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Future teachers' experiences with modelling and for teaching modelling: A Realistic Mathematics Education perspective

Carolina Guerrero-Ortiz¹ and Chris Rasmussen²

¹Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile;
c_guerrero@yahoo.com.mx

²San Diego State University, San Diego CA, United States

The purpose of this communication is to begin discussion on how the instructional design theory of Realistic Mathematics Education (RME) can be leveraged as a framework for describing the evolution of the mathematics teacher's knowledge about the learning and teaching of mathematical modelling. Drawing on data from a semester-long seminar on modelling for future secondary mathematics teachers, we theorise how participants transition from experiencing modelling for themselves to developing knowledge for teaching modelling. We adapt the RME emergent model design heuristic where preservice teachers' experimenting with modelling activities initially serves as a model-of the modelling process, subsequently becoming a model-for their design of activities for teaching modelling to their future (imagined) students.

Keywords: *Mathematical modelling, realistic mathematics education, teacher knowledge.*

Introduction

In recent years, mathematical modelling has been included in the curricula at different school levels, resulting in challenges for mathematics teachers (Doerr, 2007). These challenges highlight issues regarding modelling experiences and knowledge that future teachers need for teaching modelling. The discussion presented in this communication addresses the context of teacher training and their learning to teach modelling as described in previous reports (Guerrero-Ortiz, 2019; 2021). In previous studies we argued that when preservice teachers engaged in task design, they go through a sequence of research and modelling for the creation of a task (Guerrero-Ortiz & Camacho-Machín, 2022), thus configuring a scenario that favours real experimentation with modelling leading to subsequent design of modelling activities for others. In this way, it was possible to examine the processes that individuals develop in their own modelling activity as well as their knowledge and activity when they design modelling tasks for their future students. Given this context, in this report we posit an explanatory theory for interpreting the change in learning modelling to teaching modelling.

We work from the premise that if teachers are to teach modelling in powerful ways for their students, they themselves must be involved in authentic modelling experiments. This will allow them to develop the experience and knowledge to subsequently design and implement modelling activities for their students (Guerrero-Ortiz, 2021). Our goal in this communication is to theorise about how the instructional design theory of Realistic Mathematics Education (RME) can be leveraged to characterise the evolution of mathematics teachers' experience and knowledge about the teaching and learning of modelling. As a starting point in this research, we address the characterization of modelling practice in a teacher training context. To do this, we position ourselves in a modelling perspective conceived as an activity for reorganising knowledge (Gravemeijer, 1997), which we will

call an activity for reorganising knowledge for teaching modelling. From this perspective, we can understand a model as a result of organizing activities. In particular, we leverage the emergent model design heuristic where preservice teachers' experimenting with modelling activities serves as a *model-of* the modelling process, subsequently becoming a *model-for* the design of activities for teaching modelling. Such organizing activity may give rise to models related to one's own modelling process and later these models can serve as a basis for teaching modelling. By "evolution" we mean the process of transformation of knowledge that takes place when individuals go from experience modelling for themselves to teaching modelling.

To situate and contextualize the proposed RME-inspired way of conceptualising and analysing the evolution of learning modelling to teaching modelling, we first provide a brief overview of how modelling has been approached. Due to the need for representing and analysing the processes that individuals develop when approaching the study of a situation using mathematical tools, modelling has typically been conceived of as a *translation* between a situation in the world of reality and some mathematical domain in the world of mathematics (Niss & Blum, 2020). A well known representation of modelling is given by mapping "f" from selected objects in the extra-mathematical domain "D" to selected objects in a mathematical domain "M", which is represented by a triple (D, f, M) . Considering modelling as a form of translation from reality to mathematics involves thinking about reality and mathematics as two disjoint entities, a position that has certain implications since it may lead to a disconnect between the meanings of the mathematical objects that emerge as a product of the modelling activity (and of course there is a disconnect from the context that gives rise to models) (Barbosa, 2006). As described in the following section, in RME modelling is conceptualised in a different way, not in terms of a translating activity but rather in terms of an organising activity.

We next present an overview of RME, which is the lens through we theorise the evolution of the mathematics teacher's experience and knowledge by considering the RME heuristic of emergent models. Subsequently, since this discussion is framed in the context of teacher training where the modelling process that participants develop as well as the one that they intend for students is not exempt from a didactic intention, we show some essential elements of the teacher's knowledge for teaching modelling. Finally, we exemplify our theoretical framing with an example that highlights preservice teachers modelling process and the process encouraged for their future (imagined) students.

Modelling from a Realistic Mathematics Education perspective

From an RME perspective, mathematical concepts, structures, and ideas are inventions that humans create to organise the phenomena of the physical, social, and mental world (Freudenthal, 1973). Mathematics as an a priori collection of concepts and skills is secondary. Thus, from this perspective, students should learn mathematics through their own constructive activity (Gravemeijer, 1999). In RME-inspired curricula, students are provided opportunities to reinvent important ideas and methods for solving problems. The starting point for their journey is grounded in experientially real situations, which ultimately leads to the development of formal mathematics. Experientially real situations are different from the real-world situations. Experientially real situations can be related to real life, for example kids getting on and off a double decker bus (Cobb et al., 1997), but they can also be

mathematical contexts or even imaginary situations such as a young girl taking a ride on a magic carpet (Wawro et al., 2021). What is necessary for a situation to be experientially real is that they must offer learners an anchoring point for them to enact situation-specific strategies that make use of student-generated models.

The notion of “model” in RME, however, is related to but slightly different than how models are conceived of in the previous section. Zandieh and Rasmussen in their RME-inspired work define models as

student-generated ways of organizing their activity with observable and mental tools. By observable tools we mean things in their environment, such as graphs, diagrams, explicitly stated definitions, physical objects, etc. By mental tools we mean the ways in which students think and reason as they solve problems—their mental organizing activity. (Zandieh & Rasmussen, 2010, p. 58)

Thus, from an RME perspective, students develop models that initially function as a *model-of* their activity and subsequently become *models-for* more sophisticated activity. Not every transition between models constitutes a *model-of/ model-for* transition, but rather this is reserved for instances in which the learner shifts their thinking, from that which derives meaning from the context situation, to thinking about mathematical relations and a new mathematical reality. This transition is captured in what is referred to as the Emergent Model heuristic (Gravemeijer, 1999).

The Emergent Model heuristic can be further specified with four levels of activity: situational, referential, general, and formal. Situational activity refers to acting in a particular task setting that is experientially real for students. Referential activity involves *models-of* that refers to activity in the original task setting. General activity involves *models-for* that facilitate a focus on interpretations and solutions independent of situation specific reasoning. Finally, Formal activity involves reasoning that is no longer dependent on the support of *models-for* mathematical activity. The *model-of* to *model-for* transition in relation to the four levels of activity is discussed throughout the subsequent sections.

An example of the *model-of/model-for* transition for the case of autonomous differential equations can be found in Rasmussen et al. (2019). In this case, population data was first depicted with a table of values. Then students estimated rate of change values and created graphs of autonomous differential equations to both determine the symbolic form of the differential equation and to make predictions about long-term population outcomes. After that, students used graphs of autonomous differential equations and phase lines to argue about the solution space. Graphs of autonomous differential equations began to take relevance on their own when students investigate a population growth situation with a varying parameter. In this way, graphs of autonomous differential equations emerged as a *model-of* the solution space structure and later function as a *model-for* reasoning about the changing structure of solutions.

Essential aspects in teacher training and teaching modelling

A central question for research on teacher knowledge is the examination of how teachers' systems of interpretation for teaching mathematics develops (Doerr, 2007). Systems of interpretation are what “teachers use to see students' ways of thinking, to respond to students' ideas, to differentiate the

nuances of contexts in their practice, to see generalised understandings that cut across contexts, and to revise their own thinking in light of their experiences” (p. 70). For teaching modelling, we highlight that teacher’s systems of interpretations (or from an RME perspective their *model-for* teaching modelling) rest on the ways that they conceptualise modelling activities. In addition, teachers' knowledge about planning, organising instruction, interaction with students, specifications of the curriculum, and their development of modelling skills related to their pedagogical content knowledge (Doerr, 2007; Greefrath et al., 2022; Guerrero-Ortiz, 2019) determine the orientation of their instruction. Some researchers point out that teacher's knowledge for teaching modelling is also related to knowledge about modelling, task-related ability to solve, analyse and create modelling tasks, ability to plan and execute modelling lessons and, ability to identify phases in pupils’ modelling processes (Borromeo Ferri & Blum, 2009; Greefrath, et al., 2022).

In light of the above ideas, when designing tasks teachers make decisions, which depend on several aspects related to their experience and knowledge such as conceptions about the type of tasks and what it means to model (Guerrero-Ortiz & Reyes-Rodríguez, 2021). This is precisely where RME provides a unique window into the evolution of learning modelling for teaching modelling.

From experimenting with modelling to creating modelling tasks

We outline our reinterpretation of RME by framing how preservice teachers transition from modelling a situation for themselves to creating a modelling task for students. To do this we reinterpret an example from a task design scenery that was featured in Guerrero-Ortiz (2019). The context of this study was a semester-long seminar on modelling in mathematics education for future secondary mathematics teachers. The seminar’s goal was to support participants to experience modelling for themselves and to develop their knowledge for teaching modelling.

To exemplify the proposed theoretical framing, we present the work of one group of two preservice teachers, which has been also studied from the development of teacher knowledge approach (Guerrero-Ortiz, 2019). During the semester participants were involved in the discussion of topics related to calculus, differential equations, and mathematical modelling, including theoretical and task-related dimensions (Borromeo Ferri & Blum, 2009). The theoretical dimension involved knowledge about modelling cycles, goals/perspectives for modelling, and types of modelling tasks. The task-related dimension considered the ability to solve, analyse, and create modelling tasks. The preservice teachers experimented with several modelling tasks and designed a task for teaching modelling. Thus, they experienced modelling for themselves as they had to research about a situation and model that situation. Then, they made decisions for creating a task for their future students to model. The overall process that individuals followed is shown in Figure 1.

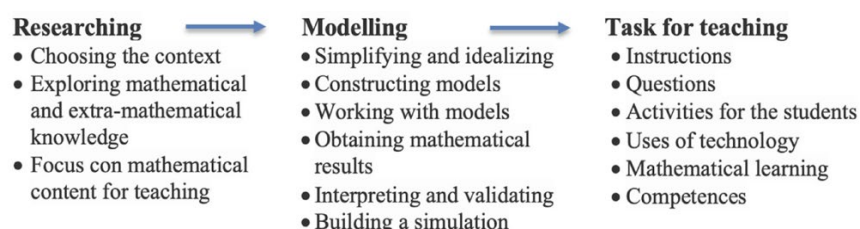


Figure 1: Tasks design process (Guerrero-Ortiz & Camacho-Machín 2022, p. 7)

Before the participants designed a task for their future students, they explored the growth of Magellanic penguin populations reported by the research literature, where the case was analysed by statistical methods (Guerrero-Ortiz, 2019). From an RME-inspired perspective, this first phase of their work is characteristic of *situational activity* because of the high relevancy of the penguin context. Then, relevant information was identified and used to explore some analytical mathematical models, which constitutes their *referential activity* as they often alluded to their analysis of the penguin situation. Finally, after this experience participants engaged in *general activity* by creating a task for teaching modelling to future students. It is general activity in the sense that their task design no longer relied on the original situational context and now the activity of the participants is focused on the design, no longer on the modelling process.

When the participants explored the situational penguin context they become aware of its characteristics and laws that determine the behaviour within the context. These features may include global information or local information, which comprises their extra mathematical knowledge. Local information refers to what is known about the situation that will allow the development of the process to address it mathematically (for example, population with border limits, elements affecting growth of population, nesting seasons and successful breeding, birth and mortality rates). The global information is that which is interesting to the situation, but not all of it is useful to solve it. For example, locations, population in the world, species of penguins, predators, and pollution. Subsequently, an organising process of simplification and idealisation takes place by recognizing the importance of the carrying capacity; this information is used to determine birth and mortality rates, and to mathematically analyse four variants of the situation, giving rise to four mathematical models, the participants only delve into the study of cases 1 and 2, $\frac{dN}{dt} = kN$; $\frac{dN}{dt} = rN(1 - \frac{N}{K})$. As noted above, we frame this part of their work as referential activity. The situational and referential activity comprises the *model-of*. The *model-of* phase is shaped by all the elements that participants learn in their modelling process. As they work in modelling for themselves, the *model-of* comprises their own modelling process and then the *model-of* shifts to function as a *model-for* teaching modelling. In addition, *the model-for* phase should include pedagogical content knowledge.

After studying the situation participants moved to work in designing a task for teaching modelling, where the process they designed to guide the students' activity is given by: understanding the situation, identifying relevant information, developing mathematical work, recognising a mathematical model and generalisation. These activities, which reflect the creation of a new reality of modelling, can be considered as *general activity* because it is no longer dependent on their earlier, situation specific work.

The task that these preservice teachers created for their future students is shown in Figure 2, where the teaching goal is for their future students to analyse the behaviour of a population based on birth and mortality rates in relation to the time.

In 1991 a massive death of penguins of the Magellanic species was evidenced. To revert the situation a group of people in 1993 decided to create a Natural Reserve of Magellanic Penguins called "Seno Otway". The reserve has capacity for hosting 6500 penguins. At the beginning the reserve started with an initial population of 300 Magellanic

penguins. From a census they calculated a mortality rate of 70 per 500 penguins and a birth rate as 200 per 500 penguins. (Guerrero-Ortiz 2019, p.1179)

1) What is the percentage of birth and death?, 2) How many penguins are born and die after a year?, 3) What was the total amount in the reserve at the year 2000?, 4) How many penguins are there today?, 5) What do you think would happen if the population exceeded the capacity of the reserve?, 6) Use the data for graphing the situation, 7) Analyse what should happen when reaching the load limit.

Figure 2: Modelling task designed for future secondary school students

The first two questions are related to the identification of relevant information and understanding the situation, these processes take place within the boundaries of the task context. With the third and fourth questions, the mathematical work is directed at studying the behaviour of the penguin population. Then, with a table of values and through the search for a behaviour pattern, a simple exponential mathematical model will be sought.

Questions 5, 6, and 7 raise a critical reflection to evaluate the scope of the model to represent the situation. For example, the preservice teachers noted that “Here the students must reflect on the exponential growth of the population because up to now we have only analysed an ideal case, where has taken into account neither load limit nor predators in the area.” These questions, in turn, suggest the consideration of the load capacity to move to the exploration of a more sophisticated model that describes the situation. Moreover, the inclusion of these more critical reflection questions provides further evidence that their prior work now functions as a *model-for* designing new tasks for others.

Following the RME perspective, the shift from modelling a situation (*model-of*) to the designing modelling activities for teaching (*model-for*) can be described according to four layers of activity. To recap, *situational activity* considers first approaches of participants to the original situation, in this case they choose an interesting situation (the Magellanic penguin population) and explore mathematical and extra-mathematical aspects related to it. This situation becomes experientially real for them when they work in understanding it. *Referential activity* considers their own modelling process as developed by the participants to study the Magellanic penguin population. They recognize some processes related to modelling and identify relevant mathematical concepts. *General activity* on the support of experience gained with modelling, involves the models that the participants have created for designing a teaching task (*models-for*). Here, elements of teacher knowledge can be involved, such as pedagogical content knowledge (PCK). *Formal activity* could be interpreted as the implementation in a teaching environment of the planned modelling activity. As this course did not follow participants into their own classrooms, we have as yet no data to describe this level of activity.

While in this case, as participants were involved in a teacher training environment, the *model-of* can be shaped by: Recognising the complexity and the necessity of simplifying a situation to be modelled, recognising mathematical and extra-mathematical knowledge, taking decisions and choose relevant information, building and exploring models that can be used to represent the situation. The *model-for* is shaped by the modelling process intended for students to follow. In addition, because the design of the task has a didactic intention, the model-for can be shaped by the participants’ PCK, which can include: knowledge of curriculum, characteristics of teaching tasks, different representations of objects, sequencing or nesting of mathematical concepts, approaches to the development of

mathematical knowledge and other elements related to the teaching of modelling (Guerrero-Ortiz, 2021; Greefrath, et al., 2022).

Conclusion

In this report we posited an RME-inspired theoretical framing where working with a modelling task and learning about the theoretical and task-related dimensions of modelling functions as the situational and referential activity, resulting in a new reality (for participants) about modelling and the modelling process. This *model-of* the modelling process serves as the new ground for which they create a modelling task for future secondary school students. This shift from modelling to being a designer of a modelling task positions their work as general activity. We see formal activity as taking place once they actually have students of their own, for then they must evaluate and support their students in the modelling process.

We conclude with a brief reflection on implications for theory, practice, and for research. Theoretically, the work presented here presents a novel adaptation of RME to a new context of teacher preparation in modelling. In terms of practice, in the course we described the preservice teachers did not learn about RME as part of their coursework. In future enactments of this course we conjecture that including RME perspective as part of the theoretical and task-related dimensions they learn about would enrich their modelling design work by offering them a curriculum design heuristic useful for a wide range of content. Regarding research, a longitudinal component to this research would allow us to investigate how their modelling experiences and knowledge gained from their preservice class are (or are not) leveraged as formal activity in their own classrooms with their own students.

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