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Investigating critical mathematical thinking when applying mathematics to real-world problems

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The capacity to use mathematics critically is essential for making decisions and forming judgements about challenges facing society, including those related to the economy, health and the environment. Critical Mathematical Thinking (CMT) involves the use of mathematical techniques and reasoning to address complex real-world problems in a wide range of real-world contexts. CMT also requires the capacity to reflect on the consequences of proposed solutions (e.g., social, ethical) on individuals and society at large. In this paper, we outline a project that aims to generate new insight into teaching practices that can promote or inhibit students' CMT development. This requires attention to what teachers see when students work on CMT tasks and how they respond – teacher noticing. Planned outcomes of the project include new theoretical and practical insights as well as resources designed to promote teaching and learning practices that support students' CMT development.

Keywords: Critical thinking, critical mathematics education, critical numeracy, mathematical applications, mathematical modelling.

Introduction

The capacity to apply mathematics critically is essential for forming balanced judgements and making prudent decisions about economic, health, environmental and other challenges faced by society. This capacity is vital for an informed and contributing citizenry and the development of a Science, Technology, Engineering and Mathematics (STEM) capable workforce (Australian Government, 2019). Critical Mathematical Thinking (CMT) involves the application of mathematics to complex real-world problems in a wide range of contexts. Identifying the impact of proposed solutions to real-world problems on individuals and society and how these can be addressed is also a key dimension of CMT (Maass et al., 2019). An inability to engage CMT when applying mathematics to real-world problems leads to fewer life opportunities, especially for the marginalised and disadvantaged (D'Ambrosio & D'Ambrosio, 2013).

Despite its importance, there have been few empirical studies focused on developing CMT in school classrooms, and there is little evidence that current curriculum and pedagogical responses to this challenge have been effective. We know, however, that developing the critical capabilities needed to apply mathematics to real-world problems is a challenging goal (e.g., Geiger et al., 2015). In the

Australian context, the challenging nature of CMT is evident in the results of large-scale international testing programs, such as the Programme for International Student Assessment (PISA), which show that Australian students have consistently underperformed on items that require mathematical reasoning when addressing real-world problems (Thomson et al., 2016).

This paper reports on the underpinnings of a current project that aims to implement, evaluate and refine an original approach to promoting students' CMT by addressing both teacher development and student learning. Our previous research has demonstrated that teachers can use relevant frameworks to design tasks that support the development of students' CMT (e.g., Fielding-Wells, 2018; Geiger, 2019). A common challenge through this research, however, is that teachers find it difficult to maintain a balance between challenging and supporting students when implementing intellectually demanding tasks, including knowing when and when not to intervene and how to respond when intervening (Sullivan, 2017). In addition, it is vital that teachers believe that students can experience success when working on CMT tasks (Beswick, 2018).

How to promote students' CMT represents a theory/practice gap that requires insight into what teachers see when students work on CMT tasks and how they respond – a complex process conceptualised as teacher noticing (Sherin et al., 2011). For teachers to promote students' CMT, they must know and understand the characteristics of CMT, be able to recognise when these characteristics are evident or absent and be capable of in situ decision-making about the support students require.

Previous research indicates that Professional Learning (PL) focused on teacher noticing leads to enhanced decision-making when supporting mathematics learning (Sherin et al., 2011). This has been evident in video-studies that support teacher reflection on their own classroom practice are employed (Santagata et al., 2021). Thus, this project will provide teachers with opportunities for video-based PL focused on developing the capacity to notice and respond to students' CMT in the moment.

Conceptual framework

The study is informed by two primary concepts: Critical mathematical thinking and teacher noticing.

Critical mathematical thinking

While there is wide acceptance that CMT is an essential capability that involves mathematical and critical thinking for effective decision-making when applying mathematics to the real world, there is increasing awareness of the need for a social dimension within this form of thinking (Jablonka, 2020; Maass et al., 2019). Ernest (2002) argues that mathematics is an agent for social empowerment, enabling life opportunities in study and employment. He identifies two key aspects of mathematical competencies: power over mathematical language, skills, and practices; and the critical capabilities needed to participate fully in society. D'Ambrosio and D'Ambrosio (2013) further claim that mathematics is impoverished if restricted to basic proficiency as critical citizenship requires deliberation over the consequences and wisdom of mathematics-based solutions to real-world problems, including those that draw conclusions from the analysis of data, make inferences, and propose hypotheses. Beswick (2018) further observes that the impoverished forms of mathematics described by D'Ambrosio and D'Ambrosio (2013) are often all that are made available to students deemed less capable (Beswick, 2018). Thus, CMT is multi-dimensional, aimed at empowering

individuals to engage critically with real-world problems and mathematical applications. Table 1 summarises our perspective on the dimensions of CMT.

Table 1: Critical Mathematical Thinking (CMT) framework

Dimensions	Elaborations
Mathematical capability	Power over language, skills, and practices of using and applying mathematics, for example, understanding mathematical concepts and principles, identifying patterns and relationships, manipulating mathematical symbols and expressions
Critical capability	Examining ideas, drawing conclusions, clarifying meaning, identifying, and analysing arguments, awareness of informal and cultural knowledge (e.g., social, political, environmental) and their influence on conclusions, considering the consequences of decisions and the ethical assessment of their impact.
Evaluating	Assessing claims and arguments (e.g., assessing strength of evidence, quality of data, reliability of sources), generating questions, generating aligned problems (e.g., validity of a solution if the circumstances change).
Reasoning	Logical thinking, inferring, proposing, and checking hypotheses, generalising, interpreting different information sources, generating evidence-based arguments (including integrated mathematical and extra-mathematical practices).
Disposition	Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, and displaying perseverance.

Addressing real-world problems, such as that in Figure 1, requires the capability to apply mathematics, and make informed decisions. To do so effectively requires an understanding of underlying assumptions, biases, and limitations of mathematical methods, for example, the use of models, and the evaluation of reasoning behind a proposed solution (Jablonka, 2020). In this sense, CMT goes beyond merely applying mathematical concepts and techniques and requires critical engagement with both mathematical and extra-mathematical practices. Mathematical practices involve competence and confidence in: logical thinking and reasoning; employing rigour in their methods; exercising caution when making generalisations; and drawing conclusions from different information sources (Steen, 2001). Extra-mathematical practices, which include critical capabilities (e.g., D’Ambrosio & D’Ambrosio 2013; Ernst, 2002), are needed for effective and ethical decision-making. This includes the capacity to evaluate the validity of a proposed solution via evidence-based reasoning. Students must also believe they can use the mathematics they know to solve problems in the real world and have the confidence to develop solutions to problems involving contentious issues (Beswick, 2018).

Ride-sharing services like Uber have changed the personal transportation industry. While such services provide new ways of getting around, some claim they have also resulted in a range of disadvantages, for example, to the taxi industry and the environment (e.g., in worsening traffic congestion). Investigate the advantages and disadvantages of ride-sharing services. Use mathematical evidence and associated arguments to develop your position on whether these services should be available in an Australian capital city of your choice.

Figure 1: An example of a Year 9/10 CMT task

Developing the capability to introduce such tasks into school classrooms is challenging, however, Goos, Geiger and colleagues (e.g., Goos et al., 2019), through a decade-long research program related to critical aspects of numeracy, demonstrated teachers were able to develop real-world tasks that required students to exercise critical thinking when applying mathematics. When implementing tasks, however, teachers faced challenges associated with: judging the level of support students require; knowing when to encourage students to continue with a particular line of thinking; and recognising when, and then how, to intervene (e.g., Van Zoest et al., 2017) – issues taken up in practices associated with teacher professional noticing.

Table 2 illustrates how CMT applies to the task presented in Figure 1.

Table 2: Critical mathematical thinking in action

Mathematical capability	Gather and analyse data and statistics related to the ride-sharing industry, the taxi industry, and the environment in the chosen Australian capital city. This may also involve the development of mathematical models.
Critical capability	Drawing conclusions from the analysis of data/statistics or models. For example, compare changes in traffic congestion levels or the number of traditional taxis operating since the introduction of ride-sharing services. This might include drawing conclusions on how ride-sharing services impact traffic congestion, air quality, and greenhouse gas emissions using mathematical models and methods. The social and political implications of ride-sharing services, such as their effects on the taxi industry, employment opportunities, and accessibility for various socio-economic groups will also need to be considered.
Evaluating	Assess the strength of the evidence. For example, if ride-sharing services are causing an increase in traffic congestion, investigate the magnitude of this increase and its impact on the city's overall traffic situation. Generate questions to explore further, such as the potential impact of ride-sharing services on public transportation usage or carbon emissions.
Reasoning	Use reasoning based on both mathematical and extra-mathematical factors. For example, if analysis/models indicate that ride-sharing significantly reduces transportation costs for users but also results in increased traffic congestion and emissions, these factors will need to be considered against the need to implement greener transportation options.
Disposition	Belief that the available mathematics can be used to address the issue and the perseverance to find an appropriate solution even though the result might be contentious.

Teacher professional noticing

Research focused on how to enhance teachers' instructional decision-making when responding to students' thinking (e.g., Jacobs et al., 2010) has evolved into the sub-field of teacher professional noticing. This identifies key practices employed when interacting with students in complex classroom

situations (van Es, 2011). These practices are shaped by teachers' perception and interpretation of students' understanding of particular events and their possible instructional responses (Jacobs et al., 2010). Decisions about how to respond to students tend to be based on predicted student activity or alternative instructional strategies determined in-the-moment (Kaiser et al., 2017). Jacobs and Empson (2016), however, distinguished two distinct noticing practices: (a) noticing students' mathematical thinking; and (b) enacting strategies that support and extend students' thinking.

Teacher beliefs (sometimes classified as an element of dispositions) influence what is noticed, how this is interpreted, and what action is taken (Beswick, 2018). Beliefs are comprised of affective and cognitive (knowledge) components – two intertwined constructs distinguishable only by the consensus they attract in terms of the power to make sense of the world (Beswick, 2007). This means that the things teachers hold to be true about teaching and their students influence which classroom events are considered worthy of attention. While the relationship between noticing and an individual's construction of the world has been discussed in previous research (e.g., Scheiner, 2021), this nuanced perspective on noticing has not been applied to previous work in CMT.

Approach

The study is being conducted via a design-based research approach, suitable for educational settings known to be complex, interactive, and reflexive in nature and is particularly appropriate for the proposed study because it: (i) aims to improve educational practice; (ii) is initiated through a specific theoretical intent; (iii) involves iterative interventions; and (iv) develops and tests theory about changes in teaching practice and student learning (e.g., Cobb et al., 2003; Geiger, 2019).

The perspective of an observer influences what is noticed. This project employs two different techniques to capture these perspectives: (1) researcher identified instances of what a teacher should notice (e.g., Kaiser et al., 2015); and (2) what teachers themselves judge as noteworthy events (e.g., Jacobs et al., 2010). In (1), teachers receive researcher advice confirming the validity of their teaching strategies or missed opportunities to notice or attend to students' thinking. This perspective relies on what the researcher sees as noteworthy. Perspective (2) relates to the point-of-view observation technique involving teacher-worn cameras that capture their in-the-moment decision-making (Sherin et al., 2011). This study takes advantage of both perspectives through teacher/researcher collaboration on how to enhance classroom noticing. Two distinct data-gathering strategies will be used across the 3-year duration of the project: (a) the pre- and post-assessment of teachers' CMT noticing capabilities via CMT reference groups; and (b) case studies of teachers' in situ CMT noticing through the Research and Development (R&D) component of the study. Seventy-five teachers will be involved in the data gathering strategy (a), with 10 of these teachers in the R&D component (b).

Instrumentation and analysis

CMT noticing assessment instrument involves pre- and post-assessment instrument of teachers' CMT noticing capabilities. Development will draw on Kaiser et al.'s (2015) work for making judgements about teacher effectiveness on instructional competencies, accommodating cultural and contextual factors with authentic video vignettes drawn from Australian classrooms. Analysis will allow for judgements about (a) growth in teacher CMT noticing for those involved in the R&D component of

the study and (b) the growth in R&D teacher CMT noticing in comparison to teachers participating in only the symposium pre- and post-assessment.

The perspectives of researchers and teachers will be captured using two video-recording techniques.

Stationary video-recording and video-stimulated recall to investigate what teachers should notice. This involves sessions in which teachers view excerpts of video from previous classrooms in which CMT tasks are implemented. Excerpts will be based on what researchers perceive to be critical moments in students' attempts to solve a CMT-eliciting task.

Point-of-view video-recording/video-stimulated recall, aimed at establishing what teachers notice about students' CMT. Relevant video excerpts will be captured using wearable cameras teachers will self-activate when they notice what they perceive as instances of students' CMT.

Video data and video-stimulated recall interviews will be transcribed and analysed through thematic coding, drawing on theoretical structures developed from the relevant research literature (e.g., Geiger, 2019; Beswick, 2018; van Es, 2011). This analysis will include a focus on what researchers perceive teachers should notice compared to what teachers themselves do notice and then integrate those with teachers' responses to the CMT noticing assessment instrument to inform the development of a Framework for Teachers' CMT Noticing.

Planned and anticipated outcomes

Critical thinking has been identified as vital in both 21st Century Skills (e.g., European Commission, 2018) and STEM education agendas (e.g., Bybee, 2013). Mathematics is often underrepresented in critical reasoning activities within interdisciplinary STEM (English, 2016), which conflicts with the role of mathematics as a foundation for critical reasoning. The project directly addresses the development of CMT within the context of school mathematics – an approach aligned with the evidentiary practices described in both 21st Century Skills and STEM education agendas (Maass et al., 2019). This study will offer original insights into the promotion of CMT capability by: generating a new empirically grounded perspective on CMT; linking CMT growth to teacher noticing and dispositions; and employing a novel methodology.

Previous research into CMT, teacher noticing, and dispositions has been conducted independently. This project has a focus on the quality of decision-making when teachers notice key aspects of students' work on CMT tasks. Thus, this study builds on our previous work (e.g., Fielding-Wells, 2018; Geiger, 2019; Goos et al., 2019; Kaiser et al., 2017; Scheiner, 2021; Beswick, 2018) to generate original insight into how teachers interpret and respond when students work on CMT tasks in situ.

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References

Australian Government (2019). *Support for science, technology, engineering and mathematics (STEM)*. Retrieved from <https://www.education.gov.au/support-science-technology-engineering-and-mathematics>

- Beswick, K. (2007). Teachers' beliefs that matter in secondary mathematics classrooms. *Educational Studies in Mathematics*, 65, 95–120. <https://doi.org/10.1007/s10649-006-9035-3>
- Beswick, K. (2018). Systems perspectives on mathematics teachers' beliefs: Illustrations from beliefs about students. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 3–18). PME.
- Bybee, R. W. (2013). The next generation science standards and the life sciences. *The Science Teacher*, 80(2), 25–32.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13
- D'Ambrosio, U., & D'Ambrosio, B. S. (2013). The role of ethnomathematics in curricular leadership in mathematics education. *Journal of Mathematics Education at Teachers College*, 4(1), 19–25. <https://doi.org/10.7916/jmetc.v4i1.767>
- English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education*, 3, 3.
- Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, 15(1), 1–16.
- European Commission (2018). *Key competencies and basic skills*. <https://education.ec.europa.eu/education-levels/school-education/key-competences-and-basic-skills>
- Fielding-Wells, J. (2018). Dot plots and hat plots: Supporting young students emerging understandings of distribution, center and variability through modeling. *ZDM—Mathematics Education*, 50(7), 1125–1138. <https://doi.org/10.1007/s11858-018-0961-1>
- Geiger, V. (2019). Using mathematics as evidence supporting critical reasoning and enquiry in primary science classrooms. *ZDM—Mathematics Education*, 51(6), 929–940. <https://doi.org/10.1007/s11858-019-01068-2>
- Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM—Mathematics Education*, 47(4), 531–548. <https://doi.org/10.1007/s11858-015-0708-1>
- Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2019). *Enhancing numeracy teaching and learning across the curriculum*. Allen & Unwin.
- Jablonka, E. (2003). Mathematical literacy. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. S. K. Leung (Eds.), *Second international handbook of mathematics education* (pp. 75–102). Kluwer.
- Jablonka, E. (2020). Critical thinking in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 159–163). Springer.

- Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM—Mathematics Education*, 48(1-2), 185–197. <https://doi.org/10.1007/s11858-015-0717-0>
- Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202.
- Kaiser, G., Blömeke, S., König, J., Busse, A., Döhrmann, M., & Hoth, J. (2017). Professional competencies of (prospective) mathematics teachers—Cognitive versus situated approaches. *Educational Studies in Mathematics*, 94, 161–182. <https://doi.org/10.1007/s10649-016-9713-8>
- Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education*, 13(3), 369–387. <https://doi.org/10.1007/s10763-015-9616-7>
- Kuckartz, U., & Rädiker, S. (2019). *Analyzing qualitative data with MAXQDA*. Springer.
- Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019). The role of mathematics in interdisciplinary STEM education. *ZDM—Mathematics Education*, 51(6), 869–884. <https://doi.org/10.1007/s11858-019-01100-5>
- Santagata, R., König, J., Scheiner, T., Nguyen, H., Adleff, A. K., Yang, X., & Kaiser, G. (2021). Mathematics teacher learning to notice: A systematic review of studies of video-based programs. *ZDM—Mathematics Education*, 53(1), 119–134. <https://doi.org/10.1007/s11858-020-01216-z>
- Scheiner, T. (2021). Towards a more comprehensive model of teacher noticing. *ZDM—Mathematics Education*, 53(1), 85–94. <https://doi.org/10.1007/s11858-020-01202-5>
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing*. Routledge.
- Steen, L. A. (2001). Mathematics and numeracy: Two literacies, one language. *The Mathematics Educator*, 6(1), 10–16.
- Sullivan, P. (2017). *Challenging mathematical tasks*. Oxford University Press.
- Thomson, S., De Bortoli, L., & Underwood, C. (2016). *PISA 2015: A first look at Australia's results*. ACER.
- Van Es, E. A. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R.A. Philipp (Eds.), *Mathematics teacher noticing* (pp. 164–181). Routledge.
- Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning*, 19(1), 33–54. <https://doi.org/10.1080/10986065.2017.1259786>