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# Mathematicians' and teachers' views on the connection between mathematics and other fields

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*This study investigated: (1) what secondary school teachers, who participated in academic mathematics studies, learned about the connection between mathematics and other fields, and how this knowledge contributed to their teaching; and (2) what mathematicians, who taught in that program, wanted to teach teachers about this connection. Data sources included interviews with five research mathematicians and 14 teachers. Analysis revealed that the mathematicians wished to teach teachers about the mutual contribution between mathematics and other fields. Yet, the teachers reported on enriching their knowledge only about the contribution of mathematics to other fields but not about the contribution of different fields to the development of mathematics. Interpretations and ways to address this discrepancy are proposed in relation to two areas: mathematical applications and modeling, and the history of mathematics.*

*Keywords: Secondary mathematics teacher knowledge, mathematical applications and modelling, history of mathematics, academic mathematics.*

Academic mathematics courses are part of the professional preparation of secondary school mathematics teachers in many countries. Research suggests that such studies have the potential to advance not only knowledge of specific mathematics topics, concepts, and procedures which are usually explicitly addressed in mathematics textbooks and lectures, but also knowledge about the nature of the discipline, a more general epistemological knowledge which is usually only implicitly addressed during mathematics studies (e.g., Wasserman, 2018). Knowledge about mathematics as a scientific discipline comprises knowledge about the nature of the connection between the discipline of mathematics and other scientific domains and different life areas (e.g., Blum & Niss, 1991; Jankvist, 2009b). An important characteristic of this connection is that it is bi-directional, reflecting mutual contribution. Our study focuses on what secondary school mathematics teachers may learn about the nature of this mutual contribution in academic mathematics courses and what mathematicians, who teach in this program, want to teach teachers about it.

## **The mutual contribution between mathematics and other fields: The case of the discipline of mathematics**

The interaction between mathematics and different fields and life areas is realized in the discipline via applied mathematics. An important characteristic of this interaction is that it is bi-directional, involving mutual contribution.

The contribution of mathematics to solving problems in different fields – to which we refer as the *M-application* direction of the mutual contribution – occurs when mathematical tools and techniques are used to model and analyse different situations in non-mathematical contexts. It includes works on: spectral and dynamical problems of quantum mechanics, population genetics, image processing and

medical imaging, mathematical finance, quantitative risk management, and many more. For instance, mathematics contributed greatly to solving the immense problem of mapping the human genome by providing advanced mathematical tools and techniques that enabled researchers to locate genes in the decoded sequence of all nucleotide bases that make up the human genome (Malkevitch, 2002).

The contribution of different fields to the development of the discipline – to which we refer as the *developing-M* direction of the mutual contribution – occurs when work on solving problems in other fields and life areas raises new mathematical questions which generate new mathematical concepts, methods, theories, and areas of research. Such works characterize quite a few of past and modern-day mathematical developments (but not all – mathematical questions arise from both in- and outside mathematical contexts). For instance, the theory of probability has their origin in works that attempted to answer questions raised in the context of gambling (Rowlett, 2011).

### **The mutual contribution between mathematics and other fields: The case of mathematics education**

The interaction between mathematics and diverse fields and life areas is addressed in mathematics education primarily in relation to the teaching and learning of mathematical modeling, and occasionally, in relation to the teaching and learning of the history of mathematics.

Review of the literature suggests that when being used, either as a content or as a vehicle (Julie & Mudaly, 2007), the teaching and learning of mathematical modeling usually centres on the usefulness of mathematics to solve real world problems. Thus, it explicates the *M-application* direction of the mutual contribution. For example, Kaiser and Schwarz (2006) reported a change in the mathematical beliefs of students and teachers towards a more “application-oriented” view of the discipline following teachers’ participation in university-based mathematical modelling seminars. Maas (2013) reported a similar change in students who participated in an intervention that incorporated mathematical modelling activities. In contrast, the *developing-M* direction, although occasionally mentioned as a component of the modelling process (Blum & Niss, 1991; Lesh & Doerr, 2003), is rarely addressed in the educational literature of mathematical modelling.

The potential to highlight the *developing-M* direction of the mutual contribution is sometimes linked in the literature to the teaching of the history of mathematics. For example, Jankvist (2009b) claimed that an important goal of teaching the history of mathematics is to show students that the evolution of mathematics is “driven by internal and external forces” (p. 239). Tzanakis and Arcavi (2002) suggested that the history of mathematics provides an opportunity to show that mathematical research is “often motivated by questions and problems coming from apparently unrelated disciplines” (p. 205). Yet, missing are empirical studies that examine the contribution of studying the history of mathematics to knowledge about the evolutionary and developmental aspects of the discipline of mathematics, and consequently, about the mutual contribution between mathematics and other fields.

Our study focuses on what secondary school mathematics teachers may learn about the nature of this mutual contribution in an academic program that offers teachers opportunities to learn about applied mathematics and the history of mathematics. As learning is shaped by teaching, and teaching by teachers’ goals, we also investigate what mathematicians, who teach in this program, want to teach teachers about the nature of this mutual contribution.

## **Methods**

### **Setting and participants**

The study was situated in a unique master's program for practicing secondary school mathematics teachers, in which academic-level mathematics and research in mathematics education are the main components. The mathematics component comprised eight courses, designed and taught by research mathematicians. Four of these courses dealt with topics in the school curriculum at an advanced level: algebra, analysis, geometry, and probability and statistics. Three courses dealt with topics in applied mathematics, presenting modern use and application of mathematics in computer science, natural sciences, social sciences and everyday technologies. One course appraised the history and philosophy of mathematics. In addition, a final project was carried out under the guidance of a mathematician.

Five mathematicians who taught in the program participated in the study (M1, M2 ... M5). They taught all the mathematics courses in the program but two: algebra and the use of mathematics in computer science. The participating teachers were 14 program graduates (T1, T2 ... T14). All held a bachelor's degree in mathematics or in a mathematics-related field before starting the program. Their teaching experience ranged from 5 to 23 years in secondary schools, and all (except one) had experience with teaching the most advanced track of the national mathematics curriculum.

### **Data sources and analysis**

The main data source included individual semi-structured in-depth interviews with the mathematicians and the teachers. These interviews were conducted as part of a comprehensive research program that examine the relevance and contribution of academic mathematics studies to secondary school mathematics teachers' knowledge about the discipline of mathematics, and how that knowledge contribute to their teaching (Hoffmann & Even, 2021). The mathematicians were asked about their teaching goals in the program, first in general and then specifically regarding what mathematics is. Correspondingly, the teachers were asked whether they learned something new about what mathematics is from their mathematical studies in the program, and if they did, whether that knowledge contributed to their teaching. Additionally, the teachers were presented with eight phrases in relation to characteristics of the discipline of mathematics (e.g., What is a mathematical proof; The role of definitions in mathematics; Connections between different mathematical topics) and were asked to choose three for which their mathematical studies in the program enriched their knowledge. Besides having "Connections between mathematics and other fields" indicated on one of the cards, this aspect was not explicitly raised by the interviewer during the interviews.

Additional data sources included follow-up conversations with some of the mathematicians and the teachers to clarify ambiguous statements or examples in the interviews, participant observations by the first author in selected courses, the written course syllabi, and selected students' course notes.

For the purpose of this study, the interviews were analysed qualitatively in an iterative and comparative process, aiming to identify what, if at all, the mathematicians wished to teach teachers about the nature of the mutual contribution between mathematics and other fields, what the teachers learned, and how this new knowledge contributed to their teaching. We used directed content analysis

(Hsieh & Shannon, 2005) that employed two themes that correspond to the two directions of the mutual contribution: (1) *M-application*, and (2) *developing-M*.

### **Mathematicians: Mutual contribution**

All five participating mathematicians expressed in their interviews a wish to enrich teachers' knowledge about the connection between mathematics and different fields and life areas. They portrayed the connection between the discipline of mathematics and other fields as characterized by bi-directional, reciprocal contribution, referring to the mutual contribution as an important characteristic of mathematics as a scientific discipline. For example: "It is very important, the connection between mathematics and other fields and the mutual fertilization." (M4)

#### ***M-application* direction of the contribution**

The mathematicians stressed that they wished to expand teachers' knowledge about the practical worth of mathematics; that mathematics as a scientific discipline is not just a theoretical science with no connection to the real world. Instead, it can be used to develop the world. For example,

Mathematics is a tool to describe the world around us... Moreover, you can also use it. What does it mean? After describing the world, it can be used to make a better world. (M3)

The mathematicians mentioned in their interviews a total of 16 different examples (each mentioned 3-7) that illustrate the usefulness of mathematics in solving problems in various fields and areas of life, such as physics, medicine, economics, biology, engineering, geography, computer science, communication, navigation, etc. Data analysis identified about one-half of the examples (9 out of 16) as drawn from the interviewees' own or their colleagues' courses.

Most examples (12) were from contemporary mathematical work. They included population dynamics and the Lotka-Volterra equations (the predator-prey equations) in the context of Epidemiology, the PageRank algorithm used by Google Search, jpeg image compression, the RSA algorithm for encryption that is widely used for secure data transmission, and more. For instance,

The idea is... to show some examples of mathematics in things that they know, like GPS, like Google, like RSA... all sorts of things they encounter on a daily basis and there is math... and somehow convince them that like, math is not something abstract. That it really helps in life. (M5)

#### ***Developing-M* direction of the contribution**

The mathematicians associated the *developing-M* direction with the idea that the development of the discipline of mathematics lies in mathematical work related to questions. They stressed that such questions may originate in mathematics as well as outside of mathematics. They emphasized that mathematical work on questions originated outside of mathematics is central not only to solving problems in other fields, but also to the development of the discipline of mathematics itself:

A great many of the developments in mathematics had real motivation... questions from life, not from mathematics, and the mathematical way helped solve them, and then they developed mathematics. (M4)

The mathematicians mentioned in their interviews a total of nine different examples (each mentioned 1-3) that illustrate historical and contemporary contribution of work on questions originated outside of mathematics to the development of the discipline. About one-half of the examples (5 out of 9) were identified as drawn from their own or their colleagues' courses in the program. They included: calculating area, volume, velocity and acceleration that contributed to the development of calculus; navigation that accelerated the development of trigonometry; gambling that was a motivation for the development of probability and statistics; and computer science, electricity, and electronics that promoted the development of graph theory: "Graph theory is an example of a mathematical field that was developed on its own... but also from practical questions of applied mathematics." (M4)

### **Teachers: *M-application* direction only**

Twelve teachers (out of 14) reported in their interviews that their mathematics studies in the program contributed to their knowledge about the connection between mathematics and different fields and areas of life. Ten teachers also mentioned that this new knowledge contributed to their work as mathematics teachers by helping them to increase students' interest and motivation to learn mathematics. In most cases (10 teachers) such reports appeared at an early stage of the interview, before the interviewer explicitly raised the aspect of connection between mathematics and other fields by showing a card with this phrase. For example:

Interviewer: So, what did you really learn about what mathematics is as part of the program? Has your view about what mathematics is changed?

T12: The way I look at what mathematics is has changed, yes. First of all, I realized that mathematics is useful, far beyond what we think. I used to think that it would be useful for cutting a tile so that I have a certain length in Pythagoras, or how to calculate the volume of a dome or egg... But I realized that math is much more... Today I see that almost every topic is related to mathematics. I'm telling you, it could be math, physics, or just computers... I began to understand that in programming, besides the aspect of codes, which I completely don't understand, there is a lot of mathematics. There is induction, there are repetitive processes, there is recursion. In fact, I understand programming without knowing programming at all.

And later, after picking up the card with the phrase "the connection between mathematics and other fields" this teacher repeated:

[Reading the card] "Connections between mathematics and other fields." In this regard this program gave me a lot. Physics, philosophy, in computers... As I said, nature, science... It enabled me to see that everything in nature can be organized in a mathematical way... (T12)

In contrast to the mathematicians, the teachers referred to the *M-application* direction only. In line with the goals of the mathematicians regarding this direction, the teachers reported in the interviews that they enriched their knowledge about the practical use of mathematics, and how mathematics contributes to solving problems in different areas, as was illustrated in the interview excerpts above.

Five of the eight mathematics courses in the program were mentioned in the teachers' interviews in relation to the *M-application* direction. Almost all the teachers that reported on contribution to their knowledge in this regard associated it with two of the applied mathematics courses that presented modern use and applications of mathematics in everyday technologies (11 teacher) and in life sciences

and physics (10 teachers). Six teachers associated their new knowledge with the course on the use of mathematics in computer science. Two courses, probability and statistics, and the history and philosophy of mathematics, as well as the final project were mentioned by 3, 1, and 2 teachers (respectively).

The teachers mentioned in their interviews a total of eight different examples that illustrate the *M-application* direction – much less than the 16 given by the mathematicians. Correspondingly, whereas each mathematician gave 3-7 examples, the teachers gave fewer or no examples at all, even when explicitly requested. Eight teachers mentioned 2-3 examples each; four referred only generally to scientific fields as physics, biology or computer science. The example mentioned the most in the teacher interviews (8 teachers) was the role of ordinary differential equations in the study of population dynamics in relation to the treatment of oncological patients. The next most mentioned example was the PageRank algorithm used by Google search engine (3 teachers). The following excerpt is from an interview where both examples are mentioned, as well as image compression.

The mathematical applications course was an eye-opening course... Image compression, how it is expressed mathematically... I have never seen how there is at all a connection between mathematics and these things... Google search, how it works... and a medical problem that you can model... And how mathematics can help not only to develop thinking and the mind and to enjoy mathematics but... to discover its use. And I tell my students about it as a motivation. (T7)

Other specific examples were mentioned by 1-2 teachers. These were acceleration of a vehicle, mortgage, computer games, ballistics and the poverty index.

Finally, it is worth noting that seven teachers reported that they learned in the program that mathematics is a developing science. Consequently, they began to incorporate into their teaching historical information about famous mathematicians and the development of specific mathematical concepts. All seven teachers associated their new knowledge with the course on the history and philosophy of mathematics. Three teachers linked it also to the course on geometry, two to the course on probability and statistics, and the courses on algebra, analysis, and the final project were mentioned by one teacher each. Yet, in contrast to the mathematicians' explicit description of the development of the discipline of mathematics as rooted in mathematical work on questions that may originate in or outside mathematics, none of the teachers referred to the *developing-M* direction of the mutual contribution between mathematics and various fields and life areas.

## **Discussion**

This study focused on the potential of academic mathematics studies to advance teachers' knowledge about an important component of knowledge about the nature of mathematics as a scientific discipline, namely the mutual contribution between mathematics and various areas of thought and practice. The goals of the study were to examine what teachers may learn about the nature of the mutual contribution in an academic program that offers opportunities to learn about the connection between the discipline of mathematics and different fields and life areas, and what mathematicians, who teach in this program, want to teach teachers about the nature of this mutual contribution. The study was situated in a master's program for practicing secondary school mathematics teachers that included courses in applied mathematics and a course on the history of mathematics. These two areas

are considered in the mathematics education literature as having the potential to advance knowledge about the mutual contribution.

The results of our study suggest that the participating mathematicians viewed the mutual contribution between mathematics and other fields as an important component of teachers' knowledge which they wished to advance. They emphasized in this regard both the *M-application* and the *developing-M* directions of the contribution. In line with the mathematicians' goal regarding the *M-application* direction, the participating teachers considerably advanced their knowledge and appreciation of the vast contribution of the discipline of mathematics to other domains. The teachers linked their new knowledge regarding the *M-application* direction to six of the eight mathematics courses in the program, especially, but not exclusively, to the three courses that presented modern use and applications of mathematics. These findings support the indications from the literature (Kaiser & Schwarz, 2006; Maass, 2013) that the teaching and learning of mathematical modelling and applications can enhance teacher knowledge regarding the picture of mathematics as an applied science that contributes to problem solving in diverse areas of life. In addition, our study shows that academic mathematics studies that include an explicit focus on mathematical applications may be an effective way to achieve such results.

Still, in contrast to the mathematicians' expressed goal, the teachers did not mention in their interviews the *developing-M* direction. These results are in line with the way the interaction between mathematics and other fields is often dealt with in mathematics education, emphasizing the use of mathematics for solving problems in real-world contexts. A question then arises whether this aspect needs to be more explicitly incorporated into the professional education and preparation of mathematics teachers. Our position is that it should. Both knowledge of specific mathematics topics, concepts, and procedures, and knowledge about the nature of the discipline, are important for teachers and students. Knowledge about the nature of mathematics as a scientific discipline comprises knowledge about the nature of the connection between the discipline of mathematics and different scientific domains and life areas (e.g., Blum & Niss, 1991; Jankvist, 2009b), of which an important characteristic is that it is bi-directional, reflecting mutual contribution.

Potential avenues for advancing knowledge about the *developing-M* direction in the context of teaching mathematical applications and modelling could be, for example, to extend conceptualizations of the modelling cycle to explicitly include the *developing-M* direction. Additionally, using the teaching and learning of mathematical modelling as a tool for developing knowledge of mathematical concepts, procedures, and theorems that are new for the learner (which, in a way, is analogic to the way that diverse areas contribute to the development of the discipline of mathematics) (e.g., Lesh & Doerr, 2003) while explicitly connecting it to the way the discipline of mathematics is developed.

The teaching and learning of the history of mathematics has the potential to enhance knowledge regarding the *developing-M* direction by exposing the central contribution of work in other fields to the development of the discipline of mathematics as a scientific discipline (e.g., Jankvist, 2009b; Tzanakis & Arcavi, 2002). One enterprise of future research could be the development of rich and comprehensive teaching materials that explicitly exemplify such contributions throughout the history



of mathematics. A promising avenue in this regard is the teaching and learning of the history of modern applied mathematics (Jankvist, 2009a).

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