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1 **Technical Note: Analytical Solution for Well Water Response to Earth Tides in**
2 **Leaky Aquifers with Storage and Compressibility in the Aquitard**

3
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13
14 **Key points**

- 15 - Development of a new analytical solution for Earth tide induced well water level fluctuations in
16 semi-confined aquifers considering aquitard storage, aquitard response to tidal strain, skin and
17 wellbore storage effects
- 18 - The solution correctly reflects previously observed but unexplained amplitude-frequency
19 relationships and positive or negative phase shifts
- 20 - Diagnostic information about subsurface hydro-geomechanical properties can be derived from
21 amplitude ratio and phase shifts for both semi-diurnal and diurnal tides

22
23 **Abstract**

24 In recent years, there has been a growing interest in utilizing the groundwater response to Earth tides
25 as a means to estimate subsurface properties. However, existing analytical models have been
26 insufficient in accurately capturing realistic physical conditions. This study presents a new analytical
27 solution to calculate groundwater response to Earth tide strains, including storage and compressibility
28 of the aquitard, borehole storage and skin effects. We investigate the effects of aquifer and aquitard
29 parameters on well water response to Earth tides at two dominant frequencies (O_1 and M_2) and
30 compare our results with hydraulic parameters obtained from a pumping test. Inversion of the six
31 hydro-geomechanical parameters from amplitude response and phase shift of both semi-diurnal and
32 diurnal tides provides relevant information about aquifer transmissivity, storativity, well skin effect,
33 aquitard hydraulic conductivity and diffusivity. The new model is able to reproduce previously
34 unexplained observations of the amplitude and frequency responses. We emphasize the usefulness in
35 developing relevant methodology to use the groundwater response to natural drivers for
36 characterizing hydrogeological systems.

37



38 1. Introduction

39 Aquifer properties play a vital role in managing groundwater resources, particularly amid increasing
40 anthropogenic groundwater use and the impact of climate change. While pump testing can be costly,
41 there exists a cost-effective alternative for assessing aquifer hydraulic properties - analysing the
42 groundwater response to Earth tides or atmospheric tides (McMillan et al., 2019). Observations of
43 variations in groundwater level due to tidal fluctuations date back to the works of Klönne (1880),
44 Meinzer (1939), and Young (1913). However, it was only later that hydro-geomechanical models were
45 employed to elucidate these variations (Bredehoeft, 1967; Hsieh and Bredehoeft, 1987; Roeloffs, 1996;
46 Wang, 2000; Cutillo and Bredehoeft, 2011; Kitagawa et al., 2011; Lai et al., 2013; Wang et al., 2018).
47 This progression in understanding offers a valuable opportunity to evaluate aquifer hydraulic
48 properties through the response of groundwater to tidal fluctuations.

49 Hsieh and Bredehoeft (1987) introduced the horizontal flow model, focusing on confined conditions
50 influenced by tidal forces. Conversely, Roeloffs (1996) and Wang (2000) explored interactions within
51 vertical flow under tidal fluctuations. Wang et al. (2018) expanded on these studies by incorporating a
52 flow from an upper aquitard, albeit assuming negligible storage within it. Later, Gao et al. (2020)
53 extended these models to include borehole skin effects. Thomas et al. (2023) developed an ET-GW
54 model incorporating storage and strain response in the aquitard. They applied their model to a specific
55 site to evaluate transmissivity variations and validated it using pumping tests.

56 Numerous studies have investigated aquifer hydromechanical properties by analysing GW level
57 variations induced by Earth tides, employing the models mentioned in the previous literature
58 (Narasimhan et al., 1984; Merritt, 2004; Fuentes-Arreazola et al., 2018; Zhang et al., 2019a; Shen et
59 al., 2020). However, only a limited number of validations have been conducted, which involve
60 comparing the results with robust hydraulic assessments, such as hydraulic conductivity derived from
61 slug testing (Zhang et al., 2019b) or specific storage and transmissivity characterizations obtained
62 through long-term pumping tests (Allègre et al., 2016; Valois et al., 2022). The current evaluations
63 predominantly focus on purely confined conditions, leaving a gap in knowledge regarding tidally
64 induced GW responses in aquifers under semi-confined conditions.

65 As far as the authors are aware, the publications by Sun et al. (2020), Valois et al. (2022), and Thomas
66 et al. (2023) are the sole references addressing a comparison for a leaky aquifer. Sun et al. (2020) found
67 significant discrepancies between transmissivities obtained from Earth tide fluctuations and those
68 derived from slug or pump tests. However, it is worth noting that the comparison may be subject to
69 discussion, as the authors employed a leaky aquifer model for analysing tidally induced fluctuations,
70 whereas they used a confined aquifer model for conducting slug and pump tests. In the study
71 conducted by Valois et al. (2022), the existing Earth-Tide GroundWater (ET-GW) models, as described



72 earlier, were unable to reproduce a low semi-diurnal to diurnal amplitude ratio with positive phase
73 shifts in conjunction with pumping test transmissivity data. This discrepancy highlights the complexity
74 and challenges in modelling tidally induced groundwater responses in leaky aquifers and the need for
75 further investigation in this area.

76 We note that our previous attempts to model the observed substantial amplitude decrease from O_1 to
77 M_2 frequency, combined with phase shifts close to zero, proved unsuccessful when using Earth tide
78 models found in the literature. None of the existing models could provide satisfactory results. The
79 pursuit of an explanation led to the realisation that analytical solutions with more realistic assumptions
80 are required. For example, aquifers are widely recognized to be influenced by aquitards, which often
81 consist of highly porous and compressible clay materials, contributing significant amounts of stored
82 water to the aquifer (Moench, 1985). Moreover, these aquitards are also impacted by Earth tide strains
83 (Bastias et al., 2022).

84 Our first objective is to develop an analytical solution considering storage and strain in the aquitard.
85 Unlike Thomas et al. (2023), our model incorporates borehole skin effects and allows for a fixed
86 hydraulic head at the top of the aquitard, broadening its applicability to a broader range of
87 hydrogeological conditions. The second motivation of our study is to develop a model that better fits
88 the observed results by considering aquitard storage, as evident in the pumping tests. Third, we
89 compare the results obtained from our new ET-GW model with those derived from a pumping test in
90 leaky aquifers with storage in the aquitard. Fourth, since most publications have predominantly
91 focused on assessing hydraulic properties using the semi-diurnal tide (M_2), our third motivation is to
92 demonstrate the potential of using diurnal tides (O_1) in combination with the semi-diurnal (M_2 or N_2)
93 to provide a more comprehensive characterization of aquifer and aquitard hydro-mechanical
94 properties. Our new development offers the potential to enhance hydraulic and geomechanical
95 subsurface characterization by employing a more realistic model for the groundwater response to
96 natural forces.

97

98

99 **2. Groundwater response to Earth tides in a leaky aquifer with aquitard**

100 **storage and strain**

101 Hantush (1960) pioneered the modelling of aquitard storage by modifying the leaky aquifer theory to
102 account for storage in the aquitard. In our study, we consider a semi-confined configuration (Figure 1)
103 where the target aquifer is overlain by an aquitard that allows for storage, strain, and vertical flux. Both
104 layers are assumed to be slightly compressible, spatially homogenous, infinite laterally, and have
105 constant thickness. Building upon the work of Wang et al. (2018), our research incorporates Earth Tide



106 (ET) fluctuations into the leaky aquifer equations proposed by Moench (1985). Additionally, we
 107 incorporate the skin effect, as described by Gao et al. (2020).

108 Groundwater flow and storage in an aquifer overlain by an aquitard can be described as:

$$109 \quad T \left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \right) = S \left(\frac{\partial h}{\partial t} - \frac{BK_u}{\rho g} \frac{\partial \varepsilon}{\partial t} \right) - K' \frac{\partial h'}{\partial z} \quad (1)$$

$$110 \quad \left(\frac{\partial^2 h'}{\partial z^2} \right) = \frac{1}{D'} \left(\frac{\partial h'}{\partial t} - \frac{B'K'_u}{\rho g} \frac{\partial \varepsilon}{\partial t} \right) \quad (2)$$

111 Here, h (m) and h' (m) are the hydraulic heads in the aquifer and the aquitard respectively; h'_f (m) is
 112 the fixed hydraulic head at the top of the aquitard, r (m) is the radial distance from the studied well; T
 113 (m^2/s) and S are the aquifer transmissivity and storativity; B , B' , K_u (Pa), K'_u (Pa) are the Skempton's
 114 coefficient and the undrained bulk modulus of the aquifer and aquitard respectively; ρ (kg/m^3) and g
 115 (m/s^2) are the water density and gravity constant; ε is the volumetric Earth tide strain; K' (m/s) is the
 116 aquitard hydraulic conductivity; S'_s (m^{-1}) is the specific storage of the aquitard; D' (m^2/s) is aquitard
 117 hydraulic diffusivity (K'/S'_s ratio). Any natural regional groundwater flow is considered negligible.

118 Borehole drilling causes a zone of damage with a radius r_s (see Figure 1) that is responsible for the skin
 119 effect (Van Everdingen, 1953). A negative skin can be caused by a greater hydraulic conductivity around
 120 the well because of the material damaged by the drilling, while a positive skin can be associated by
 121 porosity clogging caused by the drilling mud. This is reflected in the well's pressure head Δh_s . The skin
 122 factor (sk) can be defined as:

$$123 \quad sk = \frac{\Delta h_s}{\left(r \frac{\partial h}{\partial r} \right)_{r=r_w}} \quad (3)$$

124 Following above assumptions, the boundary conditions are

$$125 \quad h(r, t) = h_\infty(t) \text{ at } r = \infty \quad (4)$$

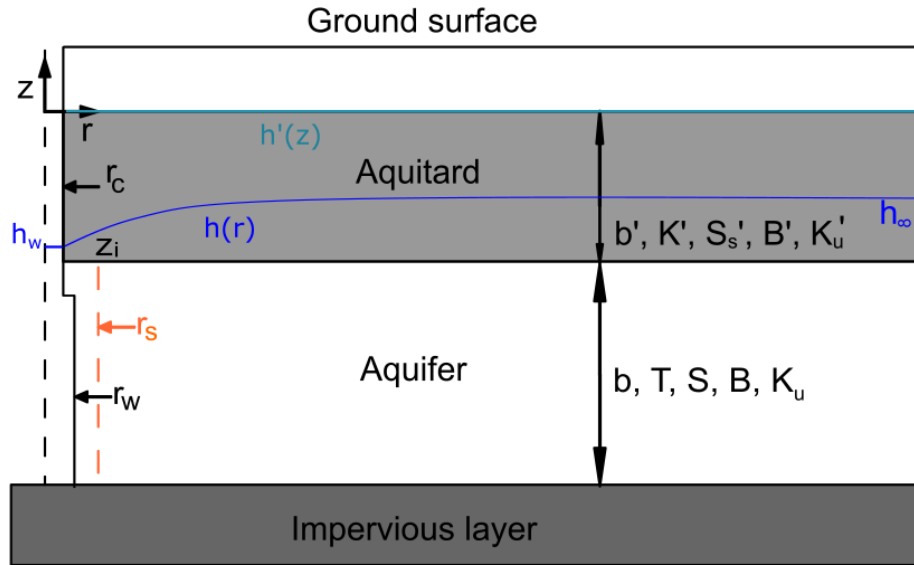
$$126 \quad h_w(t) = h(r, t) - sk \left(r \frac{\partial h(r, t)}{\partial r} \right) \text{ at } r = r_w \quad (5)$$

$$127 \quad 2\pi r_w T \left(\frac{\partial h}{\partial r} \right)_{r=r_w} = \pi r_c^2 \frac{\partial h_w}{\partial t} \quad (6)$$

$$128 \quad h' = h \text{ at } z = z_i \quad (7)$$

$$129 \quad h' = h'_j \text{ at } z = 0 \quad (8)$$

130 Here, t is the time (s); r_w and r_c are the radius of the well screened portion and the radius of well casing
 131 in which water level fluctuates; z_i is the aquifer-aquitard interface elevation (see Figure 1) and h_w is the
 132 hydraulic head at r_w .



133

134

Figure 1: Semi-confined system with a compressible aquitard with storage

135 Following Hsieh et al. (1987) and Wang et al. (2018), complex numbers were used to facilitate harmonic
 136 model development and the solution is obtained by first solving Equation 2 in the aquitard, then
 137 deriving the head response in the aquifer far away from the well (h_∞) which is independent of the
 138 radial distance. Then, the well effect on the aquifer response is considered by using a flux condition at
 139 the well that accounts for wellbore storage. Since h' , h , ε , h_w , h_∞ are all periodic functions, they can be
 140 expressed as:

141
$$\varepsilon(t) = \varepsilon_0 e^{i\omega t} \quad (9)$$

142
$$h_\infty(t) = h_{\infty,0} e^{i\omega t} \quad (10)$$

143
$$h_w(t) = h_{w,0} e^{i\omega t} \quad (11)$$

144
$$h(r, t) = h_0(r) e^{i\omega t} \quad (12)$$

145
$$h'(z, t) = h'_0(z) e^{i\omega t} \quad (13)$$

146 Here, $i = \sqrt{-1}$; ε_0 (m) is the ET strain amplitude and ω (s^{-1}) is the angular frequency. In this case,
 147 Equation 2 becomes:

148
$$\left(\frac{\partial^2 h'_0}{\partial z^2}\right) = \frac{1}{D'} \left(i\omega h'_0 - i\omega \frac{B'K'_u}{\rho g} \varepsilon_0 \right) \quad (14)$$



149 According to Wang (2000) and Roeloffs (1996) and as detailed in appendix A, the solution of Equation
 150 2 is:

$$151 \quad h'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} + \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (15)$$

152 where $\delta = \left(\frac{2D'}{\omega}\right)^{1/2}$. Thus, at the interface between the aquifer and the aquitard ($z=z_i$), we have
 153 pressure continuity as $h'(z_i, t) = h_0 e^{i\omega t} = h(t)$ which leads to:

$$154 \quad \left(\frac{\partial h'}{\partial z}\right)_{z=z_i} = \frac{1+i}{\delta} \left(\frac{h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right) e^{i\omega t} \quad (16)$$

155 Equation 1 can be solved far away from the well using h_∞ which is independent of the radial distance
 156 from the well and by using the source term from h' as follows

$$157 \quad 0 = \frac{\partial h_\infty}{\partial t} - \frac{BK_u}{\rho g} \frac{\partial \varepsilon}{\partial t} - \frac{K'}{S} \frac{1+i}{\delta} \left(\frac{h_\infty - \frac{B'K'_u}{\rho g} \varepsilon_0}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right) e^{i\omega t} \quad (17)$$

$$158 \quad h_{\infty,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} \frac{B'K'_u}{BK_u} \left(\frac{-1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)}} \quad (18)$$

159 The disturbance in water level due to the well can be expressed as:

$$160 \quad s(r, t) = h(r, t) - h_\infty(t) \quad (19)$$

161 Equation 1 becomes:

$$162 \quad T \left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) = S \left(\frac{\partial s}{\partial t} \right) - K' \frac{1+i}{\delta} S \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \quad (20)$$

163 with the boundary conditions:

$$164 \quad s(r \rightarrow \infty) = 0 \quad (21)$$

$$165 \quad h_{w,0} - h_{\infty,0} = s - sk \left(r \frac{\partial s}{\partial r} \right) \quad \text{at } r = r_w \quad (22)$$

$$166 \quad 2\pi r_w T \left(\frac{\partial s}{\partial r} \right)_{r=r_w} = i\omega \pi r_w^2 h_{w,0} \quad (23)$$

167 The solution of this differential equation is $s(r) = C_1 I_0(\beta r) + C_2 K_0(\beta r)$ (Wang *et al.*, 2018), where I_0
 168 and K_0 are the modified Bessel functions of the first and second kind and the zeroth order, respectively.

169 Further,



$$\beta = \left(\frac{i\omega S}{T} - \frac{K' (1+i)}{T} \frac{1}{\delta \tanh\left(\frac{1+i}{\delta} z_i\right)} \right)^{1/2} \quad (24)$$

The boundary conditions lead to $C_1=0$ and $C_K = -\frac{i\omega r_c^2 h_{w,0}}{2T\beta r_w K_1(\beta r_w)}$ because $\frac{dK_0(r)}{dr} = -K_1(r)$. Therefore, the final solution for the well water level is:

$$h_{w,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} \frac{B'K'_u}{BK_u} \left(\frac{-1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_{w,0} \frac{\rho g}{B'K'_u \varepsilon_0} - 1}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{\sigma \left(Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \right)} \quad (25)$$

where

$$\sigma = 1 + \frac{i\omega r_c^2 K_0(\beta r_w)}{2T\beta r_w K_1(\beta r_w)} + \frac{i\omega r_c^2}{2T} sk \quad (26)$$

By assuming $h'_z = 0$ (i.e. the hydraulic head at the top of the aquitard corresponds to the unsaturated-saturated interface at $z=0$), Equation 25 can be reorganized to

$$h_{w,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} R_{KuB} \left(\frac{1 - \cosh\left(\frac{1+i}{\delta} z_i\right)}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{\sigma \left(Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \right)} \quad (27)$$

where

$$R_{KuB} = \frac{K'_u B'}{K_u B} \quad (28)$$

By disregarding $\frac{BK_u}{\rho g}$ product which only controls the amplitude, the solution has six independent parameters which are T, S, K', D', sk and the R_{KuB} ratio.

Let us now define the amplitude response (or amplitude ratio), A , and phase shift, α , of the GW response to ET fluctuations:

$$A = \left| h_{w,0} / \frac{BK_u}{\rho g} \varepsilon_0 \right| \quad (29)$$

$$\alpha = \text{arg} \left[h_{w,0} / \frac{BK_u}{\rho g} \varepsilon_0 \right] \quad (30)$$

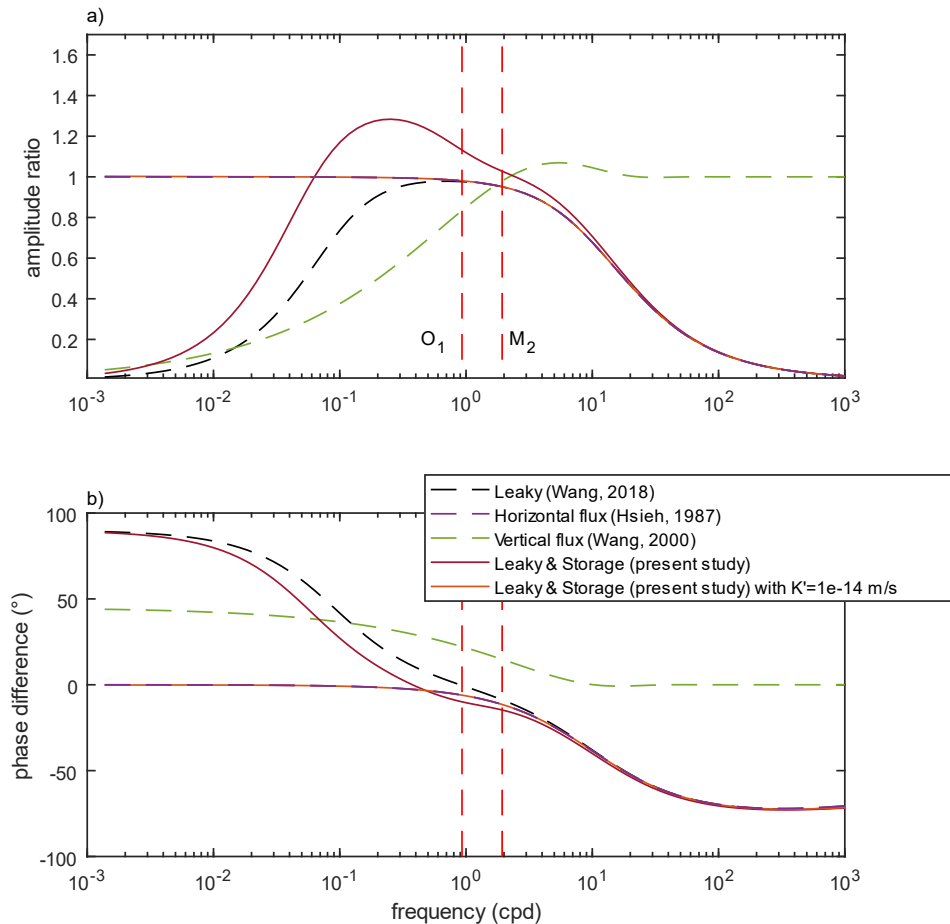
Figure 2 shows the amplitude response and phase shift as a function of frequency using our new solution in comparison to key models reported in the literature. Aquitard parameters were set according to Batlle-Aguilar et al. (2016), while aquifer parameters were chosen according to the field application below. We validate the solution using a very low aquitard conductivity (10^{-14} m/s), so that we can



191 compare it to the horizontal flux with wellbore storage model (Hsieh et al., 1987). It shows a perfect
192 match.

193 Because both horizontal and vertical flux models are associated with opposite phase shift signs (Figure
194 2b), the latter can offer valuable insights for model selection (Allègre et al., 2014). Positive phase shifts
195 in the vertical flux model are related to an increasing amplitude ratio with frequency, whereas the
196 wellbore storage model exhibits the opposite behaviour. Wang et al. (2018) developed a leaky model
197 capable of demonstrating both positive and negative phase shifts, where positive phase shifts
198 correspond to an increasing amplitude ratio with frequencies, and negative phase shifts are linked to
199 a decreasing amplitude ratio.

200 Our new model showcases positive or negative phase shifts with an increasing or decreasing amplitude
201 ratio over frequency, even allowing for amplitude ratios greater than one. Notably, Wang (2000)
202 observed a similar characteristic in the vertical flux model, with an amplitude just above one (1.06) for
203 very specific conditions. At high frequencies, our model displays amplitude ratios and phase shifts
204 similar to those of the leaky and wellbore storage models, reflecting the attenuation of high-frequency
205 pore pressure fluctuations in the aquifer by well water.



206

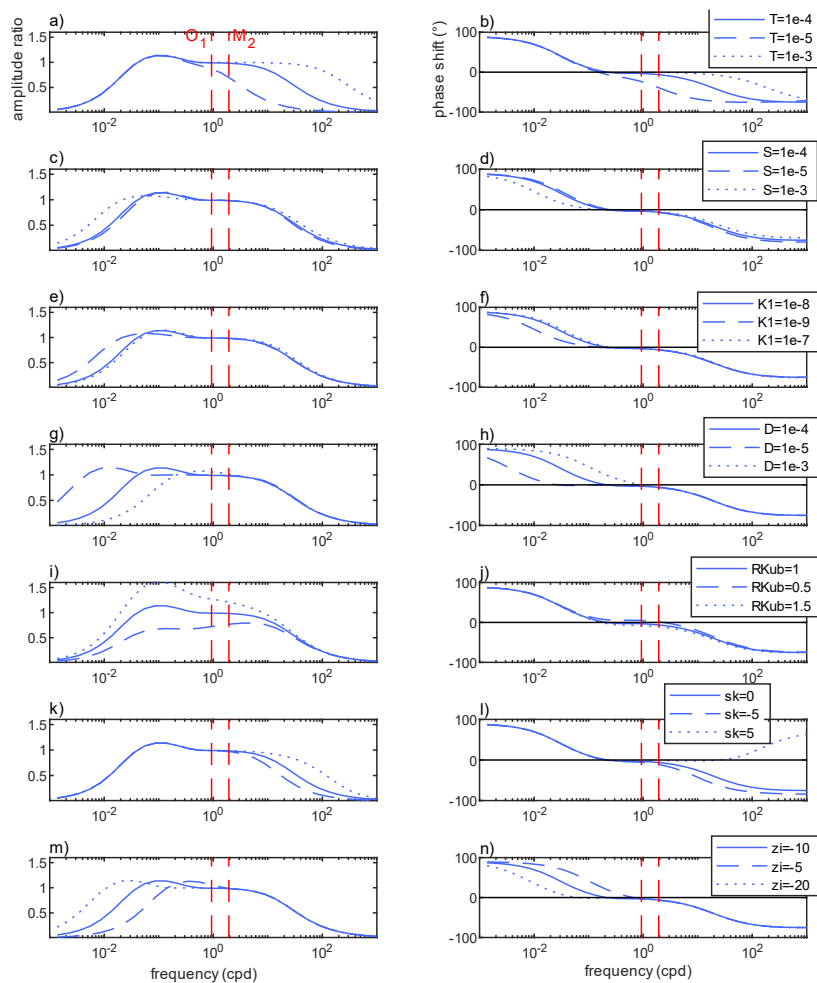
207 *Figure 2: Frequency variation of amplitude response and phase shift of the groundwater response to Earth tides. The*
 208 *transmissivity (T) is 10^{-5} m²/s, storativity (S) is 10^{-4} , hydraulic conductivity of the aquitard (K') is 10^{-8} m/s, aquitard hydraulic*
 209 *diffusivity (D') is 10^{-4} m²/s, skin factor (sk) is 0, R_{KuB} to 1.4, well casing radius (r_c) and screen radius (r_w) is 6.03 cm. b' was set*
 210 *to 5 m. Screen depth (z) was set to 23 m for the vertical flow model of Wang (2000).*

211

212 We explored the parameter space by focussing on the frequency-dependant amplitude response and
 213 phase shift responses for different sets of parameter values. The reference parameter set is the one
 214 described above. T , R_{KuB} and z_1 have a large impact on model shapes. As also observed by Hsieh et al.
 215 (1987), S does not have a major impact on the results (Fig3c and 3d). The skin effect does not play a
 216 large role in the useful frequency band for amplitudes, but its influence is larger for the phase shifts
 217 when compared to the reference parameter set. K' does not significantly influence the results with



218 respect to the reference parameter set used in the study (Figures 3e to 3f). K' has the opposite role of
 219 S and they appear to compensate each others effects, because of their respective role in Equation 27.



220

221 *Figure 3: Illustration of the amplitude response (left column) and phase shift (right column) as a function of frequency for*
 222 *various parameters compared to the reference parameter set.*

223



224 **3. Application of the new model to a groundwater monitoring dataset from Cambodia**

225 **3.1 Field site and previous results**

226 The field site in Northwest Cambodia comprises three boreholes drilled into the subsurface, consisting
227 of mudstone, claystone, siltstone, and sandstone. Time series and pumping test results have been
228 reported in Valois et al. (2022), while details of the lithology can be found in Vouillamoz et al. (2012;
229 2016) and Valois et al. (2017; 2018; 2022). Pumping from the aquifer is limited by a very low specific
230 yield, attributed to the presence of fine deposits such as clay and mudstone (Vouillamoz et al. 2012;
231 Valois et al., 2018).

232 The boreholes were drilled to a depth of 31 meters with a radius of 6 inches and equipped with 4-inch
233 PVC casing from top to bottom, featuring a 9-meter long screen at the hole's base. The aquifer is
234 situated within a hard rock media, comprising either claystone or sandstone, located beneath a 10-
235 meter thick clay layer.

236 For the pumping tests, the wells were pumped for three days, and water levels were allowed to recover
237 for four days in two observation wells. The interpretation of the pumping tests utilized
238 AQTESOLV™/Pro v4.5 software, employing the leaky aquifer with aquitard storage model (Moench,
239 1985) or a 3D flow using the generalized radial flow model (Barker, 1988). The selected solutions,
240 compared to other models (Theiss, Hantush without aquitard storage), demonstrated the best fit with
241 a Root Mean Square Error (RMSE) of 0.02 m for Cambodia.

242

243 **3.2 Well sensitivities and phase shifts to Earth tides**

244 Between 2010 and 2015, well water levels were measured at 20, 40, or 60-minute intervals using
245 absolute pressure sensors (Diver data loggers, Eijkelkamp Soil & Water, NL). To compensate for
246 barometric pressure (BP) effects, data from a barometer located a few kilometres away from the field
247 site were utilized (Eijkelkamp Soil & Water, NL). A zero-phase Butterworth filter was employed to
248 eliminate low-frequency content (periods longer than 10 days) from both groundwater (GW) and BP
249 data.

250 For each site's geolocation (latitude, longitude, and height), ET strain time series were computed at
251 20-minute intervals using SPOTL software (Agnew, 2012). The time series were then modelled using
252 Harmonic Least-Squares (HALS; Schweizer et al., 2021) with eight frequencies corresponding to the
253 major tides (Table 1) following Merritt's description (2004). HALS provides amplitude and phase
254 estimations for each tidal component and record.

255



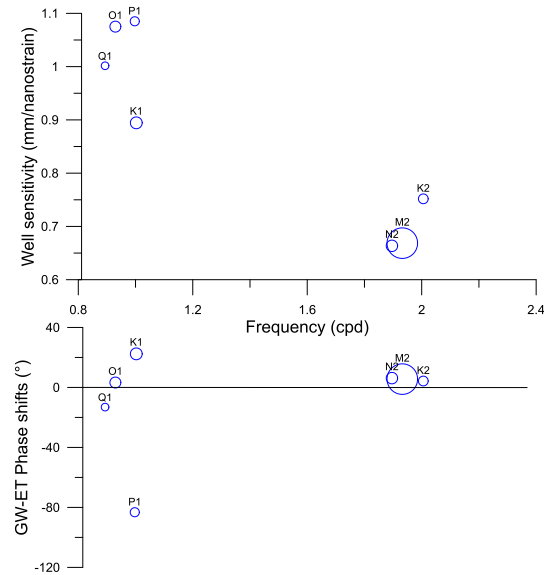
256 **Table 1.** Dominant tidal components that are generally found in groundwater measurements
257 (adapted from MacMillan et al., 2019)

Darwin Name	Frequency (cpd)	Attribution
Q_1	0.89365	Earth
O_1	0.929536	Earth
P_1	0.997262	Earth
S_1	1.000000	Atmosphere
K_1	1.002738	Earth
N_2	1.895982	Earth
M_2	1.932274	Earth
S_2	2.000000	Earth/Atmosphere
K_2	2.005476	Earth

258

259 The results obtained from HALS were utilized to calculate the amplitude response and phase shift
260 between groundwater (GW) and Earth tide (ET) for each tidal component. These amplitude responses
261 are commonly known as "well sensitivities" to Earth tide strains (Rojstaczer and Agnew, 1989) and are
262 summarized in Figure 4 alongside the corresponding phase shifts.

263 The well sensitivities to tides exhibit a frequency-dependent behaviour, resulting in similar values for
264 neighbouring frequencies and a generally decreasing magnitude. The amplitudes of M_2 and N_2 are
265 relatively straightforward to assess due to their significant magnitudes (11.2 and 2.1 mm, respectively),
266 and their amplitude responses and phase shifts are highly similar. The phase shifts for the tides of
267 interest (O_1 , N_2 , and M_2) are positive. However, the signs of the amplitudes for the other tides can be
268 attributed to their low amplitude responses, which are challenging to characterize using HALS.



269

270 *Figure 4: Amplitude responses and phase shifts as a function of frequency for the Cambodian site. S1 and S2 were excluded*
 271 *as they are not only generated by Earth tides. The circle size is proportional to the amplitude in the well water levels.*

272

273 3.3 Fitting the M_2/O_1 amplitude response ratio and phase shifts

274 The analysis is restricted to two types of tides: the semi-diurnal and diurnal tides. This limitation arises
 275 because the magnitude of Earth tide-induced well water levels is significantly damped for higher
 276 frequencies, making it difficult to discern and analyse tides beyond these two types. Here, we use
 277 amplitude responses (A_{M_2} , A_{O_1}) and phase shifts (α_{M_2} , α_{O_1}) to estimate hydraulic subsurface properties.
 278 N_2 tide was not used since its response may be too similar to M_2 and does not help with constraining
 279 the model. Amplitudes are influenced by geomechanical parameters (BK_u) which are generally not
 280 considered in classical hydrogeology. Valois et al. (2022) previously illustrated that the M_2 to O_1
 281 amplitude response ratio can be computed because it is not directly multiplied by BK_u and because it
 282 provides useful information about model choice. This leads to a system of three equations and six
 283 parameters (T , S , K' , D' , $skin$, and R_{KuB}) by using the simplified model in Equation 27 when the geometry
 284 of the well and the aquitard-aquifer system is known:

$$285 \quad \frac{A_{M_2}}{A_{O_1}} = \left| \frac{h_{w,0,\omega=M_2}}{h_{w,0,\omega=O_2}} \right| \quad (31)$$

$$286 \quad \alpha_{M_2} = \arg \left[\frac{h_{w,0,\omega=M_2}}{h_{w,0,\omega=O_2}} \frac{BK_u}{\rho g} \varepsilon_0 \right] \quad (32)$$

$$287 \quad \alpha_{O_1} = \arg \left[\frac{h_{w,0,\omega=O_1}}{h_{w,0,\omega=O_2}} \frac{BK_u}{\rho g} \varepsilon_0 \right]. \quad (33)$$

288 Table 2 displays the data to be fitted using the three equations above.



289 *Table 2: Data to be fitted using the ET-GW model*

	$\frac{A_{M2}}{A_{O1}}$	α_{M2} (°)	α_{O1} (°)
Cambodia	0.62	5.62	3.3

290

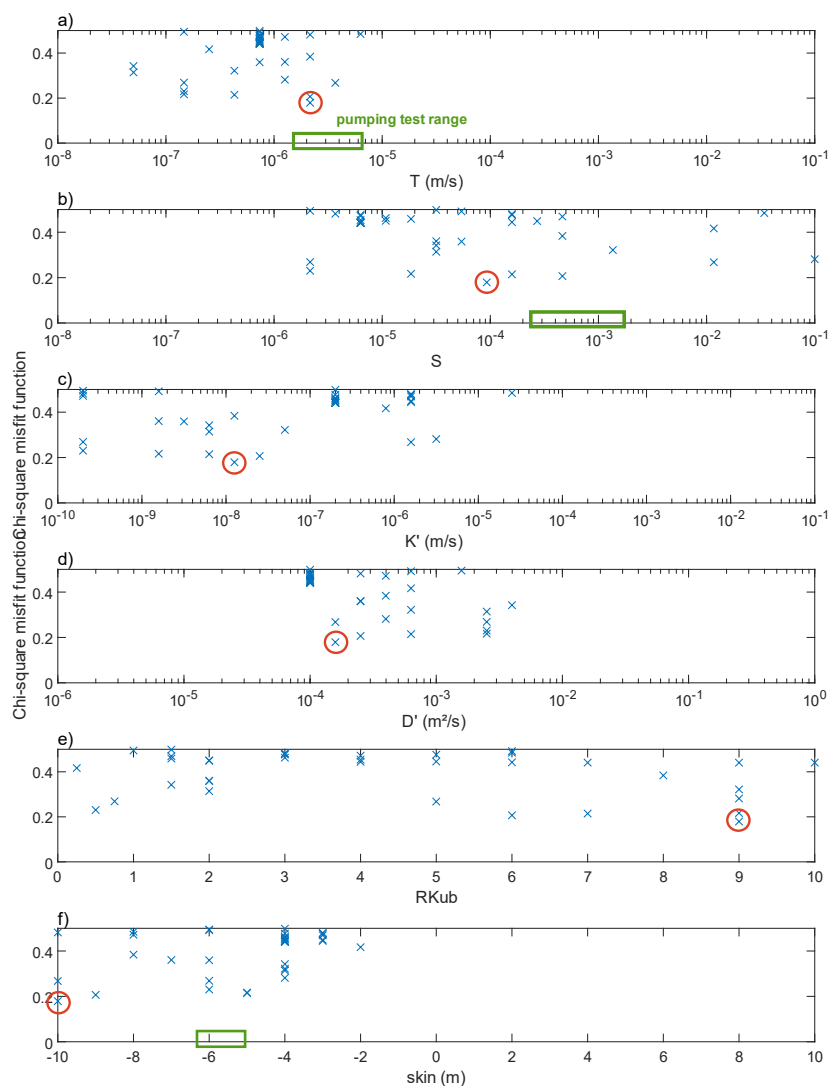
291 A systematic exploration of the entire parameter space without any constraints other than the well
 292 and aquifer geometry was carried out. Hydraulic and geomechanical property ranges are chosen
 293 according to the literature, i.e., De Marsily (1986) and Domenico and Schwartz (1998). In order to
 294 assess the goodness of fit with the three observed parameters (Eqs 31 to 33), the objective chi-square
 295 function is defined below:

296
$$\chi^2 = 1/N \sum_{i=1}^N \left(\frac{(Obs_i - Mod_i)}{Error_i} \right)^2 \quad (34)$$

297 where N is the number of observed parameters (3 here), Obs_i , Mod_i and $Error_i$ are the observed
 298 parameter, modeled parameter and their errors respectively. Thus, this objective function takes into
 299 account errors of the observed parameters (De Pasquale et al.; 2022). They were set to 0.1°, 0.5° and
 300 0.2 for α_{M2} , α_{O1} and $\frac{A_{M2}}{A_{O1}}$ respectively, according to Valois et al. (2022).

301

302 The model allows to fit both O_1 and M_2 positive phases and the low M_2 to O_1 amplitude ratio (misfit
 303 closed to 0 in Fig. 5), whereas the model of Gao et al (2020) cannot (misfit above 1 see Appendix B,
 304 and Valois et al., 2020). The T value is in good agreement with the pumping test range (Fig. 5a). S is
 305 half an order of magnitude below the pumping test range (Fig 5b) whereas the storativity best-fit for
 306 Gao et al (2020) is two orders of magnitude above (Appendix B). The skin effect also shows acceptable
 307 values as compared to the pumping test (Fig 5f). The parameter exploration shows best-fits for K' and
 308 D' , whereas it is difficult to identify a clear best-fit for the R_{KUB} parameter. The values are within the
 309 expected range for the hydrogeological configuration: The mudstone aquitard has a lower hydraulic
 310 conductivity (10^{-8} m/s) than the underlying claystone aquifer (coarser grain size than the aquitard, with
 311 a K value of about 10^{-7} m/s for an aquifer thickness of 22 m), and a diffusivity of about 10^{-4} m²/s. This
 312 is in agreement with the aquitard classification of Pacheco (2013).



313

314 *Figure 5: Results of full parameters exploration using two leaky aquifer models for the Cambodian case study.*

315

316 4. DISCUSSION

317 4.1. Uncertainties and discrepancies

318 There are several sources of uncertainty which originate from measurement and their propagation as
 319 well as uncertainties introduced by model assumptions. We believe that uncertainties linked to
 320 pressure sensor resolution (0.2 mm) and time resolution (20 minutes) as well as the HALS



321 decomposition were too low to be worth considering, at least for the semi-diurnal tides. This can be
322 deduced from the nearly identical amplitude responses at M_2 and N_2 at our field site. Because those
323 responses are indeed identical, it means that errors in the raw data set did not influence the response
324 characterization. We note that amplitude responses and phase shifts show larger discrepancies for the
325 diurnal tides. This could be linked to overall lower amplitudes which are generally more difficult to
326 characterise. We therefore conclude that errors arising from uncertainties are negligible compared to
327 the uncertainty introduced by model assumptions, in agreement with Sun *et al.* (2020).

328 Discrepancies between hydro-geomechanical properties derived from the groundwater response to
329 Earth tides (termed as “passive” and assuming a compressible matrix) and hydraulic testing (e.g., slug,
330 pump and lab testing, termed as “active” and generally assuming an incompressible matrix) have been
331 reported in the literature and have not been appropriately reconciled. By fitting amplitude response
332 ratio and phase shifts (Section 3.3), a T value discrepancy of one order of magnitude can be observed
333 between both approaches. We hypothesise that this is caused by parameter anisotropy.

334 Zhang *et al.* (2019b) pointed out differences in hydraulic conductivities of more than one order of
335 magnitude between ET analysis and slug tests and attributed this to differences in the investigated
336 scale. Allègre *et al.* (2016) reported much higher values of storativity derived from pumping test is
337 compared to ET when using the vertical flow model. Sun *et al.* (2020) showed that T values are
338 frequency-dependent with several orders of magnitude differences when comparing co-seismic, ET,
339 slug or pump test methods. The discrepancies can be explained by the different conceptual models
340 used in the active (based on perfectly confined) and passive methods (based on leaky conditions) or
341 by the frequency dependency of hydraulic parameters. The literature illustrates that transmissivity,
342 hydraulic conductivity or specific storage can indeed vary depending on the frequency of the forcing
343 (e.g., Cartwright *et al.*, 2005; Renner and Messar, 2006; Gultinan and Becker, 2015; Rabinovich *et al.*,
344 2015). This demonstrates the need for attention when assessing hydraulic parameters using passive
345 methods for semi-confined conditions. We specifically emphasise the need for using the same
346 conceptual model (i.e., confined, leaky with or without storage, vertical flow) when comparing active
347 and passive methods, as well as the need of preliminary hydrogeological knowledge of both the aquifer
348 system (i.e., presence of an aquitard with or without storage) (Bastias *et al.*, 2022) as well as the
349 borehole skin effect.

350

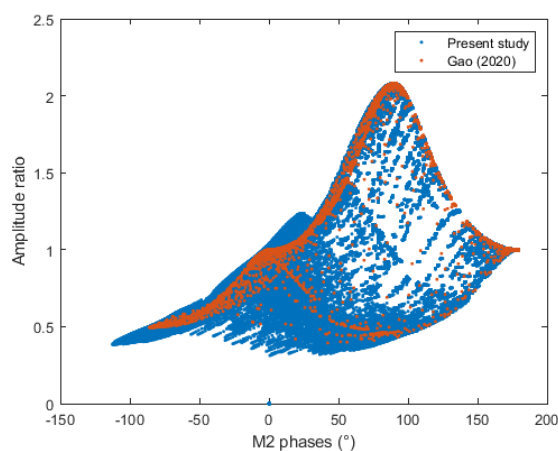
351 **4.2. The use of the leaky model with aquitard storage**

352 Our new analytical solution describing the well water level response to harmonic Earth tide strains
353 contains at least six hydro-geomechanical parameters that could be derived from only three features,
354 e.g., M_2 to O_1 amplitude response ratio and M_2 and O_1 phase shifts. Applying this model to real-world



355 cases to derive properties from amplitude responses and phase shifts provides relevant information
356 on T , S , D' , K' , and skin effect, but it is prone to non-uniqueness. Thus, a priori information may be
357 needed depending on the capacity of the inverse problem to fit observed data (phases shifts and
358 amplitude ratio). In our case study, parameter assessment would benefit from prior information on S
359 (or K') and R_{KuB} .

360 The model presented in this study can be useful when the hydrogeological configuration involves
361 storage in the aquitard with fixed head (i.e., Dirichlet) boundary conditions and for cases where phase
362 shifts and amplitude ratio do exemplify a specific pattern. For example, when compared to Gao et al
363 (2020) and using the parametrisation of the present study (Fig. 6), our solution is able to model lower
364 M_2 to O_1 amplitude ratio, lower phases, and higher amplitude ratio for phases closed to 0.



365

366 *Figure 6: Outputs of the models using the parametrization of the study ($rc=rw=6.08$ cm, $z_f=-10$ m).*

367 While the amplitudes are controlled by the product of the Skempton coefficient and the undrained
368 bulk modulus, these mechanical parameters also affect phase shifts. Therefore, further investigations
369 are needed to assess these influences using other methods or to link them empirically with the
370 hydraulic parameters. This is crucial to enhance confidence in utilizing groundwater response to Earth
371 tides as a valuable tool for better understanding and characterizing groundwater resources.

372

373 5. Conclusion

374 We have developed a new analytical solution for the well water level response to Earth tide strains.
375 This solution considers a previously unprecedented physical reality, specifically, a leaky aquifer with



376 aquitard storage, subject to Dirichlet boundary conditions under tidal strain. Additionally, our model
377 considers the influence of borehole storage and skin effects, further improving the accuracy and
378 comprehensiveness of the analysis. This model extends upon previous models and allows advanced
379 characterization of the subsurface using the groundwater response to natural forces. The new model
380 overcomes previous limitations, for example it explains very low M_2 to O_1 amplitude ratios as well as
381 large phase shift difference between M_2 and O_1 tides. The model relies on six combinations of hydro-
382 geomechanical parameters. In this study, we assess the most sensitive parameters to be the
383 transmissivity, the well skin effect, the aquitard to aquifer mechanical parameters ratio ($B'K_u'/BK_u$), as
384 well as aquitard diffusivity and aquitard conductivity to aquifer storativity ratio.
385 We apply our new model to a groundwater monitoring dataset from Cambodia and compare the
386 results with pumping tests undertaken in the same formation. We used the diurnal (O_1) and semi-
387 diurnal (M_2) tides to better constrain the model. Results illustrate significant insight into subsurface
388 properties. For example, we derive relevant information about T , S , D' , K' , and *skin effect*, when
389 compared to the pumping test results. Overall, our new model can be used to shed light on previously
390 inexplicable well water level behaviour and can be paired with other investigation methods to enhance
391 understanding of subsurface processes.

392

393 **Competing interests**

394 The contact author has declared that none of the authors has any competing interests.

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- 514



515 **Appendix**

516 **Appendix A: The analytical solution in the aquitard**

517 To solve Equation 14 with the boundary conditions in Equations 7 and 8, we define:

518
$$\widehat{h}'_0 = h'_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A1})$$

519 Thus, the equation system become:

520
$$\left(\frac{\partial^2 \widehat{h}'_0}{\partial z^2} \right) = \frac{i\omega \widehat{h}'_0}{D'} \quad (\text{A2})$$

521
$$\widehat{h}'_0(z = z_i) = h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A3})$$

522
$$\widehat{h}'_0(z = 0) = h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A4})$$

523 The solution \widehat{h}'_0 is of the form:

524
$$\widehat{h}'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} \quad (\text{A5})$$

525 It yields

526
$$A_1 = \frac{e^{\frac{(1+i)}{\delta}z_i} \left(h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \right) - \left(h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \right)}{2 \sinh \left(\frac{(1+i)}{\delta} z_i \right)} \quad (\text{A6})$$

527
$$A_2 = \frac{-e^{-\frac{(1+i)}{\delta}z_i} \left(h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \right) + \left(h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \right)}{2 \sinh \left(\frac{(1+i)}{\delta} z_i \right)} \quad (\text{A7})$$

528

529 Thus:

530
$$h'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} + \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A8})$$

531

532 **Appendix B: Additional information on parameter exploration**

533

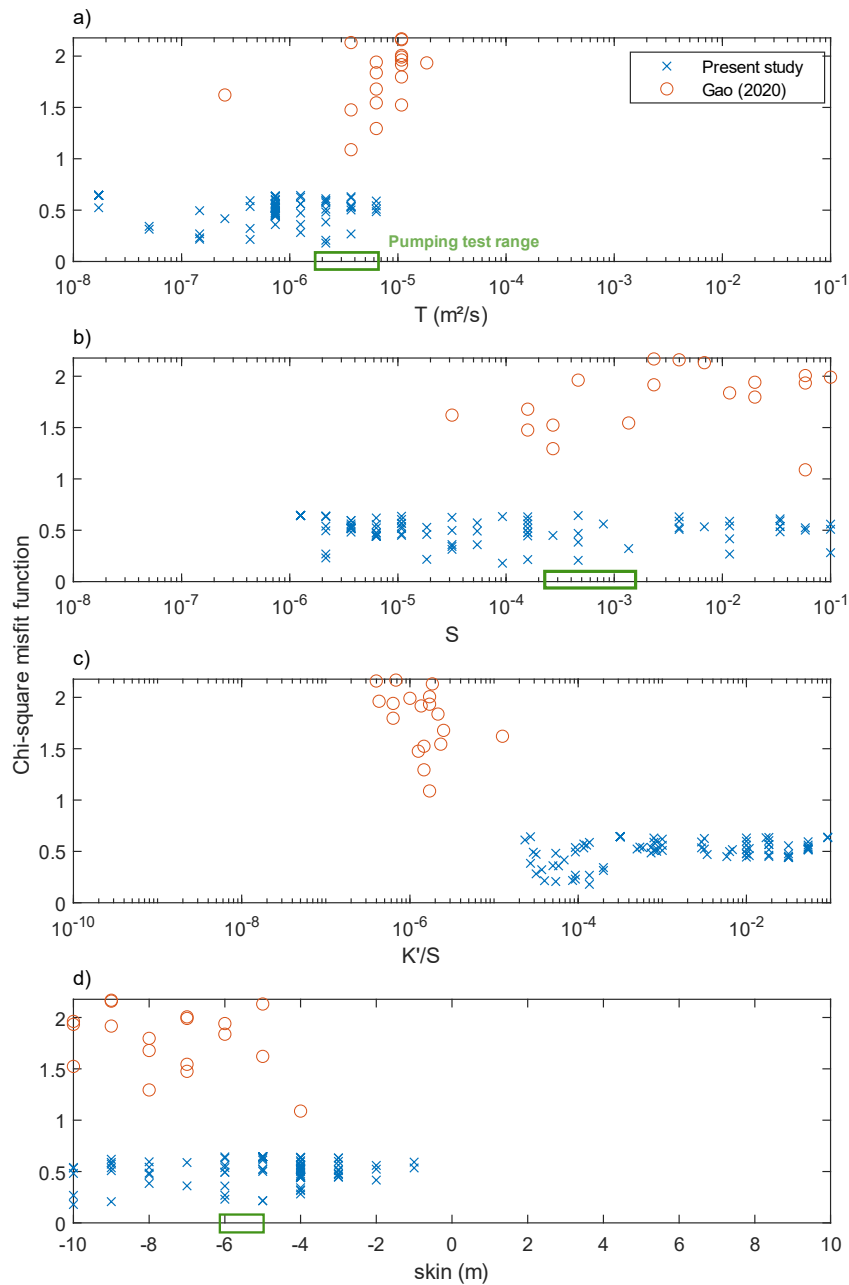
534 The figure A1 shows the misfit functions using the model developed in this study and the model of
 535 Gao et al . (2020). Misfits are clearly higher for the older model that do not consider storage and
 536 tidal response in the aquitard. Storativity best-fit using the model of Gao et al (2020) failed to
 537 reproduce pumping test values. Nevertheless, transmissivity and skin estimates fall within
 538 pumping test range.

539

540

541

542



543

544 *Figure A1: Comparison of misfit function for the present model and the one of Gao et al. (2020) for the Cambodian case study.*
545 *Only the first hundred best-fit were plotted.*

546