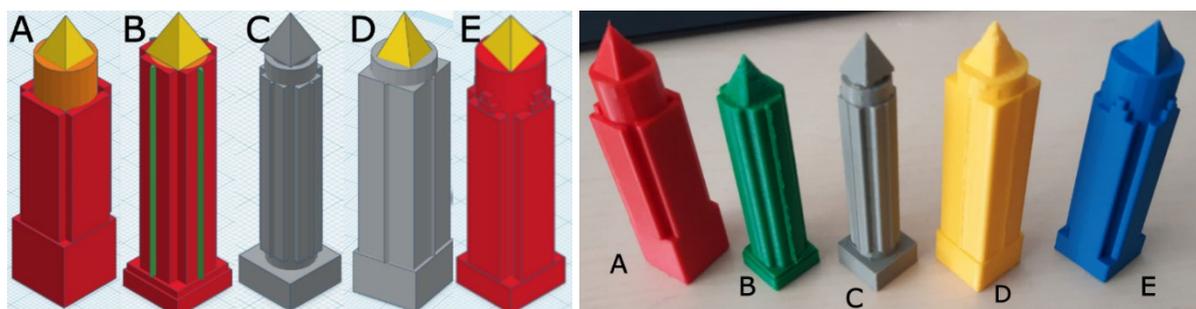


Figure 4: The 3D models of the fair tower in TinkerCad (left) and printed (right) done by the students' Groups A-E

While working with TinkerCad, the students were asked to note and sketch their step-by-step approach, in particular in terms of the geometric solids. Worksheets with an introduction of the problem, information about the tower's height, space for the sketches and a table for a documentation of the steps undertaken while recreating the tower were provided. The worksheets were used to guide and structure the students' activities, making their progress independent from the guidance of the authors during the session and allowing individual approaches. The authors supported the students in individual questions and problems with TinkerCad. At the end of session two, all groups had their tower models ready to be printed. Between session two and three, the towers were printed by the authors due to limited in-session time of the project (following Anđić et al., 2022).

In the third session, the printed models, as shown in Figure 4 (right), were handed to the students in terms of a validation. Simultaneously, all towers were printed during the session once more to

familiarize the students with the printing process itself as well. The students were asked to take a look at their model following the questions: *What went well? What could be improved?* In a second step, the students worked in mixed groups and compared their models. Based on this, they worked on a “manual” for building the 3D model of the fair tower and reflected recommendations for future 3DMP activities.

The students’ documentations during the three sessions build the basis for the analysis in terms of the formulated research question. We analyze the documentations, in particular the sketches and the undertaken steps, by means of the modelling cycle by Blum and Leiss (2007). Since the students only modelled the towers and did not print them themselves, the focus of this analysis is on the steps of *Simplifying and Structuring*, *Mathematizing* and *Validating*, which from a theoretical point of view are directly related to the creation and the validation of the 3D model (cf. Figure 2). For each group, the documents were coded in terms of the selected steps. In the context of this article, *Simplifying and Structuring* contains the recognition of characteristics and inaccuracies. *Mathematizing* is understood as the transfer into mathematics by gaining data. *Validating* includes all processes that include a check and evaluation of the final products (Blum & Leiß, 2007).

Results

Analysis of the modelling step *Simplifying and Structuring*

Figure 5 shows the sketch of Group E involving the relevant geometric solids to describe its shape (left). This example can be seen as prototypical of what all groups did during this first session: The sketch, that was made on site of the tower’s location, was the students’ first attempt to *simplify and structure* the task, i.e. the description of the tower’s shape. As presented in Figure 5, the students used geometric solids to approximate the tower’s shape which goes along with a *simplification* of the original object, e.g. the example group evaluating the windows as unimportant, but the staircase-like arrangement at the upper end of the outer facade as important. In addition, the students used the geometric solids to *structure* the tower which is on the left side marked with numbers and on the right side named as corresponding levels.

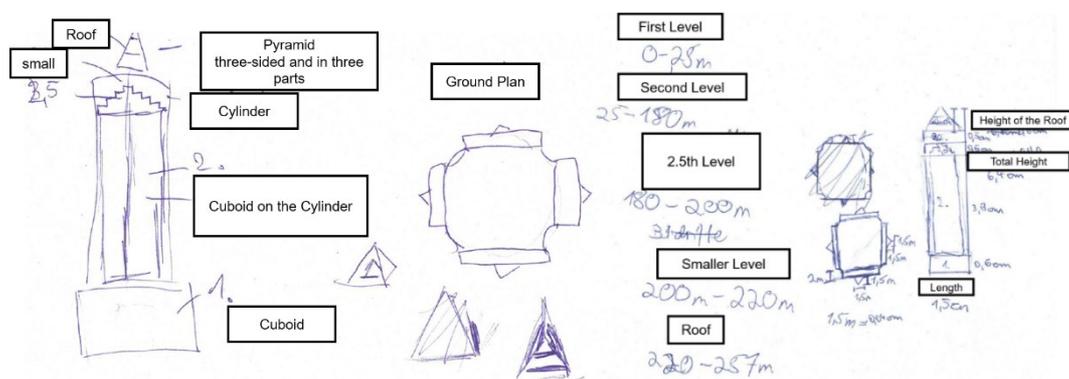


Figure 5: Students’ sketch involving geometric solids (left) and size estimations plus scaling (right)

With the involvement of TinkerCad in session two, the students were asked to *structure* their model in more detail. While *simplifications* were already mostly completed in the first session, we can observe from the notes of session two that the students refined their *structure* again in the context of

the “building plan”, e.g. in Figure 6, the group indicated how many cuboids were necessary to describe the outer facade and enriched this with small, step-by-step sketches.

Step	Geometric Body	Description and Sketch
1	Cuboid	Lower Basis 
2	Cylinder	Center of the tower 
3	Four equal sized cuboids	At the cylinder 
4	Four-sided pyramid	Roof 
5	Two cuboids	Staircases 
6	Four equal prisms	Windows at the pillars 

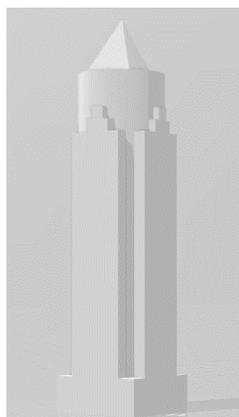


Figure 6: Students’ manual (left) and final model of the created tower (right)

Analysis of the modelling step Mathematizing

To arrive at the result shown in Figure 6 (right), the data from the construction plan had to be entered into the digital tool together with previously made *mathematizations*. In Figure 5 (right), the students *mathematized* the simplified version of the tower by estimating the size of different “levels” in terms of a former division of the tower into parts. On the right-hand side of Figure 5, it is possible to follow the students’ scaling for the 3D print, which was a necessary basis for the transformation in TinkerCad.

Analysis of the modelling step Validating

From the last session, we can also recognize the students’ *validation* activities. In all groups, the students named aspects that they thought had already worked well, as well as aspects that could still be improved. From these considerations, the main focus of the students can be recognized: Most groups were satisfied with the size and scale of the printed models in terms of their made *mathematizations*. Two groups additionally mentioned symmetry and shapes, whereas two other groups took the printing process into consideration by adding “no floating elements”. This issue was shortly addressed during the familiarization with the actual printing process and potential problems. One group, in addition, mentioned the model to be “close to reality”. For the improvements, we can see considerations concerning size and overhangs – a problem that might be relevant during the actual printing process. Hereafter, we observe individual challenges of the groups, e.g. missing elements, symmetry, holes and cutouts as well as a stronger focus on “details” which can be interpreted as improvements in the *simplifying and structuring* step.

In comparison of a final building manual in Figure 7 to the first sketches and building manuals of the students (cf. Figure 5 and 6), it can be observed that the students started to follow some of their revisions, namely (1) added information about the size of each object in terms of proportions (*mathematizing* step), (2) major role of symmetry, e.g. in step two, they mention “place it centrally on the cuboid” (*simplifying and structuring* step) (3) formulation of details, e.g. holes in terms of the building process and prisms in terms of the tower’s shape (*simplifying and structuring* step).

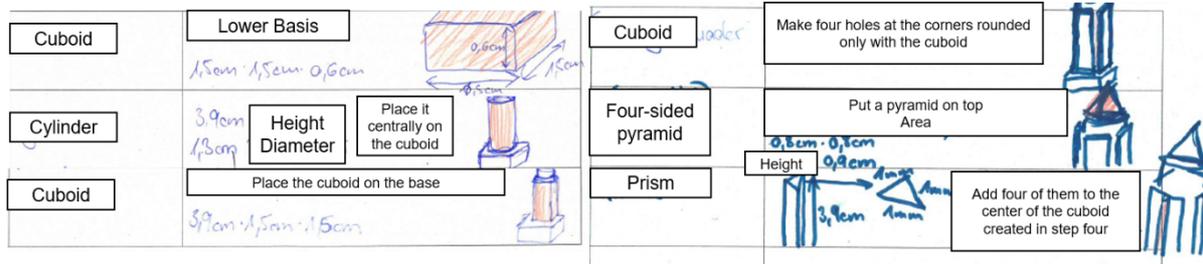


Figure 7: Excerpt from a final building manual

Discussion

In this paper, we observed selected modelling steps of students while recreating a skyscraper utilizing 3DMP. It was our aim to describe the students' activities and, in particular, characterize the modelling steps undergone. Through the students' final products and the documentations of the groups, the following conclusions can be drawn: The students followed similar approaches in the sense that they used the same basic geometric solids for their model of the tower. Still, we saw differences in the depth of including details of the tower, e.g. the staircase-like surface of prisms on all sides of the tower (cf. El Bedewy et al., 2022). With the last observation, it can be concluded that the students mainly focused on the step *simplifying and structuring* (cf. Blum & Leiss, 2007). In addition, we could see reflections in the *validation* step (cf. Blum & Leiss, 2007). Especially in the comparison of the former sketches and the final building plan, the students recognized the importance of details in the architecture, e.g. proportions and symmetry, after this reflection which is related to their considerations in the step *simplifying and structuring*.

The findings have to be interpreted in terms of the following limitations. Firstly, none of the students had any prior experience in 3DMP. Therefore, the task was structured by subtasks. This might have led to a particular focus on the modelling steps described. This can also be extended to the fact that the students did not print the models themselves. For this reason, the activities linked to the modelling step *work mathematically* were limited in advance (cf. Figure 2) and not taken into further consideration. Thus, the results presented should be seen less as focal points on selected modelling steps and much more as ways to include the emphasized modelling steps in mathematics classes by the use of 3DMP. Through alternative examples and/or different guidance in terms of provided materials, it is possible that the excluded steps in this analysis are highlighted in a more intense way. In addition, the students mentioned that they wished for more time to finish their products and/or work on their products once more after validation. This factor could have led to the consideration of more details.

In summary, the 3DMP activity can address selected steps of the modelling cycle, and in this implementation, it particularly focused on the question of details and *simplifications* as well as *validation*. On the one hand, the technical component supported the *mathematization* process and on the other hand, the actual 3D printing potentially laid the foundation for a reflective *validation*, allowing the students to complete their work. In an oral evaluation, the students highlighted this possibility to actually print their models and hold it in their own hands (cf. Dilling et al., 2021). This first impression from the students raises the question of the students' perception of the task – their perspective on the 3DMP activity will be taken up as an outlook for future research.

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