



**HAL**  
open science

## Insights into the collaborative work of expert teachers within a lesson study project

Galit Nagari-Haddif, Ronnie Karsenty, Abraham Arcavi

► **To cite this version:**

Galit Nagari-Haddif, Ronnie Karsenty, Abraham Arcavi. Insights into the collaborative work of expert teachers within a lesson study project. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04414834

**HAL Id: hal-04414834**

**<https://hal.science/hal-04414834>**

Submitted on 24 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Insights into the collaborative work of expert teachers within a lesson study project

Galit Nagari-Haddif, Ronnie Karsenty and Abraham Arcavi

Weizmann Institute of Science, Israel; [galit.nagari-haddif@weizmann.ac.il](mailto:galit.nagari-haddif@weizmann.ac.il)

*A main component of Lesson Study (LS) is a collaborative research into the mathematical topic at hand and the ensuing planning of a lesson. LS has spread worldwide, and its enactment varies according to cultural, institutional and personal implementation contexts. We report on a case study of a group of Israeli teachers, working collaboratively within a school-based lesson study community. We describe how these experienced teachers, who were novices at LS, went through the first two stages of an LS cycle, i.e., goal setting and collaborative lesson planning. Our main focus is on the discussion sessions held by the teachers and the processes that eventually shaped the agreed lesson plan. From this case we extract several key issues concerning the adaptation of LS into the local context of the study, and point to aspects of LS that were productively implemented within this adaptation and aspects that were not embraced.*

*Keywords: Lesson study, collaborative, teachers, community, adaptation.*

## Introduction

Lesson Study (LS) is a professional development approach that originated in Japan and has since been adapted and implemented in many countries around the world. The Japanese LS process consists of several steps (e.g., Fujii, 2018): goal setting, collaborative lesson planning; research lesson; post-lesson discussion and reflection. The first two steps are key stages which include the scrutinizing of mathematical ideas underlying the topic, establishing mathematical and pedagogical goals for the lesson, selecting appropriate tasks and problems to realize these goals, deciding on and specifying the teaching moves, anticipating student responses, proposals and difficulties, and considering how these may be addressed (ibid.). Doig et al. (2011) claim that while there may be insufficient attention to these stages in non-Japanese implementations of LS, they identify four types of tasks typically used in Japanese research lessons: tasks addressing a concept; tasks that develop mathematical processes; tasks chosen based on a rigorous examination of scope and sequence; and tasks addressing known misconceptions. LS enactments across the world vary according to cultural, institutional and personal implementation contexts (e.g., Quaresma et al., 2018). Educators in different countries have come a long way in developing their own understanding of how LS can be implemented to improve teaching and learning (e.g., Stigler & Hiebert, 2016). In this report, we describe and analyze the first steps of an Israeli LS group of experienced teachers, focusing on the management of their collaborative work.

## Context: The Israeli lesson study project

The context of our study is the project named Math-VALUE (Video Analysis and Lesson-study to Upgrade Expertise), the first and so far only LS program in Israel. In this project, secondary mathematics teachers take part in school-based professional learning communities (PLCs), established in order to learn about and to implement LS, with academic support and guidance. The project was initiated in 2019 at the Department of Science Teaching of the Weizmann

Institute of Science (WIS), and as of October 2020 thirteen Math-VALUE PLCs have been operating across Israel, each including 8-12 middle and high school mathematics teachers. Most PLCs continue to a second year and some also to a third year, and currently there are eight active PLCs. Each PLC meets for a total of 60 hours per school year, either face-to-face, virtually or in a hybrid mode. The participating schools represent different sectors of the Israeli society: Jewish and Arab, rural and urban, religious and secular. The facilitators of these PLCs are school-based leaders, each selected from among the local mathematics team. Prior to facilitation, they underwent an ad hoc year-long preparation course at WIS, during which they participated in a full cycle of LS and became acquainted with leadership practices. The implementation of the Math-VALUE project is followed by extensive research on aspects of the PLCs work (Nagari-Haddif et al., 2023) focusing on key issues concerning the adaptation of LS into the local context. Herein we report on a case study of experienced mathematics teachers, who experimented with LS for the first time. We analyze the discussion sessions in this PLC and the processes that shaped the agreed lesson plan, which emerged after several iterations. We then point to aspects of LS that were identified as most productive for this group and on those that were left aside.

## **Methodology**

*Setting.* This PLC operated for two years in a well-known and reputed high school, located in a large Israeli city. It consisted of 13 teachers, with an average teaching experience of 25 years. Prior to the project, these teachers typically worked individually, and their collaboration was limited to the joint planning of exams. All teachers hold a bachelor's degree, ten of them hold a master's degree, and one has a Ph.D. degree. During the first year, the teachers watched authentic videotaped lessons of unknown colleagues, analyzed them lessons using the VIDEO-LM six-lens framework (Karsenty & Arcavi, 2017), and then learned the principles of LS. After about eight two-hour sessions dedicated to these activities, the PLC teachers split into three subgroups (LS groups) in order to start planning lessons. We focus on one of these LS groups, that planned a lesson for 10<sup>th</sup> graders studying in an advanced-level track around problems that involve finding minimum or maximum values of a certain function or expression, arising from real-world situations. This subgroup had 4 teachers, Ori, Ira, Josh and Daria, with a teaching experience of 20, 16, 40 and 8 years, respectively.

*Data collection and analysis.* The working sessions of the group were videotaped and transcribed. The analysis was a “bottom-up” process, targeted at capturing pivotal ideas raised in the group, without an a-priori framework, that in our view might have directed or even biased our observations. In this way, we were able to identify and distill the main arguments and considerations that played a role in the group’s decisions, and to present an evolving narrative. We particularly noticed in the data the salient discussions on the proposed tasks, and the process of discarding tasks, designing new ones, and considering the pedagogical implications thereof.

## **Findings**

We present the narrative of the process undergone by the subgroup during the ‘study’ and ‘plan’ LS phases at each step.

*Idea 1: The popcorn problem.* Ori proposed to consider the following problem:

The Oren family wants to fold a cylindrical container out of a rectangular piece of paper and fill it with popcorn. There are two options to do this: vertically and horizontally (there is plenty of paper left to create the bottom of the cylinder). Tamir is sure that the height of the container is the most important variable, and that the taller cylinder would contain more popcorn than the wider one. Rocha, the grandmother, is sure that the wider cylinder would contain the largest amount of popcorn. The daughter, Shalva, laughs at the whole thing and tells them that since they are using the same piece of paper, no matter how they roll it, the cylinder will contain the same amount of popcorn. What do you think?

Ori shared his experience with an introductory lesson to extrema problems, that he usually teaches based on the above problem. He likes this lesson because the task can be related to extrema problems and provoke students' thinking in several ways. For example, students can first be given A4 sheets of paper, with which they can experiment and build cylindrical containers. Then, a question that can be posed is whether it matters what the containers are filled with, for example, popcorn, rice, or pretzels. Ori noted that students also build containers that are not necessarily cylindrical, such as rectangular boxes. An element that seemed important to Ori was that the problem is taken from the real world, where the idea of maximizing or minimizing a given magnitude becomes intuitively familiar. To further concretize this, he sometimes adds another real-life situation, in which a movie theatre owner who is especially stingy (or generous) wants to use papers of the same size to create containers that would hold as little (or as much) amount of popcorn as possible. The LS group agreed to continue with this task, and it appeared they perceived it as an experiential and enjoyable lesson where the realistic context can leverage students' understanding of the idea of extrema. Accordingly, they set the goal of the lesson: students should realize there might be different options for creating a cylindrical container, and that in one of these options the cylinder's volume is maximized or minimized. Interestingly, however, reconsidering the task during several discussions resulted in the group eventually rejecting it, since the task only affords two discrete positions and thus it does not reflect the continuous and dynamic aspects of extrema problems. In addition, the group realized that considering the volumes of the container implies using formulas and engaging in calculations, which moves the activity away from the idea of conveying a qualitative sense of the extremum concept. Thus, the group decided to search for another task.

*Idea 2: The dog yard problem.* To avoid complicated calculations, while maintaining a realistic context, Daria shared the following story that she sometimes uses in her lessons:

My friend wants to enclose a rectangular-shaped area in her garden for her dog, and she has a fence of a certain length. How should she use the fence so that her dog has as much space as possible?

This problem involves relatively simple mathematics and emphasizes the dynamic and continuous aspect of extrema problems. The group discussed this idea and decided to discard it, on the grounds that it loses the experiential and enjoyable nature of the previous task.

*Idea 3: Optimizing mobile phone features.* In an attempt to take into account a qualitative investigation anchored in a realistic situation, Ori suggested to start the lesson with an activity

outside the realm of mathematics. He thought of having the students take out their cellphones, then asking: What improvements would you like to make to your phones? Some expected answers would be that the phone should have a longer battery life, a larger memory, or a better image quality. It was proposed that this context allows students to engage with concepts related to optimization. Then, in the second part of the lesson, they could proceed to the popcorn problem discussed earlier. After further discussions, the LS group decided to abandon this idea, mainly since they did not see it as serving the goal of emphasizing the dynamic aspect of extrema problems and brings back calculations of volume in the popcorn containers problem.

*Idea 4: Dido's problem from ancient Roman mythology.* In the next iteration, the LS group engaged with the following classic story:

In his epic poem *Aeneid*, Publius Vergilius Maro (70–19 BC) tells the story of the Phoenician Queen Dido, who fled from her brother after he had assassinated her husband. Upon arriving in Carthage, Dido asked the King to purchase a piece of land that could be encompassed by an oxhide. When her request was granted, Dido cut the oxhide into thin strips and encircled the area of land. 2600 years later, it was mathematically proven that of all the shapes with a given perimeter, the shape with the maximum area is a circle.

The subgroup decided to plan a lesson that focusing on: (1) How many different shapes can be formed from a 3-meter-long ribbon? (2) How many shapes with *the same area* can be formed from this ribbon? In the lesson, they planned to focus on rectangles, triangles, squares, other polygons, and circles. When discussing different goals, the group first persisted with motivational objectives raised in the previous iterations, i.e., that the students' first exposure to extrema problems should be qualitative, experiential in nature and anchored in a physical context. Second, the group presented goals that can be classified as mathematical and meta-mathematical, such as emphasizing the multiplicity of possible solution options and distinguishing between a hypothesis and a solution. Third, socio-mathematical objectives were mentioned, for example to enact and cultivate mathematical discourse; to encourage students to share ideas with peers; and to stimulate productive competitiveness (in finding more shapes). Dido's problem, also known as “the isoperimetric problem”, involves two variables: the number of sides and their length. There is an infinite number of same-perimeter polygons that can be sorted according to their area. When the lesson plan inspired by the Dido's problem, was presented to the whole PLC, some reservations were expressed. For example, one PLC member noted the following:

“The lesson really deals with the dynamic aspect of extrema problems and with an infinite number of polygons, but it is very open-ended. I think it's important to think beforehand where the lesson could go, and the directions in which students could take it, and somehow be ready with answers. That is, check if the task is well-defined and where you want to go... it might make sense to narrow down the problem and still deal with the dynamic aspect”.

Additional difficulties were raised, e.g., the fact that the circle is the shape with the maximum area is non-intuitive; calculating the area of various shapes cannot always be performed with the tools available to students. To address this last concern, it was suggested to use the dynamic software [Desmos](#). This resulted in the next iteration of the discussion.

*Idea 5: Integrating a dynamic software to help with area calculations.* The group discussed the following question: Given a specific perimeter, what are the constraints and conditions for constructing polygons? They realized that formulating constraints for constructing rectangles is relatively easy, yet constructing triangles requires additional considerations, such as *the triangle inequality*. Ori delved into this by experimenting with Desmos, and shared his insights:

“Following comments [from the PLC plenary discussion], I had an idea in a completely different direction. I tried to see how complex this story is and suddenly I figured that I want students to work more like mathematicians and less like high school students... I started to write the constraints of these triangles in Desmos to see when there is a solution to this problem. This takes us in a slightly different direction, because we are not dealing with an extrema problem; we are examining whether such polygons exist ... When students have Desmos, all they have to do is enter the constraints and see whether there is a solution to the problem. This could be a very interesting investigation that prepares the students for the topic of extrema problems”.

At this point, the planning of the lesson took a sharp turn. The main aspiration goal became to provide an opportunity for students to experience the spirit of the way in which mathematicians work. The lesson plan became experiential in a completely different sense than what was proposed in the first iteration. Contrary to the goals that the group had stated earlier, dealing with higher mathematics became an important goal of the lesson. After further discussion, they decided to reduce the problem and focus on extrema problems in relation to triangles.

*Idea 6 (final): Focusing on triangles.* The LS group decided to center on triangles and to abandon reaching the concluding that of all the same-perimeter shapes, the circle has the maximum area. The final lesson plan is shown in Figure 1. The function for the area of the triangle with a given perimeter (e.g., 12) and a given side (e.g., 5) is expressed by means of Heron’s formula as  $S(x) = \sqrt{6(6-x)(x-1)(6-5)}$ . The group agreed that although the students have no knowledge of derivatives yet, they are sufficiently familiar with the quadratic function and with the properties of the square root, thus they are able to handle the task. By means of some directive questions the teacher can ask (e.g., “for which x values is the root function defined / not defined / constant), students will be able to sketch the graph of  $S(x)$  by themselves, without a dynamic graphical tool. In contrast to the previous iteration, the group could elicit a main drawback of using Desmos here - it may decrease students’ productive struggle by making the sketch of the function immediate - which overturned the previous decision to use the dynamic software.

Notably, the introduction of the extremum was left to the last item of the worksheet, after students experience the dynamic aspect of the geometrical situation and sense how the maximum value for the triangle’s area is one among an infinite number of other possibilities represented by the function. This iteration seemed to satisfy the goals pursued by the LS group, while at the same time avoiding the pitfalls seen in the previous ideas. During the discussions around idea 6, the teachers stopped the planning and engaged in a purely mathematical exploration for its own sake. They explored the locus of the vertices of the variable triangles (with perimeter of length 12, one side of length 5 and two variable sides) and found it to be an ellipse, with 5 as the distance between its focal points and 12 the length of its main axis. The

planned lesson was taught by Ori and was considered very successful by the PLC members and by other mathematics educators who watched it. The richness of the mathematical ideas and the interconnections therein were noted, as well as the active participation of students and their observable agency in asking questions and proposing insights. However, during the planning process, except for some indirect allusions, the group deliberately refrained from engaging in discussions about predicted student reactions, and the final lesson plan they presented to the PLC members included only the final task (Figure 1).

**I. Preparation:**  
Introducing and proving Heron's formula for calculating the area of a triangle:  
 $S(x) = \sqrt{p(p-a)(p-b)(p-c)}$  ( $p$  = semi perimeter;  $a$ ,  $b$ ,  $c$  denote the side lengths).

**II. Administering a worksheet with the following items:**

- (1) Consider a triangle with a perimeter of 12 length units and a side of 7 length units.  
Does such a triangle exist? If so, how many such triangles are there? If not, explain.
- (2) Consider a triangle with a perimeter of 12 length units and a side of 5 length units.  
Does such a triangle exist? If so, how many such triangles are there? If not, explain.
- (3) Construct a function that represents the triangle's area.
- (4) Sketch the graph of the function.
- (5) How many different triangles **have** the same area? Explain.
- (6) Find the triangle with the maximum area. Can you characterize this triangle?

**Figure 1: The final lesson plan as was presented by the LS group**

In an interview with Ori about various aspects of LS (Nagari-Haddif et al., 2023) he related to his preparations towards the teaching of this lesson and referred to this issue explicitly:

“I had this basic structure in mind. I mean I had a problem with clear sub-problems that I planned to do with them in a particular order. I knew at what point I wanted to start and where I wanted to end, but I did not know how the lesson would develop in the middle... I mostly build my lessons based on my experience and from what comes up in class ... This lesson was relatively meticulous in the way it was planned because that was the goal of the LS project, but I didn't know what the students would bring to my doorstep. So in that sense, there was a very neat and organized frame, and I had the goals I wanted to achieve, but I was hoping, and I know it works for me, to achieve these points ... In retrospect, planning the lesson in more detail would have been meaningless. I think the lesson is built on trusting my intuition and hoping the students will not disappoint and get to very interesting points. Of course, I could not anticipate everything that happened there. But because the students are so good and curious, I could count on them to take the lesson in interesting directions.”

## Discussion

Some important issues can be pointed out regarding the “mise-en-scène” of lesson study in a specific context so far removed from its original habitat.

*Collaborative work.* The case portrays how highly experienced teachers, who were not used to collaborative activities, became fully involved in jointly planning a lesson. The concatenation of their decisions regarding the suitability of six different proposed ideas to achieve certain learning goals, brought to bear rich considerations that seldom could have occurred to a single teacher working in isolation. Thus, the very fact that teachers deeply engaged in the kind of

exchanges reported is in itself an evidence of the importance of such a forum to exist, thrive and persist.

*Centrality of tasks.* Task selection and analysis (and at a higher-level, task design), proves time and again to be the backbone of LS, around which many mathematical and pedagogical ideas can emerge and flourish, enhancing professional learning even for teachers “who have seen it all”. Incidental “mathematical discoveries”, such as the ellipse that was discovered during the group discussion, are not uncommon when teachers discuss in depth the underlying mathematics of tasks (e.g., Karsenty & Arcavi, 2017). The centrality of the tasks appeared both in the group discussion and in the final lesson plan is consistent with Doig et al. (2011), that pointed to the increasing attention being paid to task design in Western contexts. Selecting and analyzing tasks may elicit discussions on both the mathematical ideas and about the goals to pursue. In our case, as we have shown, teachers juggled with three intertwined types of goals: (1) mathematical and meta-mathematical goals (e.g., conveying the intuitive, qualitative and dynamic nature of the notion of extrema in certain problem situations; reducing computational complexity; connecting seemingly disparate mathematical topics such as areas and graphs of functions); (2) motivational goals (e.g., focusing on interesting real life phenomena); and (3) socio-mathematical goals (e.g., engaging students with authentic mathematical explorations and cultivating their mathematical discourse). The interplay between these goals and their attainment was a main tool for the teachers to make decisions, to discard ideas, to look for new ones and to finally converge to an agreed task for the lesson. During the group's discussions, goals were added and even completely replaced with others. By discussing the various tasks brought up, the group ultimately identified the desired goals for the lesson, sharpened and even reversed their priorities. Thus, while at the beginning it was important for them that the task would be linked to everyday life and would not involve complex mathematics, the final task eventually did include many mathematical ideas and was not directly connected to a realistic context. In the process, the definition of what the group considered as an ‘experiential activity’ has changed. The initial intention was for students to work with their hands, that is, construct containers for popcorn, or use a ribbon to create different shapes. After Ori experimented with Desmos, this changed to having students work like mathematicians, implying an experiential lesson in a completely different sense. In practice, the term *experiential* attained a new meaning in the eyes of the teachers, with the experience becoming completely mathematical. One of the PLC members described this after watching the videotaped lesson: “This is a group that is working and having fun. For 90 minutes, students enjoy doing math”. In other words, although the group had initially thought that complex mathematics would limit the experience and the students’ enjoyment, the lesson they ended up planning did include complex mathematics and yet was very experiential.

*(In)Fidelity to the LS model.* It is well known that when LS migrates to a different culture or to a specific institutional context within the same culture, adaptations are inevitable (e.g., Quaresma et al., 2018). In the case we reported here, teachers engaged deeply and seriously in the research part of the LS. However, another crucial component of the planning process refers to the explicit anticipation of students’ answers, ideas, proposals and potential difficulties. This component was not taken up by the LS group. As Ori clearly said, he likes to rely on student



responses, especially unanticipated ones, and he trusts his abilities to incorporate these into the flow of the lesson. It is possible that this spirit has affected the choice of the group not to meticulously predict and plan for students' reactions.

## Epilogue

In Japan, LS is a powerful and a sustainable approach to lifelong professional development across the nation. Its rationale, implementation and institutionalization, as well as the model lessons produced, made it a source of inspiration and a model for many other countries. In our case, the implementation of the Math-VALUE project yields insights regarding the adaptability of LS not only to the culture of the nation, but also to the culture of the local institution in which it is enacted, and to the characteristics of specific teacher populations. Our aim is to pursue further research, which will enlighten us in order to improve LS across the country, with its different variations, yet with the aim of becoming the major hub for ongoing professional development in each and every school in which mathematics is taught.

## Acknowledgments

This study was supported by the Israel Science Foundation (Grant #1219/20) and the Weizmann Institute of Science.

## References

- Doig, B., Groves, S., and Fujii, T (2011). The critical role of task development in lesson study. In L. C. Hart, A. S. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education*. Springer, 181–199. [https://doi.org/10.1007/978-90-481-9941-9\\_15](https://doi.org/10.1007/978-90-481-9941-9_15)
- Fujii, T. (2018). Lesson study and teaching mathematics through problem solving: the two wheels of a cart. In M. Quaresma, C. Winsløw, S. Clivaz, J. P. da Ponte, A. N. Shúilleabháin, & A. Takahashi (Eds.), *Mathematics lesson study around the world* (pp. 1–21). Springer. [https://doi.org/10.1007/978-3-319-75696-7\\_1](https://doi.org/10.1007/978-3-319-75696-7_1)
- Karsenty, R. & Arcavi, A. (2017). Mathematics, lenses and videotapes: a framework and a language for developing reflective practices of teaching. *Journal of Mathematics Teacher Education*, 20(3), 433–455. <https://doi.org/10.1007/s10857-017-9379-x>
- Nagari-Haddif, G., Karsenty, R., & Arcavi, A. (2023). Lesson study and improvisation: Can two walk together except they be agreed? In A. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 371–378). University of Haifa and PME. <https://pme46.edu.haifa.ac.il/conference-schedule/conference-proceedings>
- Quaresma, M., Winsløw, C., Clivaz, S., da Ponte, J. P., Shúilleabháin, A. N. & Takahashi, A. (Eds.) (2018). *Mathematics lesson study around the world*. Springer. <https://doi.org/10.1007/978-3-319-75696-7>
- Stigler, J. W., & Hiebert, J. (2016). Lesson study, improvement, and the importing of cultural routines. *ZDM – Mathematics Education*, 48, 581–587. <https://doi.org/10.1007/s11858-016-0787-7>