Topology of the relative motion: circular and eccentric reference orbit cases

Jordi Fontdecaba I Baig, Gilles Metris, Pierre Exertier

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Abstract

This paper deals with the topology of the relative trajectories in flight formations. The purpose is to study the different types of relative trajectories, their degrees of freedom, and to give an adapted parameterization. The paper also deals with the research of local circular motions. Even if the exist only when the reference orbit is circular, we extrapolate initial conditions to the eccentric reference orbit case.This alternative approach is complementary with traditional approaches in terms of cartesian coordiantes or differences of orbital elements.

1 Introduction

Flight formations have become a key technology for present and future space missions. From a dynamical point of view, they have been studied since the first spatial rendezvous operations [2]. There is a certain number of publications in the literature presenting different analytical theories for temporal evolution of the relative motion [4], [5]. Different authors have explored the possibility of finding periodic relative orbits under the effect of perturbations [1], [7]. We consider that, before taking into account perturbations, there is a field of investigation which consists in describing all the possible relative trajectories for the non-perturbed case. This paper tackles this task. We lay stress on circular or near circular local trajectories because circular local trajectories have special properties which make them specially useful for space missions (interferometry missions, LISA mission, ...).
First, we study the circular reference orbit case. Since the relative trajectory is, under some assumptions, an ellipse, we propose to study it using "local orbital elements" defined in [3]. We are explaining that the six local orbital elements are not independent, and that the relative ellipse has only four degrees of freedom. Second, when the reference orbit is slowly eccentric, the relative motion is no more an ellipse. Our local orbital elements are always a well-suited parametrization because the resulting relative motion is a kind of perturbed ellipse. For highly eccentric references orbits, we propose a parameterization based, only, on four parameters, two for the inplane motion, and two for the out-of-plane motion.
The paper has the following structure: second section is dedicated to the statement of the problem. Third section deals with the topology of relative motion while section four is devoted to the search of circular relative motions.

2 The equations of the relative motion

2.1 The statement of the problem

In the whole text, we use the well-known keplerian elements: the semi-major axis a, the eccentricity e, the inclination i, the right ascension of ascending node Ω, the argument of perigee ω, and the mean anomaly M. The motion, studied in an inertial reference frame denoted IJK, is described through temporal series of keplerian elements as well as of positions $\overrightarrow{r}|_{IJK}$ and velocities $\overrightarrow{v}|_{IJK}$. We use the following notations:

$$\begin{aligned}\overrightarrow{EO} &= (a, e, i, \Omega, \omega, M)^T \\ \overrightarrow{x}|_{IJK} &= \begin{pmatrix} \overrightarrow{r}|_{IJK} \\ \overrightarrow{v}|_{IJK} \end{pmatrix}\end{aligned}$$

From now on, we will consider a reference orbit which will be described by its orbital elements or by its position and velocity. This reference orbit can be the orbit of one of the satellites of the formation or it can correspond to a fictitious point. For simplicity, we will name it the reference satellite indicated by the subscript $_r$. Inertial position and velocity can also be projected into the orbital local frame (RTN) defined by the given reference orbit. Relations between projections into IJK frame and RTN frame are quite simple:

$$\overrightarrow{x}|_{IJK} = \begin{pmatrix} \mathcal{R}(\overrightarrow{EO}_r) & 0 \\ 0 & \mathcal{R}(\overrightarrow{EO}_r) \end{pmatrix} \overrightarrow{x}|_{RTN}$$

where the matrix $\mathcal{R}(\overrightarrow{EO})$ is:

$$\mathcal{R}(\overrightarrow{EO}) = \begin{pmatrix} \cos\Omega\cos u - \sin\Omega\sin u\cos i & -\cos\Omega\sin u - \sin\Omega\cos u\cos i & \sin\Omega\sin i \\ \sin\Omega\cos u + \cos\Omega\sin u\cos i & -\sin\Omega\sin u + \cos\Omega\cos u\cos i & -\cos\Omega\sin i \\ \sin u\sin i & \cos u\sin i & \cos i \end{pmatrix}$$

with $u = \omega + f$, where f is the true anomaly.

The relative motion between a satellite and the reference satellite can be described by the difference of position and velocity projected in the orbital frame of the reference orbit:

$$\Delta\overrightarrow{x}|_{RTN} = \overrightarrow{x}|_{RTN} - \overrightarrow{x}_r|_{RTN} = \begin{pmatrix} \Delta\overrightarrow{r}|_{RTN} \\ \Delta\overrightarrow{v}|_{RTN} \end{pmatrix} \tag{1}$$

or by the differences of orbitals elements between the two orbits:

$$\Delta\overrightarrow{EO} = \overrightarrow{EO} - \overrightarrow{EO}_r \tag{2}$$

We will also use the following notation:

$$\begin{aligned}\Delta\overrightarrow{r}|_{RTN} &\equiv (\Delta R, \Delta T, \Delta N)^T \\ \Delta\overrightarrow{v}|_{RTN} &\equiv (\Delta V_R, \Delta V_T, \Delta V_N)^T\end{aligned}$$

2.2 The temporal evolution of the relative motion

In order to obtain easy analytical expressions of the relative motion, we study the linear, non-perturbed problem. As it is proved in [3], the equations of the motion of this problem are:

$$\begin{aligned}\Delta R(t) &= \frac{r_r}{a_r}\Delta a_0 - a_r\cos f_r\Delta e_0 + \frac{a_r e_r}{\eta_r}\sin f_r\Delta M_0 - \frac{3}{2}\frac{n_r e_r}{\eta_r}\sin f_r(t-t_0)\Delta a_0 \\ \Delta T(t) &= a_r\left(1 + \frac{1}{\eta_r^2}\frac{r_r}{a_r}\right)\sin f_r\Delta e_0 + r_r\cos i_r\Delta\Omega_0 + r_r\Delta\omega_0 + \frac{a_r^2\eta_r}{r_r}\Delta M_0 - \frac{3}{2}\frac{a_r n_r\eta_r}{r_r}(t-t_0)\Delta a_0 \\ \Delta N(t) &= r_r\sin u_r\Delta i_0 - r_r\sin i_r\cos u_r\Delta\Omega_0\end{aligned} \tag{3}$$

These equations give temporal evolution of relative position projected in the local orbital frame as function of six parameters: $\Delta\overrightarrow{EO}(t_0)$. They are equivalent to the Lawden's equations [6]. Lawden's equations are parameterized with initial relative position and velocity while our equations are parameterized with initial differences of orbital elements. Our parametrization gives more compact expressions and enables a better physical understanding of the role of each initial condition. Thereafter, we describe resulting relative motion in three cases: (i) when the reference orbit is circular, (ii) when the reference orbit is slightly eccentric, (iii) when the reference orbit is strongly eccentric.

3 The topology of the relative motion

3.1 The circular reference orbit case

When the reference orbit is circular, we parameterize the two orbits with a set of non-singular elements: $\overrightarrow{ENS} = (a, C, i, \Omega, S, \lambda)^T$, where the elements (e, ω, M) have been replaced by the elements (C, S, λ) defined by:

$$\begin{aligned} C &= e\cos\omega && (4)\\ S &= e\sin\omega && (5)\\ \lambda &= \omega + M && (6) \end{aligned}$$

We also define the differences of non-singulars elements as:

$$\Delta\overrightarrow{ENS} = \overrightarrow{ENS} - \overrightarrow{ENS}_r \quad (7)$$

The particularization of equations (3), after the introduction of non-singular elements, gives:

$$\begin{aligned} \Delta R(t) &= \Delta a_0 - a_r\left(\cos\lambda\Delta C_0 + \sin\lambda\Delta S_0\right) && (8)\\ \Delta T(t) &= a_r\left(2\sin\lambda\Delta C_0 - 2\cos\lambda\Delta S_0 + \Delta\lambda_0 - \cos i_r\Delta\Omega_0 - \frac{3}{2}n(t-t_0)\Delta a_0\right)\\ \Delta N(t) &= a_r\left(\sin\lambda\Delta i_0 - \sin i_r\cos\lambda\Delta\Omega_0\right) \end{aligned}$$

This solution is equivalent to the well-known Clohessy-Wilthsire equations for the non-perturbed motion. We note that these equations correspond with the standard parametrical equation of an ellipse centered on the origin of the local frame except for two kind of terms:

- *terms providing from a difference of semi-major axis* : a different semi-major axis gives different orbital frequencies which produce a secular growth of the difference in the T component. As this effect destroys the formation in a short period of time, we do the hypothesis: $\Delta a_0 = 0$. This hypothesis may not be true for docking or rendezvous operations, but it is always fulfilled in flight formations.
- *constant term on the T axis*: these terms, $(\Delta\lambda_0 - \cos i_r\Delta\Omega_0)$, can be removed just by a shift on the origin of the axis. We do not take care of these terms.

Disgarding foregoing terms, equations (8) rewrite:

$$\begin{aligned} \Delta R &= -a_r\Delta C_0\cos\lambda - a_r\Delta S_0\sin\lambda && (9)\\ \Delta T &= -2a_r\Delta S_0\cos\lambda + 2a_r\Delta C_0\sin\lambda\\ \Delta N &= -a_r\Delta\Omega_0\cos\lambda + a_r\Delta i_0\sin\lambda \end{aligned}$$

which correspond to the elliptical motion. As it is usually done on the two-body problem, the elliptical motion will be parameterized through a set of orbital elements called *local orbital elements* $(\overrightarrow{eo}_l)$: semi-major axis (a_l), eccentricity (e_l), inclination (i_l), longitude of ascending node (Ω_l), longitude of perigee (ω_l) and the phase (φ_l), defined as the difference of phase between the reference anomaly, and the anomaly of the local orbit.

There are two main differences with respect to classical keplerian elliptical motion (i) the origin of the axis does not correspond to a focus of the ellipse, but with the center. This leads to an ambiguity on the definition of the longitude of the perigee which is solved establishing that $\omega \in [0, \pi]$ (ii) the angular velocity is not dependent on the distance to the origin, it is constant and equal to the angular velocity of the reference frame. That is, the period of any local orbit corresponds to the period of the

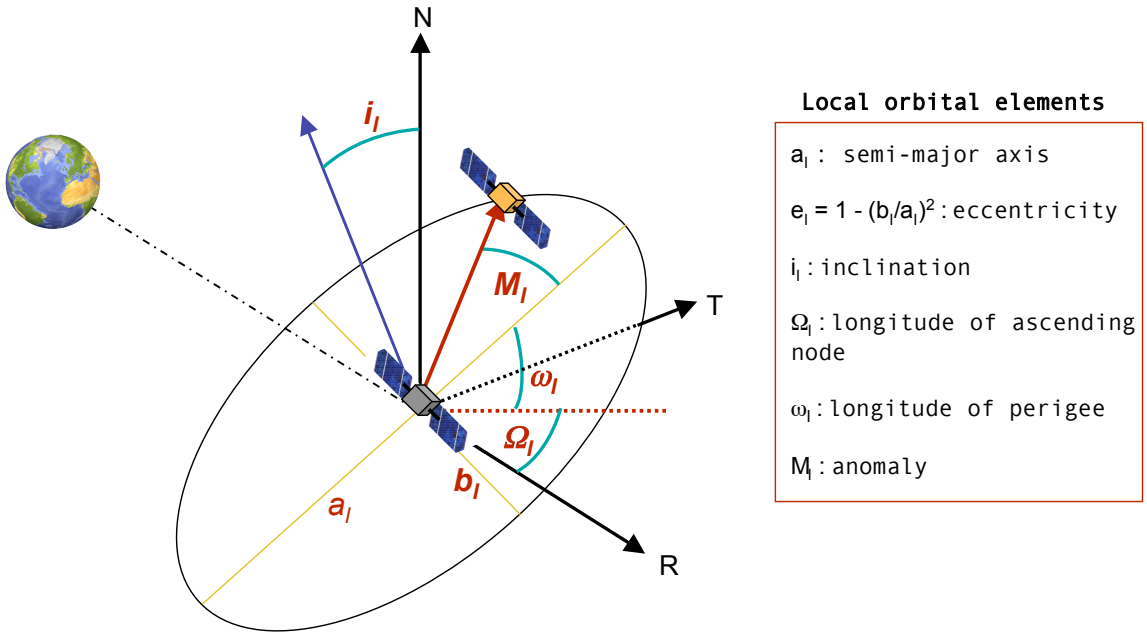

Figure 1: *definition of local orbital elements*

reference frame.

It is possible to obtain relations between the local orbital elements and the initial conditions as it is done in [3]. It is also possible to prove that only four local orbital elements are independents. The size (a_l) and the constant phase (φ_l) can always be chosen to our convenience. The other four parameters may be separated in two groups: the elements which give the form of the local orbit (e_l, ω_l), and the elements which give the orientation of the plane of the local orbit (i_l, Ω_l). One group determines the other. We have decided to express the local eccentricity and the local perigee as function of other variables (i_l, Ω_l). Tedious but simple algebraic manipulations yield:

$$\frac{e_l^4}{(2-e_l^2)^2} = \frac{\left[9 + 6\tan^2 i_l(4\cos^2\Omega_l - \sin^2\Omega_l) + \tan^4 i_l(4\cos^2\Omega_l + \sin^2\Omega_l)^2\right]}{\left(5 + \tan^2 i_l(4\cos^2\Omega_l + \sin^2\Omega_l)\right)^2} \tag{10}$$

and:

$$\tan^2\omega_l - \frac{3(\cos^2\Omega_l - \sin^2\Omega_l) + \sin^2 i_l(\cos^2\Omega_l + 4\sin^2\Omega_l)}{3\cos i_l \cos\Omega_l \sin\Omega_l}\tan\omega_l - 1 = 0 \tag{11}$$

3.2 The low eccentric reference orbit case

When the reference orbit is sligthly eccentric, we simplify equations (3) by means of an expansion in powers of the eccentricity up to first order:

$$\begin{aligned}
\frac{\Delta R}{a_r} &= -\cos\lambda\,(\Delta C_0 + S_r\Delta\lambda_0) - \sin\lambda\,(\Delta S_0 - C_r\Delta\lambda_0) + \mathcal{O}(e^2) \qquad (12)\\
\frac{\Delta T}{a_r} &= \sin\lambda\,[2\Delta C_0 + S_r\,(\Delta\lambda_0 - \Delta\Omega_0\cos i_r)] + \cos\lambda\,[-2\Delta S_0 + C_r\,(\Delta\lambda_0 - \Delta\Omega_0\cos i_r)]\\
&\quad + (\Delta\lambda_0 - \Delta\Omega_0\cos i_r) - \Delta e_0\frac{e_r}{2}\,(\sin 2\lambda\cos 2\omega_r - \cos 2\lambda\sin 2\omega_r) + \mathcal{O}(e^2)
\end{aligned}$$

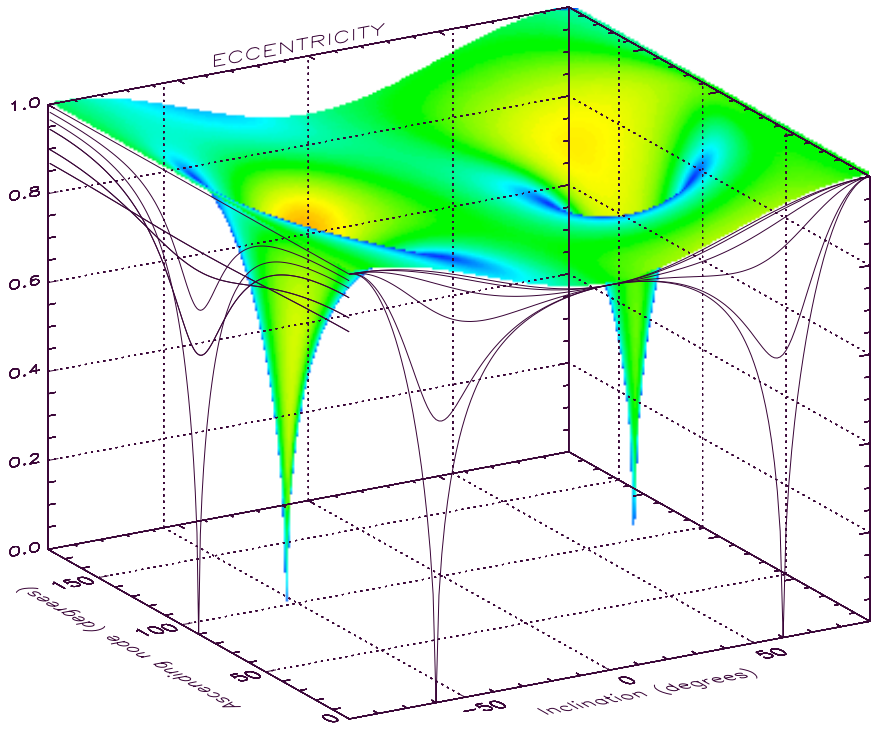

Figure 2: *Eccentricity as function of the inclination and the longitude of the ascending node*

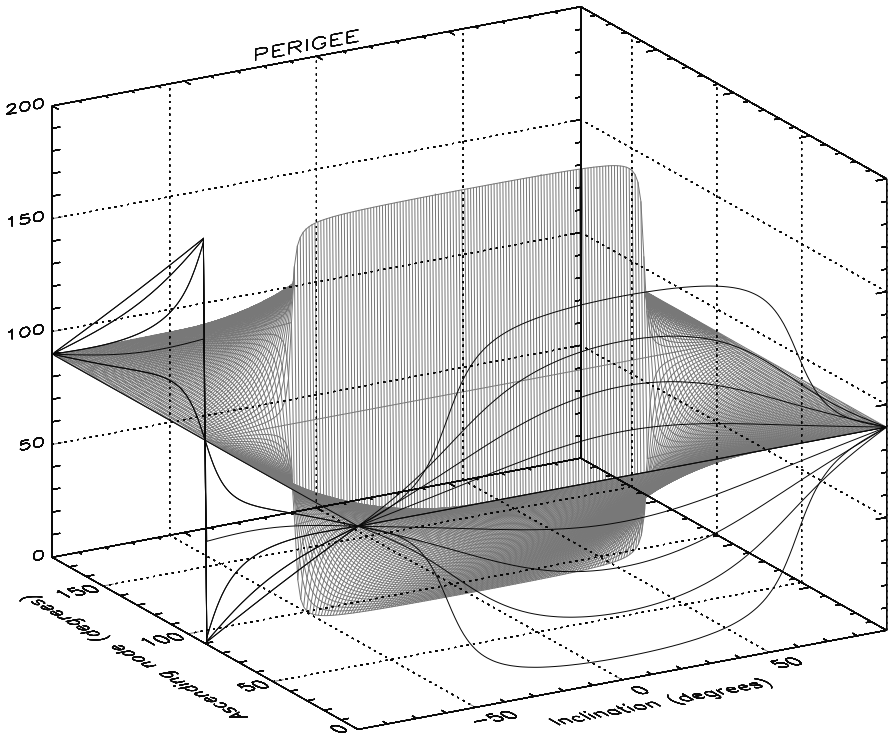

Figure 3: *Perigee as function of the inclination and the longitude of the ascending node*

$$\begin{aligned}\frac{\Delta N}{a_r} &= \sin\lambda\Delta i_0 - \cos\lambda\sin i_r\Delta\Omega_0 + \frac{e_r}{2}\left(\cos\omega\sin i_r\Delta\Omega_0 - \sin\omega_r\Delta i_0\right)\\ &+\frac{e_r}{2}\cos 2\lambda\left(\cos\omega_r\sin i_r\Delta\Omega_0 + \sin\omega_r\Delta i_0\right) + \frac{e_r}{2}\sin 2\lambda\left(\sin\omega_r\sin i_r\Delta\Omega_0 - \cos\omega_r\Delta i_0\right) + \mathcal{O}(e^2)\end{aligned}$$

In this set of equations we identify following terms:

- *constant terms*: there are constant terms not only in T axis (as in the circular case) but also in the N axis. Once again, they can be cancelled by changing the origin of the axis.
- *elliptical terms*: the terms in $\cos\lambda$ and $\sin\lambda$. The coefficients differs from the coefficients of the circular reference orbit case. The local orbital elements can be computed using the same relations as in the circular reference orbit case but with new parameters $\overrightarrow{P}$:

$$\begin{aligned}A &= -a_r\left(\Delta C_0 + S_r\Delta\lambda_0\right) & B &= -a_r\left(\Delta S_0 - C_r\Delta\lambda_0\right)\\ C &= a_r\left(-2\Delta S_0 + C_r\left(\Delta\lambda_0 - \Delta\Omega_0\cos i_r\right)\right) & D &= a_r\left(2\Delta C_0 + S_r\left(\Delta\lambda_0 - \Delta\Omega_0\cos i_r\right)\right)\\ E &= -a_r\sin i_r\Delta\Omega_0 & F &= a_r\Delta i_0\end{aligned}$$

- *double orbital frequency terms*: relative motion is no more an ellipse because of these terms which do not correspond to an elliptical motion.

From a topological point of view there is a major modification with respect to the circular case: $\Delta\lambda$ plays a new role. While in the circular case it determines only a constant, in the low eccentricity case it plays a role on the ellipse determination and on the double orbital frequency terms. The modifications with respect to circular case are of the order of the eccentricity, since the eccentricity is low, the difference are not very important.

3.3 The high eccentric reference orbit case

Our departure point is equations (3). For the same precedent reasons, we impose $\Delta a = 0$, obtaining equations:

$$\begin{aligned}\Delta R(t) &= -a_r\cos f_r\Delta e_0 + \frac{a_r e_r}{\eta_r}\sin f_r\Delta M_0\\ \Delta T(t) &= a_r\left(1 + \frac{1}{\eta_r^2}\frac{r_r}{a_r}\right)\sin f_r\Delta e_0 + r_r\cos i_r\Delta\Omega_0 + r_r\Delta\omega_0 + \frac{a_r^2\eta_r}{r_r}\Delta M_0\\ \Delta N(t) &= r_r\sin u_r\Delta i_0 - r_r\sin i_r\cos u_r\Delta\Omega_0\end{aligned} \tag{13}$$

The in-plane and out-of-plane motions can be decoupled doing the following changing of variables:

$$\begin{aligned}\Delta\omega' &= \Delta\omega + \Delta\Omega\cos i_r\\ \Delta\Omega' &= \Delta\Omega\sin i_r\end{aligned} \tag{14}$$

After, we break down the variable $u = \omega + f$ into his components:

$$\begin{aligned}\Delta i'' &= \sin\omega_r\Delta i - \cos\omega_r\Delta\Omega'\\ \Delta\Omega'' &= \cos\omega_r\Delta i + \sin\omega_r\Delta\Omega'\end{aligned} \tag{15}$$

Finally, we obtain:

$$
\begin{aligned}
x &= \frac{\Delta R}{a_r \Delta e} = -\cos f_r + eK_1 \sin f_r \\
y &= \frac{\Delta T}{a_r \Delta e} = \frac{2 + e\cos f_r}{1 + e\cos f_r} \sin f_r + K_1 \left(1 + e\cos f_r\right) + \frac{K_2}{1 + e\cos f_r} \\
z &= \frac{\Delta N}{a_r \Delta e} = \frac{1}{1 + e\cos f_r} \left(K_3 \cos f_r + K_4 \sin f_r\right)
\end{aligned} \tag{16}
$$

with the coefficients:

$$
\begin{aligned}
K_1 &= \frac{\Delta M}{\eta_r} \\
K_2 &= \eta^2 \left(\Delta\omega + \cos i_r \Delta\Omega\right) \\
K_3 &= \eta_r^2 \left(\sin\omega_r \Delta i - \cos\omega_r \sin i_r \Delta\Omega\right) \\
K_4 &= \eta_r^2 \left(\cos\omega_r \Delta i + \sin\omega_r \sin i_r \Delta\Omega\right)
\end{aligned} \tag{17}
$$

These equations express all the possible relative trajectories as function of four parameters. K_1, K_2 parameterize the in-plane motion and K_3, K_4 the out-of-plane motion. The out-of-plane motion is given by the sum of a sinus and a cosinus functions divided by $(1 + e\cos f_r)$. The in-plane motion is more complicated. K_1 produces a circular motion:

$$
\begin{aligned}
x &= -\cos f_r + eK_1 \sin f_r \\
y - K_1 &= eK_1 \cos f_r + \sin f_r
\end{aligned} \tag{18}
$$

and $K2$ produces a motion only on y axis:

$$
y - K_2 = \frac{-eK_2 \cos f_r}{1 + e\cos f_r}
$$

The values of these constants determine the form of the in-plane motion. In figure (4) we plot the form of the in-plane motion for different values of K_1, K_2, when the reference orbit eccentricity is 0.6. We verify that for high values of K_1 the motion is circular while K_2 gives linear motions on T axis.

4 Looking for local circular motions

The planes where the local motion is circular, have a very interesting property: due to the fact that the local orbital frequency is independent of the distance to the origin, all the satellites rotate at the same angular velocity around the center of the circle and their relatives distances do not change. As a consequence, all initial configurations remain invariant all along the trajectory. These planes represent very interesting possibilities for flight formation where it is necessary to keep inter-relative distances constant, as in interferometry missions or the future LISA mission. Actually, residual variations of the distance will be produced by perturbations and non-linear effects that have been neglected.

4.1 When the reference orbit is circular

There are two points in figure (2) where local eccentricity is zero. Using equations (10) and (11), it is possible to determine their local plane:

$$
\begin{aligned}
i_{l1} &= 60^\circ \qquad \Omega_{l1} = 90^\circ \\
i_{l2} &= -60^\circ \qquad \Omega_{l2} = 90^\circ
\end{aligned}
$$

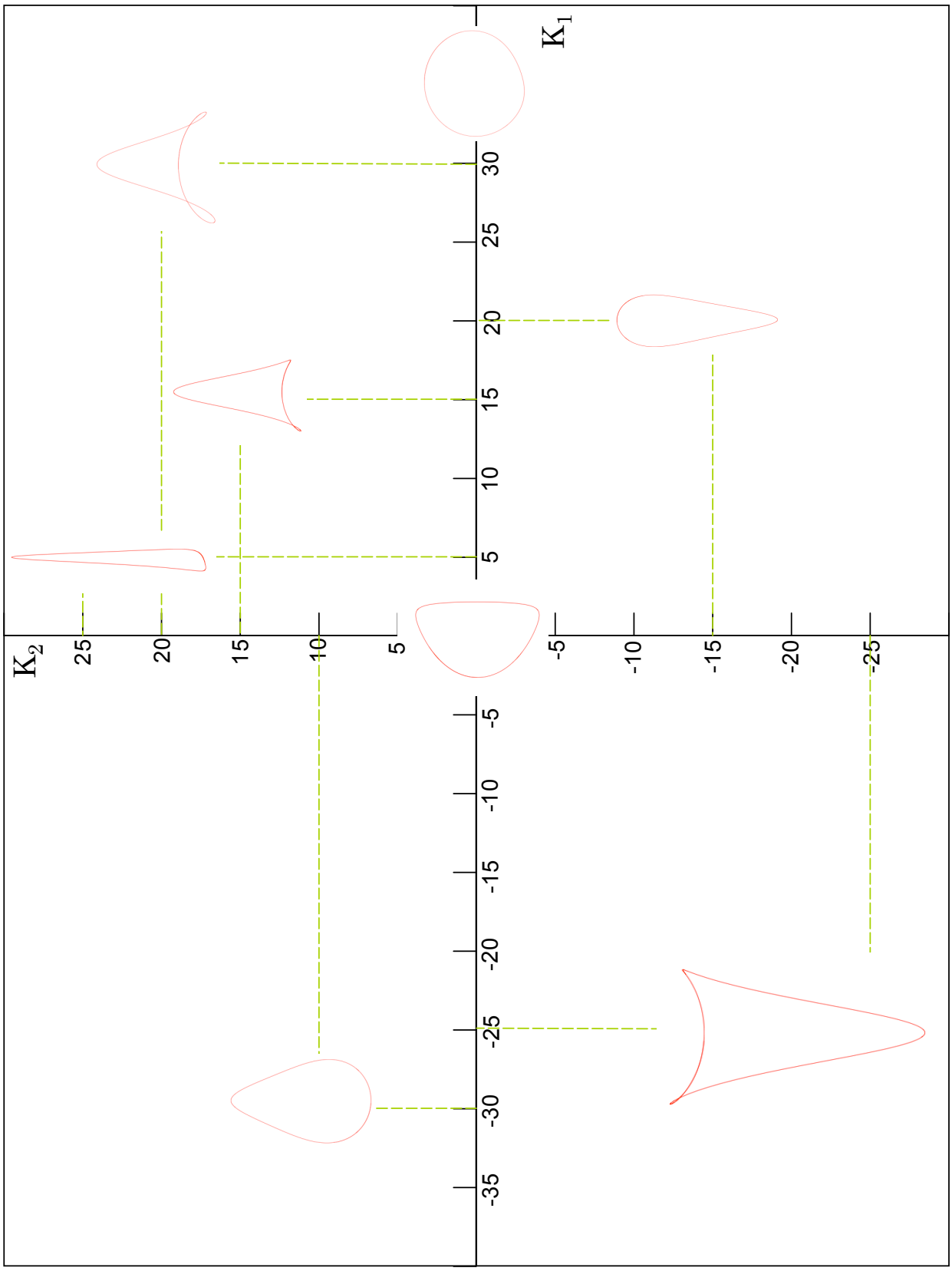

Figure 4: *In-plane motion as function of* K_1 *and* K_2

with the corresponding initial conditions:

$$\begin{aligned}\Delta\overrightarrow{ENS}_1 &= (0, \Delta C, -\sqrt{3}\Delta S, \sqrt{3}\Delta C, \Delta S, \Delta\lambda)^T \\ \Delta\overrightarrow{ENS}_2 &= (0, \Delta C, \sqrt{3}\Delta S, -\sqrt{3}\Delta C, \Delta S, \Delta\lambda)^T\end{aligned} \tag{19}$$

We recall that, due to equations (9), ΔC and ΔS determine the size (a_l) and the constant phase of the ellipse (φ_l).

4.2 When the reference orbit is slightly eccentric

When the reference orbit is not circular, the set of equations (12) reveals that it is not possible to find perfect circular local motions. But, our analysis of these equations enables to establish the necessary conditions to provide local motions as close as possible to a circle. When changing the parameterization (12) by using differential orbital elements we obtain:

$$\begin{aligned}\frac{\Delta R}{a_r} &= -\cos f_r \Delta e + e_r \sin f_r \Delta M \\ \frac{\Delta T}{a_r} &= 2\sin f_r \Delta e - e_r \cos f_r \sin f_r \Delta e + \left(\Delta M + \Delta\omega'\right) + e_r \cos f_r \left(\Delta M - \Delta\omega'\right) \\ \frac{\Delta N}{a_r} &= \left(\Delta i' \cos f_r + \Delta\Omega' \sin f_r\right) - e_r \cos f_r \left(\Delta i' \cos f_r + \Delta\Omega' \sin f_r\right)\end{aligned} \tag{20}$$

where we have introduced the following variables:

$$\begin{aligned}\Delta\omega' &= \Delta\omega + \Delta\Omega \cos i_r \\ \Delta i' &= \sin\omega_r \Delta i - \cos\omega_r \sin i_r \Delta\Omega \\ \Delta\Omega' &= \cos\omega_r \Delta i + \sin\omega_r \sin i_r \Delta\Omega\end{aligned} \tag{21}$$

Considering only elliptical terms, the following initial conditions enable to obtain the same circular motion as in the circular reference orbit case (same local inclination and ascending node):

- *SET 1*

$$\begin{aligned}&\Delta a = 0 && \Delta e = \Delta e && \Delta i = \sqrt{3}\sin\omega_r \Delta e \\ &\Delta\Omega = -\sqrt{3}\frac{\cos\omega_r}{\sin i_r}\Delta e && \Delta\omega = \sqrt{3}\frac{\cos\omega_r}{\tan i_r}\Delta e && \Delta M = 0\end{aligned}$$

- *SET 2*

$$\begin{aligned}&\Delta a = 0 && \Delta e = 0 && \Delta i = \sqrt{3} e_r \cos\omega_r \Delta M \\ &\Delta\Omega = \sqrt{3} e_r \frac{\sin\omega_r}{\sin i_r}\Delta M && \Delta\omega = -\left(1 + \sqrt{3} e_r \frac{\sin\omega_r}{\tan i_r}\right)\Delta M && \Delta M = \Delta M\end{aligned}$$

The first set of initial conditions is parameterized by a difference of eccentricity, while the second set is parameterized by a difference of anomaly. Thereafter, we refer to the set 1 as "difference of eccentricity" and to set 1 as "difference of anomaly". Resulting motion is not circular because of double orbital frequency terms. In figure (5) we compare the variation of the distance from the origin for each set of initial conditions. Both sets of conditions produce double orbital frequency terms in the N axis, but, Δe produces also these terms in the T axis. That's why the variations are more important in the left part of the figure (5).

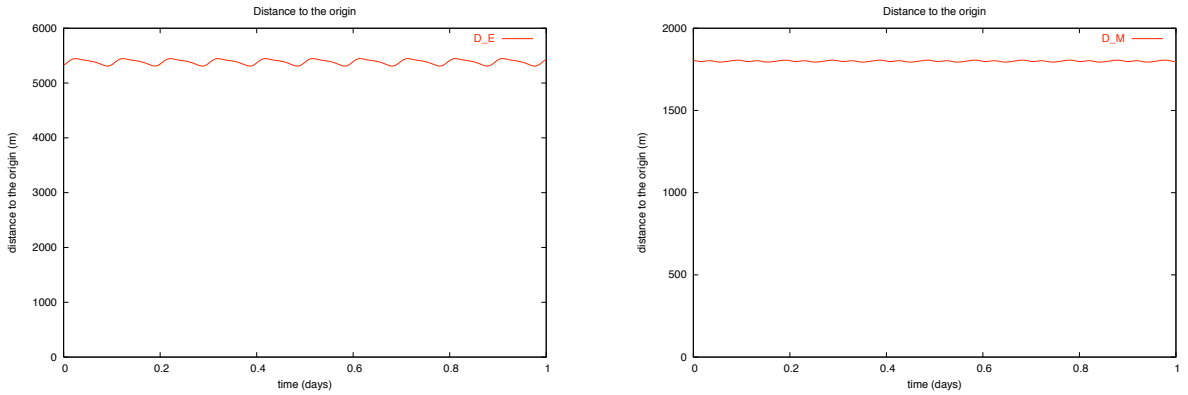

Figure 5: *Distance to the origin for different initial conditions (left: difference of eccentricity, right: difference of anomaly)*

4.3 When the reference orbit is highly eccentric

It is possible to arrange equations (13) to obtain following form:

$$\begin{aligned}
\frac{\Delta R}{a_r} &= -\cos f\Delta e + \frac{e}{\eta}\sin f\Delta M \\
\frac{\Delta T}{a_r} &= 2\sin f\Delta e + e\cos f\left(\frac{\Delta M}{\eta} - \frac{\eta^2\Delta\omega'}{1+e\cos f}\right) + \left(\eta^2\Delta\omega' + \frac{\Delta M}{\eta}\right) - e\frac{\cos f\sin f}{1+e\cos f}\Delta e \\
\frac{\Delta N}{a_r} &= \frac{\eta^2}{1+e\cos f}\left(\Delta i'\cos f + \Delta\Omega''\sin f\right)
\end{aligned} \tag{22}$$

This arrangement allows the comparison with the low eccentric form of the equations 20. Comparing the two sets, we identify the transformation of elliptical terms when the eccentricity of the reference orbit grows. We identify the following terms:

$$\begin{aligned}
-\cos f + e\sin f &\rightarrow -\cos f + \frac{e}{\eta}\sin f \\
2\sin f\Delta e + e\cos f\left(\Delta M - \Delta\omega'\right) &\rightarrow 2\sin f\Delta e + \left(\frac{\Delta M}{\eta} - \eta^2\Delta\omega'\right)e\cos f \\
\left(\Delta i'\cos f + \Delta\Omega''\sin f\right) &\rightarrow \eta^2\left(\Delta i'\cos f + \Delta\Omega''\sin f\right)
\end{aligned}$$

We impose the same previous conditions to new terms. Resulting motions are not circular because of double orbital frequency terms and non linear terms in eccentricity, that become very important. Anyway, these initial conditions produce a class of relative motions that is as close as possible to a circle.

- *SET 1*

$$\Delta a = 0 \qquad \Delta e = \Delta e \qquad \Delta i = \frac{\sqrt{3}}{\eta^2}\sin\omega\Delta e$$

$$\Delta\Omega = -\frac{\sqrt{3}}{\eta^2}\frac{\cos\omega}{\sin i}\Delta e \qquad \Delta\omega = \frac{\sqrt{3}}{\eta^2}\frac{\cos\omega}{\tan i}\Delta e \qquad \Delta M = 0$$

- *SET 2*

$$\Delta a = 0 \qquad \Delta e = 0 \qquad \Delta i = \sqrt{3}\frac{e_r}{\eta^3}\cos\omega_r\Delta M$$

$$\Delta\Omega = \sqrt{3}\frac{e_r}{\eta^3}\frac{\sin\omega_r}{\sin i_r}\Delta M \qquad \Delta\omega = -\frac{1}{\eta^3}\left(1+\sqrt{3}e_r\frac{\sin\omega}{\tan i_r}\right)\Delta M \qquad \Delta M = \Delta M$$

In the following figures (6) and (7) we have drawn the evolution of the circular motion when the eccentricity of the reference orbit grows. The other parameters of the reference orbit play no role on the form of the trajectory (for the keplerian case). Figures (6) and (7) show how, for very big reference orbit eccentricities, motion is far from being circular.

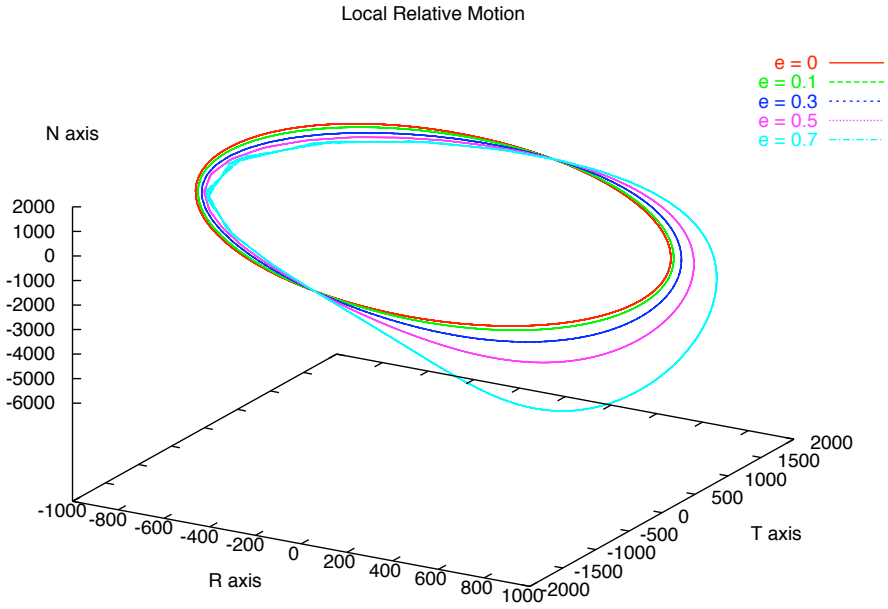

Figure 6: *Evolution of the local circular motion produced by a difference of eccentricity when the eccentricity of the reference orbit grows*

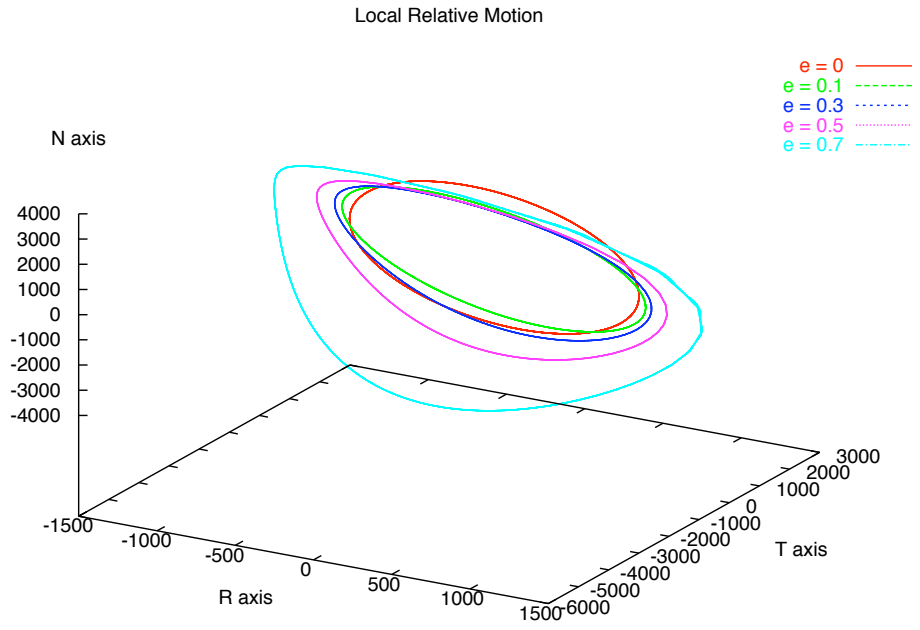

Figure 7: *Evolution of the local circular motion produced by a difference of anomaly when the eccentricity of the reference orbit grows*

5 Conclusions

This paper explores a field in relative motions which is nowadays not well-known . We characterize all the possible relative motions for all type of eccentricity of the reference orbit. We minimize the number of relevant parameters thanks to a detailed analytical study. In the case of the circular reference orbit case, we take the advantage of the representation in terms of local orbital elements. In the case of very high eccentric reference orbit it is not possible to use the local orbital elements anymore, but we separate the motion as a sum of four elementary effects.
Local circular motion has very interesting properties for space purposes. That's why we have identified the configurations that produce this motion in the circular reference orbit case. When the reference orbit is eccentric, there are no more local circular motions, but we have found the motions that are closest to a circle.
Future work will be centered in the introduction of perturbations and their effects on the relative motion. Another interesting work is to find the correct parametrization of relative motion when reference orbit is highly eccentric.

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