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Rémi Valois, Agnès Rivière, Jean-Michel Vouillamoz, Gabriel Rau

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1     **Technical Note: Analytical Solution for Well Water Response to Earth Tides in**  
2             **Leaky Aquifers with Storage and Compressibility in the Aquitard**

3  
4             Rémi Valois<sup>1,2,\*</sup>, Agnès Rivière<sup>3</sup>, Jean-Michel Vouillamoz<sup>4</sup>, Gabriel C. Rau<sup>5</sup>

5  
6                             <sup>1</sup>French Red-Cross, 4 rue Diderot, Paris, France

7             <sup>2</sup>Hydrogeology Lab, UMR EMMAH, University of Avignon, 74 rue Louis Pasteur, Avignon, France

8                             <sup>3</sup>Geosciences Department, Mines Paris - PSL, 75272 Paris, France

9             <sup>4</sup>Univ. Grenoble Alpes, IRD, CNRS, INRAE, Grenoble INP, IGE, 38000 Grenoble, France

10             <sup>5</sup>School of Environmental and Life Sciences, The University of Newcastle, Callaghan, Australia

11  
12                             \*Correspondence: [remi.valois1@gmail.com](mailto:remi.valois1@gmail.com); Tel.: +33-6-8434-0779 (R.V.)

13  
14     **Key points**

- 15     - Development of a new analytical solution for Earth tide induced well water level fluctuations in  
16         semi-confined aquifers considering aquitard storage, aquitard response to tidal strain, skin and  
17         wellbore storage effects
- 18     - The solution correctly reflects previously observed but unexplained amplitude-frequency  
19         relationships and positive or negative phase shifts
- 20     - Diagnostic information about subsurface hydro-geomechanical properties can be derived from  
21         amplitude ratio and phase shifts for both semi-diurnal and diurnal tides

22  
23     **Abstract**

24     In recent years, there has been a growing interest in utilizing the groundwater response to Earth tides  
25     as a means to estimate subsurface properties. However, existing analytical models have been  
26     insufficient in accurately capturing realistic physical conditions. This study presents a new analytical  
27     solution to calculate groundwater response to Earth tide strains, including storage and compressibility  
28     of the aquitard, borehole storage and skin effects. We investigate the effects of aquifer and aquitard  
29     parameters on well water response to Earth tides at two dominant frequencies ( $O_1$  and  $M_2$ ) and  
30     compare our results with hydraulic parameters obtained from a pumping test. Inversion of the six  
31     hydro-geomechanical parameters from amplitude response and phase shift of both semi-diurnal and  
32     diurnal tides provides relevant information about aquifer transmissivity, storativity, well skin effect,  
33     aquitard hydraulic conductivity and diffusivity. The new model is able to reproduce previously  
34     unexplained observations of the amplitude and frequency responses. We emphasize the usefulness in  
35     developing relevant methodology to use the groundwater response to natural drivers for  
36     characterizing hydrogeological systems.

37



## 38 **1. Introduction**

39 Aquifer properties play a vital role in managing groundwater resources, particularly amid increasing  
40 anthropogenic groundwater use and the impact of climate change. While pump testing can be costly,  
41 there exists a cost-effective alternative for assessing aquifer hydraulic properties - analysing the  
42 groundwater response to Earth tides or atmospheric tides (McMillan et al., 2019). Observations of  
43 variations in groundwater level due to tidal fluctuations date back to the works of Klönne (1880),  
44 Meinzer (1939), and Young (1913). However, it was only later that hydro-geomechanical models were  
45 employed to elucidate these variations (Bredehoeft, 1967; Hsieh and Bredehoeft, 1987; Roeloffs, 1996;  
46 Wang, 2000; Cutillo and Bredehoeft, 2011; Kitagawa et al., 2011; Lai et al., 2013; Wang et al., 2018).  
47 This progression in understanding offers a valuable opportunity to evaluate aquifer hydraulic  
48 properties through the response of groundwater to tidal fluctuations.

49 Hsieh and Bredehoeft (1987) introduced the horizontal flow model, focusing on confined conditions  
50 influenced by tidal forces. Conversely, Roeloffs (1996) and Wang (2000) explored interactions within  
51 vertical flow under tidal fluctuations. Wang et al. (2018) expanded on these studies by incorporating a  
52 flow from an upper aquitard, albeit assuming negligible storage within it. Later, Gao et al. (2020)  
53 extended these models to include borehole skin effects. Thomas et al. (2023) developed an ET-GW  
54 model incorporating storage and strain response in the aquitard. They applied their model to a specific  
55 site to evaluate transmissivity variations and validated it using pumping tests.

56 Numerous studies have investigated aquifer hydromechanical properties by analysing GW level  
57 variations induced by Earth tides, employing the models mentioned in the previous literature  
58 (Narasimhan et al., 1984; Merritt, 2004; Fuentes-Arreazola et al., 2018; Zhang et al., 2019a; Shen et  
59 al., 2020). However, only a limited number of validations have been conducted, which involve  
60 comparing the results with robust hydraulic assessments, such as hydraulic conductivity derived from  
61 slug testing (Zhang et al., 2019b) or specific storage and transmissivity characterizations obtained  
62 through long-term pumping tests (Allègre et al., 2016; Valois et al., 2022). The current evaluations  
63 predominantly focus on purely confined conditions, leaving a gap in knowledge regarding tidally  
64 induced GW responses in aquifers under semi-confined conditions.

65 As far as the authors are aware, the publications by Sun et al. (2020), Valois et al. (2022), and Thomas  
66 et al. (2023) are the sole references addressing a comparison for a leaky aquifer. Sun et al. (2020) found  
67 significant discrepancies between transmissivities obtained from Earth tide fluctuations and those  
68 derived from slug or pump tests. However, it is worth noting that the comparison may be subject to  
69 discussion, as the authors employed a leaky aquifer model for analysing tidally induced fluctuations,  
70 whereas they used a confined aquifer model for conducting slug and pump tests. In the study  
71 conducted by Valois et al. (2022), the existing Earth-Tide GroundWater (ET-GW) models, as described



72 earlier, were unable to reproduce a low semi-diurnal to diurnal amplitude ratio with positive phase  
73 shifts in conjunction with pumping test transmissivity data. This discrepancy highlights the complexity  
74 and challenges in modelling tidally induced groundwater responses in leaky aquifers and the need for  
75 further investigation in this area.

76 We note that our previous attempts to model the observed substantial amplitude decrease from  $O_1$  to  
77  $M_2$  frequency, combined with phase shifts close to zero, proved unsuccessful when using Earth tide  
78 models found in the literature. None of the existing models could provide satisfactory results. The  
79 pursuit of an explanation led to the realisation that analytical solutions with more realistic assumptions  
80 are required. For example, aquifers are widely recognized to be influenced by aquitards, which often  
81 consist of highly porous and compressible clay materials, contributing significant amounts of stored  
82 water to the aquifer (Moench, 1985). Moreover, these aquitards are also impacted by Earth tide strains  
83 (Bastias et al., 2022).

84 Our first objective is to develop an analytical solution considering storage and strain in the aquitard.  
85 Unlike Thomas et al. (2023), our model incorporates borehole skin effects and allows for a fixed  
86 hydraulic head at the top of the aquitard, broadening its applicability to a broader range of  
87 hydrogeological conditions. The second motivation of our study is to develop a model that better fits  
88 the observed results by considering aquitard storage, as evident in the pumping tests. Third, we  
89 compare the results obtained from our new ET-GW model with those derived from a pumping test in  
90 leaky aquifers with storage in the aquitard. Fourth, since most publications have predominantly  
91 focused on assessing hydraulic properties using the semi-diurnal tide ( $M_2$ ), our third motivation is to  
92 demonstrate the potential of using diurnal tides ( $O_1$ ) in combination with the semi-diurnal ( $M_2$  or  $N_2$ )  
93 to provide a more comprehensive characterization of aquifer and aquitard hydro-mechanical  
94 properties. Our new development offers the potential to enhance hydraulic and geomechanical  
95 subsurface characterization by employing a more realistic model for the groundwater response to  
96 natural forces.

97

98

## 99 **2. Groundwater response to Earth tides in a leaky aquifer with aquitard**

### 100 **storage and strain**

101 Hantush (1960) pioneered the modelling of aquitard storage by modifying the leaky aquifer theory to  
102 account for storage in the aquitard. In our study, we consider a semi-confined configuration (Figure 1)  
103 where the target aquifer is overlain by an aquitard that allows for storage, strain, and vertical flux. Both  
104 layers are assumed to be slightly compressible, spatially homogenous, infinite laterally, and have  
105 constant thickness. Building upon the work of Wang et al. (2018), our research incorporates Earth Tide



106 (ET) fluctuations into the leaky aquifer equations proposed by Moench (1985). Additionally, we  
 107 incorporate the skin effect, as described by Gao et al. (2020).

108 Groundwater flow and storage in an aquifer overlain by an aquitard can be described as:

$$109 \quad T \left( \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \right) = S \left( \frac{\partial h}{\partial t} - \frac{BK_u}{\rho g} \frac{\partial \varepsilon}{\partial t} \right) - K' \frac{\partial h'}{\partial z} \quad (1)$$

$$110 \quad \left( \frac{\partial^2 h'}{\partial z^2} \right) = \frac{1}{D'} \left( \frac{\partial h'}{\partial t} - \frac{B'K'_u}{\rho g} \frac{\partial \varepsilon}{\partial t} \right) \quad (2)$$

111 Here,  $h$  (m) and  $h'$  (m) are the hydraulic heads in the aquifer and the aquitard respectively;  $h'_j$  (m) is  
 112 the fixed hydraulic head at the top of the aquitard,  $r$  (m) is the radial distance from the studied well;  $T$   
 113 ( $\text{m}^2/\text{s}$ ) and  $S$  are the aquifer transmissivity and storativity;  $B$ ,  $B'$ ,  $K_u$  (Pa),  $K'_u$  (Pa) are the Skempton's  
 114 coefficient and the undrained bulk modulus of the aquifer and aquitard respectively;  $\rho$  ( $\text{kg}/\text{m}^3$ ) and  $g$   
 115 ( $\text{m}/\text{s}^2$ ) are the water density and gravity constant;  $\varepsilon$  is the volumetric Earth tide strain;  $K'$  ( $\text{m}/\text{s}$ ) is the  
 116 aquitard hydraulic conductivity;  $S'_s$  ( $\text{m}^{-1}$ ) is the specific storage of the aquitard;  $D'$  ( $\text{m}^2/\text{s}$ ) is aquitard  
 117 hydraulic diffusivity ( $K'/S'_s$  ratio). Any natural regional groundwater flow is considered negligible.

118 Borehole drilling causes a zone of damage with a radius  $r_s$  (see Figure 1) that is responsible for the skin  
 119 effect (Van Everdingen, 1953). A negative skin can be caused by a greater hydraulic conductivity around  
 120 the well because of the material damaged by the drilling, while a positive skin can be associated by  
 121 porosity clogging caused by the drilling mud. This is reflected in the well's pressure head  $\Delta h_s$ . The skin  
 122 factor ( $sk$ ) can be defined as:

$$123 \quad sk = \frac{\Delta h_s}{\left( r \frac{\partial h}{\partial r} \right)_{r=r_w}} \quad (3)$$

124 Following above assumptions, the boundary conditions are

$$125 \quad h(r, t) = h_\infty(t) \text{ at } r = \infty \quad (4)$$

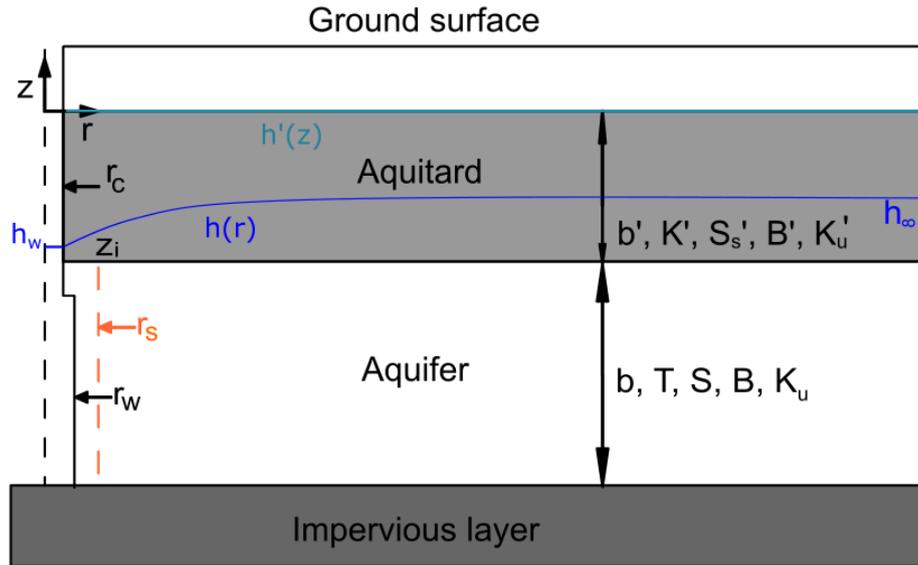
$$126 \quad h_w(t) = h(r, t) - sk \left( r \frac{\partial h(r, t)}{\partial r} \right) \text{ at } r = r_w \quad (5)$$

$$127 \quad 2\pi r_w T \left( \frac{\partial h}{\partial r} \right)_{r=r_w} = \pi r_c^2 \frac{\partial h_w}{\partial t} \quad (6)$$

$$128 \quad h' = h \text{ at } z = z_i \quad (7)$$

$$129 \quad h' = h'_j \text{ at } z = 0 \quad (8)$$

130 Here,  $t$  is the time (s);  $r_w$  and  $r_c$  are the radius of the well screened portion and the radius of well casing  
 131 in which water level fluctuates;  $z_i$  is the aquifer-aquitard interface elevation (see Figure 1) and  $h_w$  is the  
 132 hydraulic head at  $r_w$ .



133

134

Figure 1: Semi-confined system with a compressible aquitard with storage

135 Following Hsieh et al. (1987) and Wang et al. (2018), complex numbers were used to facilitate harmonic  
 136 model development and the solution is obtained by first solving Equation 2 in the aquitard, then  
 137 deriving the head response in the aquifer far away from the well ( $h_\infty$ ) which is independent of the  
 138 radial distance. Then, the well effect on the aquifer response is considered by using a flux condition at  
 139 the well that accounts for wellbore storage. Since  $h'$ ,  $h$ ,  $\varepsilon$ ,  $h_w$ ,  $h_\infty$  are all periodic functions, they can be  
 140 expressed as:

141 
$$\varepsilon(t) = \varepsilon_0 e^{i\omega t} \quad (9)$$

142 
$$h_\infty(t) = h_{\infty,0} e^{i\omega t} \quad (10)$$

143 
$$h_w(t) = h_{w,0} e^{i\omega t} \quad (11)$$

144 
$$h(r, t) = h_0(r) e^{i\omega t} \quad (12)$$

145 
$$h'(z, t) = h'_0(z) e^{i\omega t} \quad (13)$$

146 Here,  $i = \sqrt{-1}$ ;  $\varepsilon_0$  (m) is the ET strain amplitude and  $\omega$  ( $s^{-1}$ ) is the angular frequency. In this case,  
 147 Equation 2 becomes:

148 
$$\left(\frac{\partial^2 h'_0}{\partial z^2}\right) = \frac{1}{D'} \left( i\omega h'_0 - i\omega \frac{B'K'_u}{\rho g} \varepsilon_0 \right) \quad (14)$$



149 According to Wang (2000) and Roeloffs (1996) and as detailed in appendix A, the solution of Equation  
 150 2 is:

$$151 \quad h'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} + \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (15)$$

152 where  $\delta = \left(\frac{2D'}{\omega}\right)^{1/2}$ . Thus, at the interface between the aquifer and the aquitard ( $z=z_i$ ), we have  
 153 pressure continuity as  $h'(z_i, t) = h_0 e^{i\omega t} = h(t)$  which leads to:

$$154 \quad \left(\frac{\partial h'}{\partial z}\right)_{z=z_i} = \frac{1+i}{\delta} \left( \frac{h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right) e^{i\omega t} \quad (16)$$

155 Equation 1 can be solved far away from the well using  $h_\infty$  which is independent of the radial distance  
 156 from the well and by using the source term from  $h'$  as follows

$$157 \quad 0 = \frac{\partial h_\infty}{\partial t} - \frac{BK_u}{\rho g} \frac{\partial \varepsilon}{\partial t} - \frac{K'}{S} \frac{1+i}{\delta} \left( \frac{h_\infty - \frac{B'K'_u}{\rho g} \varepsilon_0}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right) e^{i\omega t} \quad (17)$$

$$158 \quad h_{\infty,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} \frac{B'K'_u}{BK_u} \left( \frac{-1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)}} \quad (18)$$

159 The disturbance in water level due to the well can be expressed as:

$$160 \quad s(r, t) = h(r, t) - h_\infty(t) \quad (19)$$

161 Equation 1 becomes:

$$162 \quad T \left( \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) = S \left( \frac{\partial s}{\partial t} \right) - K' \frac{1+i}{\delta} S \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \quad (20)$$

163 with the boundary conditions:

$$164 \quad s(r \rightarrow \infty) = 0 \quad (21)$$

$$165 \quad h_{w,0} - h_{\infty,0} = s - sk \left( r \frac{\partial s}{\partial r} \right) \quad \text{at } r = r_w \quad (22)$$

$$166 \quad 2\pi r_w T \left( \frac{\partial s}{\partial r} \right)_{r=r_w} = i\omega \pi r_w^2 h_{w,0} \quad (23)$$

167 The solution of this differential equation is  $s(r) = C_1 I_0(\beta r) + C_2 K_0(\beta r)$  (Wang *et al.*, 2018), where  $I_0$   
 168 and  $K_0$  are the modified Bessel functions of the first and second kind and the zeroth order, respectively.

169 Further,



170 
$$\beta = \left( \frac{i\omega S}{T} - \frac{K'(1+i)}{T} \frac{1}{\delta \tanh\left(\frac{1+i}{\delta} z_i\right)} \right)^{1/2} . \quad (24)$$

171 The boundary conditions lead to  $C_1=0$  and  $C_K = -\frac{i\omega r_c^2 h_{w,0}}{2T\beta r_w K_1(\beta r_w)}$  because  $\frac{dK_0(r)}{dr} = -K_1(r)$ . Therefore,  
 172 the final solution for the well water level is:

173 
$$h_{w,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} \frac{B'K'_u}{BK_u} \left( \frac{-1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} - \frac{h'_0 \frac{\rho g}{B'K'_u \varepsilon_0} - 1}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{\sigma \left( Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \right)} , \quad (25)$$

174 where

175 
$$\sigma = 1 + \frac{i\omega r_c^2 K_0(\beta r_w)}{2T\beta r_w K_1(\beta r_w)} + \frac{i\omega r_c^2}{2T} sk . \quad (26)$$

176 By assuming  $h'_0 = 0$  (i.e. the hydraulic head at the top of the aquitard corresponds to the  
 177 unsaturated-saturated interface at  $z=0$ ), Equation 25 can be reorganized to

178 
$$h_{w,0} = \frac{BK_u}{\rho g} \varepsilon_0 \frac{Si\omega + K' \frac{1+i}{\delta} R_{KuB} \left( \frac{1 - \cosh\left(\frac{1+i}{\delta} z_i\right)}{\sinh\left(\frac{1+i}{\delta} z_i\right)} \right)}{\sigma \left( Si\omega - K' \frac{1+i}{\delta} \frac{1}{\tanh\left(\frac{1+i}{\delta} z_i\right)} \right)} , \quad (27)$$

179 where

180 
$$R_{KuB} = \frac{K'_u B'}{K_u B} . \quad (28)$$

181 By disregarding  $\frac{BK_u}{\rho g}$  product which only controls the amplitude, the solution has six independent  
 182 parameters which are  $T, S, K', D', sk$  and the  $R_{KuB}$  ratio.

183 Let us now define the amplitude response (or amplitude ratio),  $A$ , and phase shift,  $\alpha$ , of the GW  
 184 response to ET fluctuations:

185 
$$A = \left| h_{w,0} / \frac{BK_u}{\rho g} \varepsilon_0 \right| \quad (29)$$

186 
$$\alpha = \text{arg} \left[ h_{w,0} / \frac{BK_u}{\rho g} \varepsilon_0 \right] . \quad (30)$$

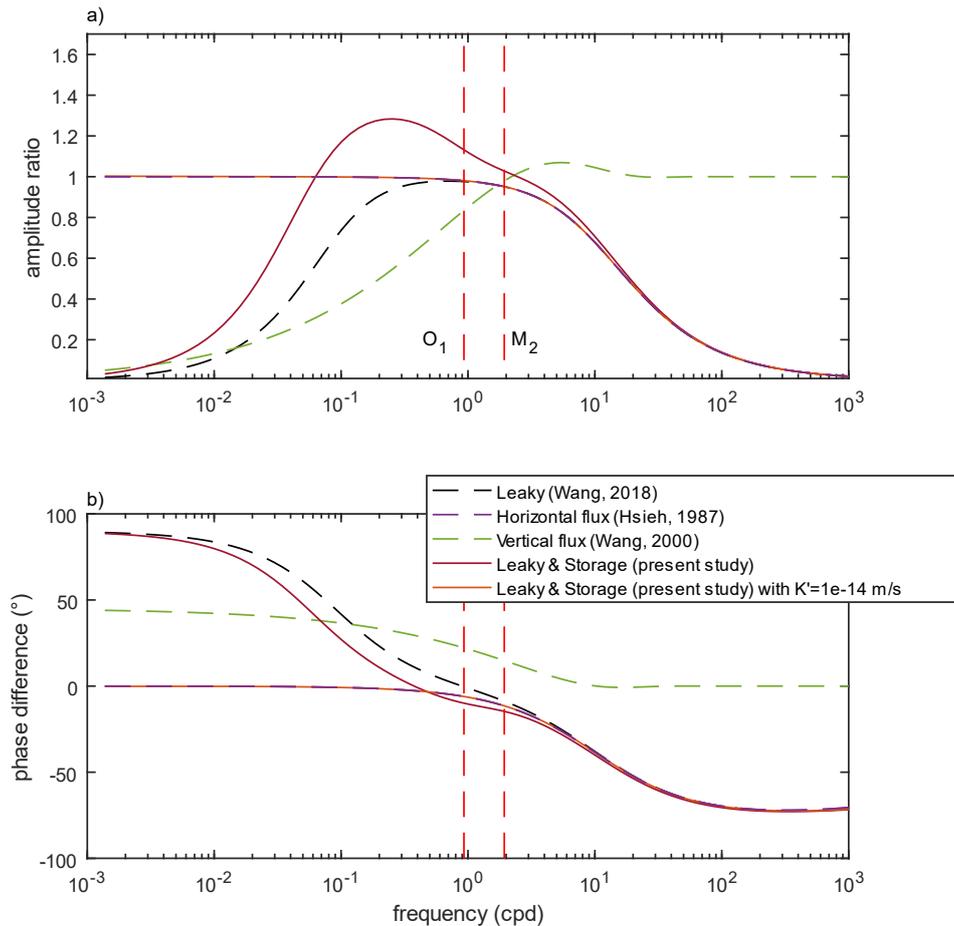
187 Figure 2 shows the amplitude response and phase shift as a function of frequency using our new  
 188 solution in comparison to key models reported in the literature. Aquitard parameters were set according  
 189 to Batlle-Aguilar et al. (2016), while aquifer parameters were chosen according to the field application  
 190 below. We validate the solution using a very low aquitard conductivity ( $10^{-14}$  m/s), so that we can



191 compare it to the horizontal flux with wellbore storage model (Hsieh et al., 1987). It shows a perfect  
192 match.

193 Because both horizontal and vertical flux models are associated with opposite phase shift signs (Figure  
194 2b), the latter can offer valuable insights for model selection (Allègre et al., 2014). Positive phase shifts  
195 in the vertical flux model are related to an increasing amplitude ratio with frequency, whereas the  
196 wellbore storage model exhibits the opposite behaviour. Wang et al. (2018) developed a leaky model  
197 capable of demonstrating both positive and negative phase shifts, where positive phase shifts  
198 correspond to an increasing amplitude ratio with frequencies, and negative phase shifts are linked to  
199 a decreasing amplitude ratio.

200 Our new model showcases positive or negative phase shifts with an increasing or decreasing amplitude  
201 ratio over frequency, even allowing for amplitude ratios greater than one. Notably, Wang (2000)  
202 observed a similar characteristic in the vertical flux model, with an amplitude just above one (1.06) for  
203 very specific conditions. At high frequencies, our model displays amplitude ratios and phase shifts  
204 similar to those of the leaky and wellbore storage models, reflecting the attenuation of high-frequency  
205 pore pressure fluctuations in the aquifer by well water.



206

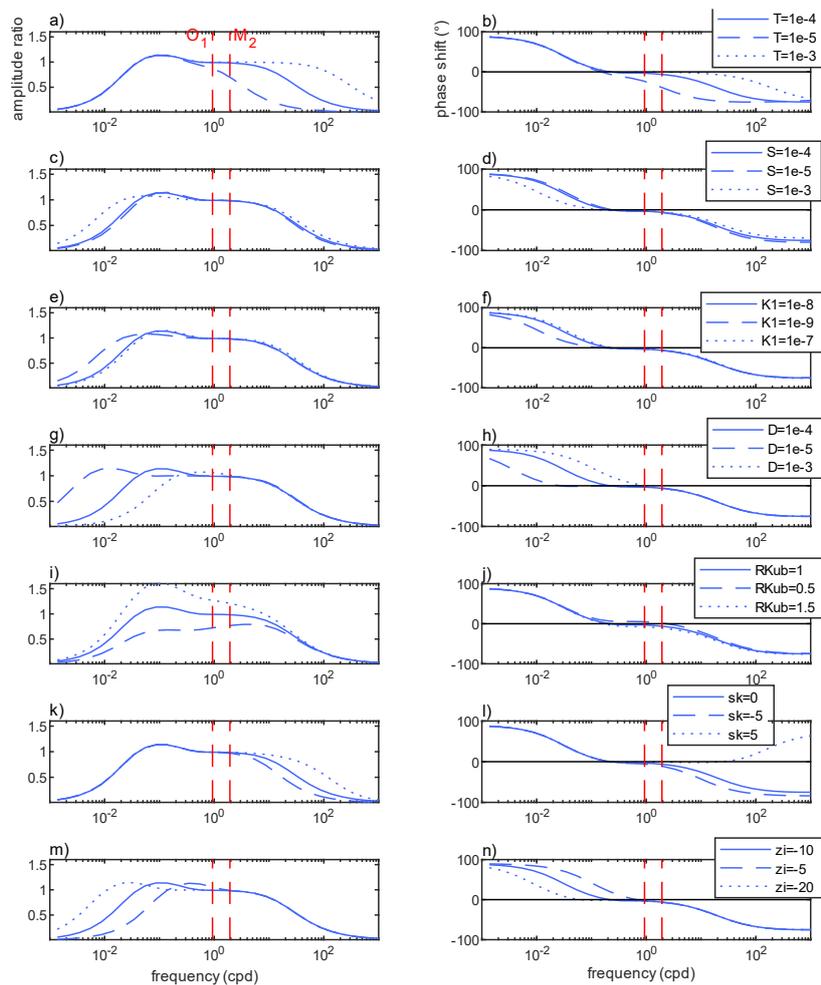
207 *Figure 2: Frequency variation of amplitude response and phase shift of the groundwater response to Earth tides. The*  
 208 *transmissivity ( $T$ ) is  $10^{-5} \text{ m}^2/\text{s}$ , storativity ( $S$ ) is  $10^{-4}$ , hydraulic conductivity of the aquitard ( $K'$ ) is  $10^{-8} \text{ m/s}$ , aquitard hydraulic*  
 209 *diffusivity ( $D'$ ) is  $10^{-4} \text{ m}^2/\text{s}$ , skin factor ( $sk$ ) is 0,  $R_{KuB}$  to 1.4, well casing radius ( $r_c$ ) and screen radius ( $r_w$ ) is 6.03 cm.  $b'$  was set*  
 210 *to 5 m. Screen depth ( $z$ ) was set to 23 m for the vertical flow model of Wang (2000).*

211

212 We explored the parameter space by focussing on the frequency-dependant amplitude response and  
 213 phase shift responses for different sets of parameter values. The reference parameter set is the one  
 214 described above.  $T$ ,  $R_{KuB}$  and  $z_1$  have a large impact on model shapes. As also observed by Hsieh et al.  
 215 (1987),  $S$  does not have a major impact on the results (Fig3c and 3d). The skin effect does not play a  
 216 large role in the useful frequency band for amplitudes, but its influence is larger for the phase shifts  
 217 when compared to the reference parameter set.  $K'$  does not significantly influence the results with



218 respect to the reference parameter set used in the study (Figures 3e to 3f).  $K'$  has the opposite role of  
 219  $S$  and they appear to compensate each others effects, because of their respective role in Equation 27.



220

221 *Figure 3: Illustration of the amplitude response (left column) and phase shift (right column) as a function of frequency for*  
 222 *various parameters compared to the reference parameter set.*

223



### 224 **3. Application of the new model to a groundwater monitoring dataset from Cambodia**

#### 225 **3.1 Field site and previous results**

226 The field site in Northwest Cambodia comprises three boreholes drilled into the subsurface, consisting  
227 of mudstone, claystone, siltstone, and sandstone. Time series and pumping test results have been  
228 reported in Valois et al. (2022), while details of the lithology can be found in Vouillamoz et al. (2012;  
229 2016) and Valois et al. (2017; 2018; 2022). Pumping from the aquifer is limited by a very low specific  
230 yield, attributed to the presence of fine deposits such as clay and mudstone (Vouillamoz et al. 2012;  
231 Valois et al., 2018).

232 The boreholes were drilled to a depth of 31 meters with a radius of 6 inches and equipped with 4-inch  
233 PVC casing from top to bottom, featuring a 9-meter long screen at the hole's base. The aquifer is  
234 situated within a hard rock media, comprising either claystone or sandstone, located beneath a 10-  
235 meter thick clay layer.

236 For the pumping tests, the wells were pumped for three days, and water levels were allowed to recover  
237 for four days in two observation wells. The interpretation of the pumping tests utilized  
238 AQTESOLV™/Pro v4.5 software, employing the leaky aquifer with aquitard storage model (Moench,  
239 1985) or a 3D flow using the generalized radial flow model (Barker, 1988). The selected solutions,  
240 compared to other models (Theiss, Hantush without aquitard storage), demonstrated the best fit with  
241 a Root Mean Square Error (RMSE) of 0.02 m for Cambodia.

242

#### 243 **3.2 Well sensitivities and phase shifts to Earth tides**

244 Between 2010 and 2015, well water levels were measured at 20, 40, or 60-minute intervals using  
245 absolute pressure sensors (Diver data loggers, Eijkelkamp Soil & Water, NL). To compensate for  
246 barometric pressure (BP) effects, data from a barometer located a few kilometres away from the field  
247 site were utilized (Eijkelkamp Soil & Water, NL). A zero-phase Butterworth filter was employed to  
248 eliminate low-frequency content (periods longer than 10 days) from both groundwater (GW) and BP  
249 data.

250 For each site's geolocation (latitude, longitude, and height), ET strain time series were computed at  
251 20-minute intervals using SPOTL software (Agnew, 2012). The time series were then modelled using  
252 Harmonic Least-Squares (HALS; Schweizer et al., 2021) with eight frequencies corresponding to the  
253 major tides (Table 1) following Merritt's description (2004). HALS provides amplitude and phase  
254 estimations for each tidal component and record.

255



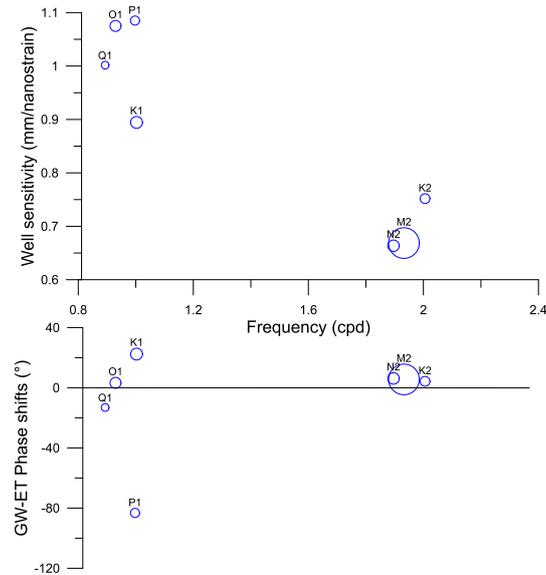
256 **Table 1.** Dominant tidal components that are generally found in groundwater measurements  
257 (adapted from MacMillan et al., 2019)

Darwin Name	Frequency (cpd)	Attribution
$Q_1$	0.89365	Earth
$O_1$	0.929536	Earth
$P_1$	0.997262	Earth
$S_1$	1.000000	Atmosphere
$K_1$	1.002738	Earth
$N_2$	1.895982	Earth
$M_2$	1.932274	Earth
$S_2$	2.000000	Earth/Atmosphere
$K_2$	2.005476	Earth

258

259 The results obtained from HALS were utilized to calculate the amplitude response and phase shift  
260 between groundwater (GW) and Earth tide (ET) for each tidal component. These amplitude responses  
261 are commonly known as "well sensitivities" to Earth tide strains (Rojstaczer and Agnew, 1989) and are  
262 summarized in Figure 4 alongside the corresponding phase shifts.

263 The well sensitivities to tides exhibit a frequency-dependent behaviour, resulting in similar values for  
264 neighbouring frequencies and a generally decreasing magnitude. The amplitudes of  $M_2$  and  $N_2$  are  
265 relatively straightforward to assess due to their significant magnitudes (11.2 and 2.1 mm, respectively),  
266 and their amplitude responses and phase shifts are highly similar. The phase shifts for the tides of  
267 interest ( $O_1$ ,  $N_2$ , and  $M_2$ ) are positive. However, the signs of the amplitudes for the other tides can be  
268 attributed to their low amplitude responses, which are challenging to characterize using HALS.



269

270 *Figure 4: Amplitude responses and phase shifts as a function of frequency for the Cambodian site. S1 and S2 were excluded*  
 271 *as they are not only generated by Earth tides. The circle size is proportional to the amplitude in the well water levels.*

272

### 273 3.3 Fitting the $M_2/O_1$ amplitude response ratio and phase shifts

274 The analysis is restricted to two types of tides: the semi-diurnal and diurnal tides. This limitation arises  
 275 because the magnitude of Earth tide-induced well water levels is significantly damped for higher  
 276 frequencies, making it difficult to discern and analyse tides beyond these two types. Here, we use  
 277 amplitude responses ( $A_{M_2}$ ,  $A_{O_1}$ ) and phase shifts ( $\alpha_{M_2}$ ,  $\alpha_{O_1}$ ) to estimate hydraulic subsurface properties.  
 278  $N_2$  tide was not used since its response may be too similar to  $M_2$  and does not help with constraining  
 279 the model. Amplitudes are influenced by geomechanical parameters ( $BK_u$ ) which are generally not  
 280 considered in classical hydrogeology. Valois et al. (2022) previously illustrated that the  $M_2$  to  $O_1$   
 281 amplitude response ratio can be computed because it is not directly multiplied by  $BK_u$  and because it  
 282 provides useful information about model choice. This leads to a system of three equations and six  
 283 parameters ( $T$ ,  $S$ ,  $K'$ ,  $D'$ ,  $skin$ , and  $R_{KuB}$ ) by using the simplified model in Equation 27 when the geometry  
 284 of the well and the aquitard-aquifer system is known:

285 
$$\frac{A_{M_2}}{A_{O_1}} = |h_{w,0,\omega=M_2}/h_{w,0,\omega=O_2}| \quad (31)$$

286 
$$\alpha_{M_2} = \arg \left[ h_{w,0,\omega=M_2} / \frac{BK_u}{\rho g} \varepsilon_0 \right] \quad (32)$$

287 
$$\alpha_{O_1} = \arg \left[ h_{w,0,\omega=O_1} / \frac{BK_u}{\rho g} \varepsilon_0 \right]. \quad (33)$$

288 Table 2 displays the data to be fitted using the three equations above.



289 *Table 2: Data to be fitted using the ET-GW model*

	$\frac{A_{M2}}{A_{O1}}$	$\alpha_{M2}$ (°)	$\alpha_{O1}$ (°)
Cambodia	0.62	5.62	3.3

290

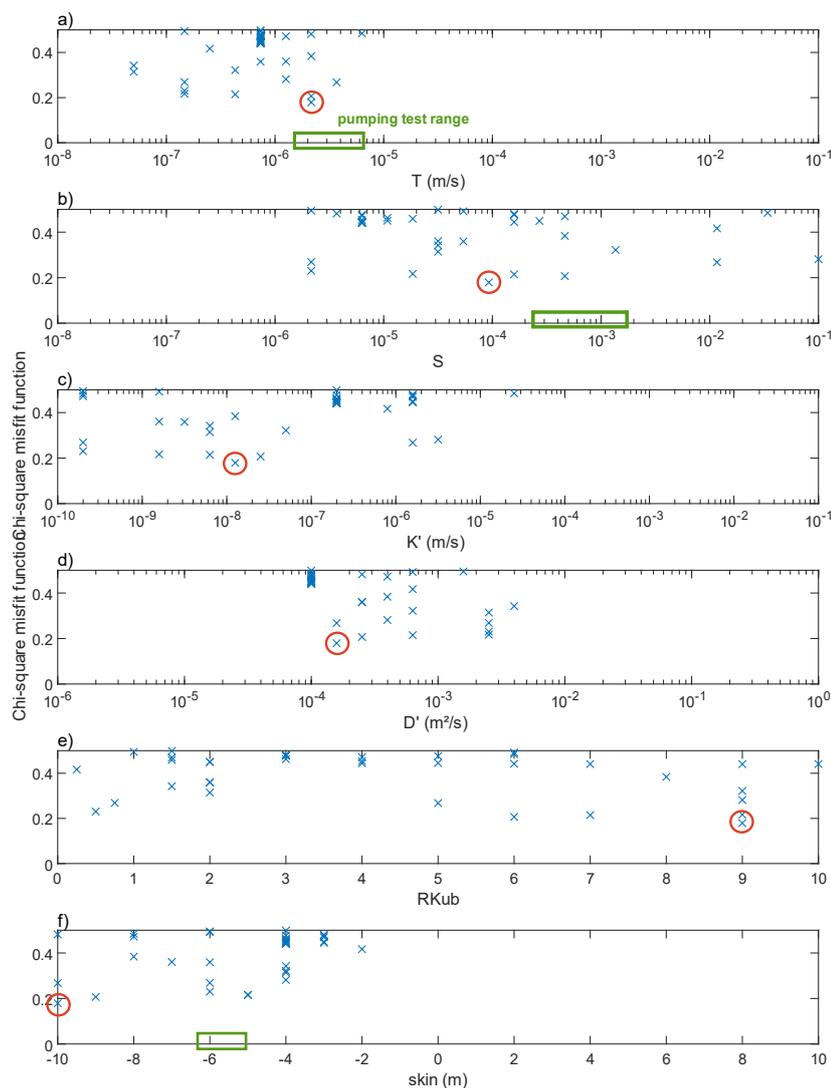
291 A systematic exploration of the entire parameter space without any constraints other than the well  
 292 and aquifer geometry was carried out. Hydraulic and geomechanical property ranges are chosen  
 293 according to the literature, i.e., De Marsily (1986) and Domenico and Schwartz (1998). In order to  
 294 assess the goodness of fit with the three observed parameters (Eqs 31 to 33), the objective chi-square  
 295 function is defined below:

296 
$$\chi^2 = 1/N \sum_{i=1}^N \left( \frac{(Obs_i - Mod_i)}{Error_i} \right)^2 \quad (34)$$

297 where  $N$  is the number of observed parameters (3 here),  $Obs_i$ ,  $Mod_i$  and  $Error_i$  are the observed  
 298 parameter, modeled parameter and their errors respectively. Thus, this objective function takes into  
 299 account errors of the observed parameters (De Pasquale et al.; 2022). They were set to 0.1°, 0.5° and  
 300 0.2 for  $\alpha_{M2}$ ,  $\alpha_{O1}$  and  $\frac{A_{M2}}{A_{O1}}$  respectively, according to Valois et al. (2022).

301

302 The model allows to fit both  $O_1$  and  $M_2$  positive phases and the low  $M_2$  to  $O_1$  amplitude ratio (misfit  
 303 closed to 0 in Fig. 5), whereas the model of Gao et al (2020) cannot (misfit above 1 see Appendix B,  
 304 and Valois et al., 2020). The  $T$  value is in good agreement with the pumping test range (Fig. 5a).  $S$  is  
 305 half an order of magnitude below the pumping test range (Fig 5b) whereas the storativity best-fit for  
 306 Gao et al (2020) is two orders of magnitude above (Appendix B). The skin effect also shows acceptable  
 307 values as compared to the pumping test (Fig 5f). The parameter exploration shows best-fits for  $K'$  and  
 308  $D'$ , whereas it is difficult to identify a clear best-fit for the  $R_{KUB}$  parameter. The values are within the  
 309 expected range for the hydrogeological configuration: The mudstone aquitard has a lower hydraulic  
 310 conductivity ( $10^{-8}$  m/s) than the underlying claystone aquifer (coarser grain size than the aquitard, with  
 311 a  $K$  value of about  $10^{-7}$  m/s for an aquifer thickness of 22 m), and a diffusivity of about  $10^{-4}$  m<sup>2</sup>/s. This  
 312 is in agreement with the aquitard classification of Pacheco (2013).



313

314 *Figure 5: Results of full parameters exploration using two leaky aquifer models for the Cambodian case study.*

315

## 316 4. DISCUSSION

### 317 4.1. Uncertainties and discrepancies

318 There are several sources of uncertainty which originate from measurement and their propagation as  
319 well as uncertainties introduced by model assumptions. We believe that uncertainties linked to  
320 pressure sensor resolution (0.2 mm) and time resolution (20 minutes) as well as the HALS



321 decomposition were too low to be worth considering, at least for the semi-diurnal tides. This can be  
322 deduced from the nearly identical amplitude responses at  $M_2$  and  $N_2$  at our field site. Because those  
323 responses are indeed identical, it means that errors in the raw data set did not influence the response  
324 characterization. We note that amplitude responses and phase shifts show larger discrepancies for the  
325 diurnal tides. This could be linked to overall lower amplitudes which are generally more difficult to  
326 characterise. We therefore conclude that errors arising from uncertainties are negligible compared to  
327 the uncertainty introduced by model assumptions, in agreement with Sun *et al.* (2020).

328 Discrepancies between hydro-geomechanical properties derived from the groundwater response to  
329 Earth tides (termed as “passive” and assuming a compressible matrix) and hydraulic testing (e.g., slug,  
330 pump and lab testing, termed as “active” and generally assuming an incompressible matrix) have been  
331 reported in the literature and have not been appropriately reconciled. By fitting amplitude response  
332 ratio and phase shifts (Section 3.3), a  $T$  value discrepancy of one order of magnitude can be observed  
333 between both approaches. We hypothesise that this is caused by parameter anisotropy.

334 Zhang *et al.* (2019b) pointed out differences in hydraulic conductivities of more than one order of  
335 magnitude between ET analysis and slug tests and attributed this to differences in the investigated  
336 scale. Allègre *et al.* (2016) reported much higher values of storativity derived from pumping test is  
337 compared to ET when using the vertical flow model. Sun *et al.* (2020) showed that  $T$  values are  
338 frequency-dependent with several orders of magnitude differences when comparing co-seismic, ET,  
339 slug or pump test methods. The discrepancies can be explained by the different conceptual models  
340 used in the active (based on perfectly confined) and passive methods (based on leaky conditions) or  
341 by the frequency dependency of hydraulic parameters. The literature illustrates that transmissivity,  
342 hydraulic conductivity or specific storage can indeed vary depending on the frequency of the forcing  
343 (e.g., Cartwright *et al.*, 2005; Renner and Messar, 2006; Gultinan and Becker, 2015; Rabinovich *et al.*,  
344 2015). This demonstrates the need for attention when assessing hydraulic parameters using passive  
345 methods for semi-confined conditions. We specifically emphasise the need for using the same  
346 conceptual model (i.e., confined, leaky with or without storage, vertical flow) when comparing active  
347 and passive methods, as well as the need of preliminary hydrogeological knowledge of both the aquifer  
348 system (i.e., presence of an aquitard with or without storage) (Bastias *et al.*, 2022) as well as the  
349 borehole skin effect.

350

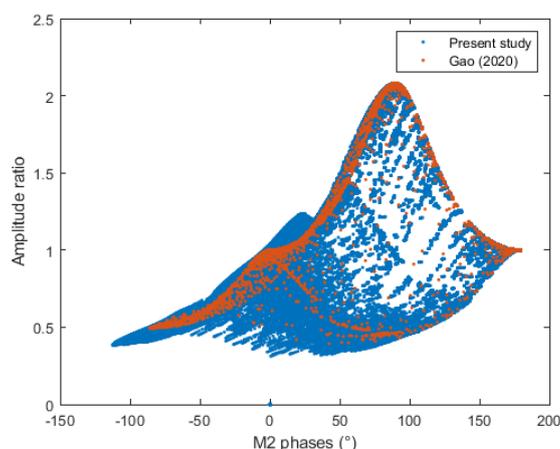
#### 351 **4.2. The use of the leaky model with aquitard storage**

352 Our new analytical solution describing the well water level response to harmonic Earth tide strains  
353 contains at least six hydro-geomechanical parameters that could be derived from only three features,  
354 e.g.,  $M_2$  to  $O_1$  amplitude response ratio and  $M_2$  and  $O_1$  phase shifts. Applying this model to real-world



355 cases to derive properties from amplitude responses and phase shifts provides relevant information  
356 on  $T$ ,  $S$ ,  $D'$ ,  $K'$ , and skin effect, but it is prone to non-uniqueness. Thus, a priori information may be  
357 needed depending on the capacity of the inverse problem to fit observed data (phases shifts and  
358 amplitude ratio). In our case study, parameter assessment would benefit from prior information on  $S$   
359 (or  $K'$ ) and  $R_{KuB}$ .

360 The model presented in this study can be useful when the hydrogeological configuration involves  
361 storage in the aquitard with fixed head (i.e., Dirichlet) boundary conditions and for cases where phase  
362 shifts and amplitude ratio do exemplify a specific pattern. For example, when compared to Gao et al  
363 (2020) and using the parametrisation of the present study (Fig. 6), our solution is able to model lower  
364  $M_2$  to  $O_1$  amplitude ratio, lower phases, and higher amplitude ratio for phases closed to 0.



365

366 *Figure 6: Outputs of the models using the parametrization of the study ( $rc=rw=6.08$  cm,  $z_f=-10$  m).*

367 While the amplitudes are controlled by the product of the Skempton coefficient and the undrained  
368 bulk modulus, these mechanical parameters also affect phase shifts. Therefore, further investigations  
369 are needed to assess these influences using other methods or to link them empirically with the  
370 hydraulic parameters. This is crucial to enhance confidence in utilizing groundwater response to Earth  
371 tides as a valuable tool for better understanding and characterizing groundwater resources.

372

## 373 5. Conclusion

374 We have developed a new analytical solution for the well water level response to Earth tide strains.  
375 This solution considers a previously unprecedented physical reality, specifically, a leaky aquifer with



376 aquitard storage, subject to Dirichlet boundary conditions under tidal strain. Additionally, our model  
377 considers the influence of borehole storage and skin effects, further improving the accuracy and  
378 comprehensiveness of the analysis. This model extends upon previous models and allows advanced  
379 characterization of the subsurface using the groundwater response to natural forces. The new model  
380 overcomes previous limitations, for example it explains very low  $M_2$  to  $O_1$  amplitude ratios as well as  
381 large phase shift difference between  $M_2$  and  $O_1$  tides. The model relies on six combinations of hydro-  
382 geomechanical parameters. In this study, we assess the most sensitive parameters to be the  
383 transmissivity, the well skin effect, the aquitard to aquifer mechanical parameters ratio ( $B'K_u'/BK_u$ ), as  
384 well as aquitard diffusivity and aquitard conductivity to aquifer storativity ratio.  
385 We apply our new model to a groundwater monitoring dataset from Cambodia and compare the  
386 results with pumping tests undertaken in the same formation. We used the diurnal ( $O_1$ ) and semi-  
387 diurnal ( $M_2$ ) tides to better constrain the model. Results illustrate significant insight into subsurface  
388 properties. For example, we derive relevant information about  $T$ ,  $S$ ,  $D'$ ,  $K'$ , and *skin effect*, when  
389 compared to the pumping test results. Overall, our new model can be used to shed light on previously  
390 inexplicable well water level behaviour and can be paired with other investigation methods to enhance  
391 understanding of subsurface processes.

392

### 393 **Competing interests**

394 The contact author has declared that none of the authors has any competing interests.

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- 514



515 **Appendix**

516 **Appendix A: The analytical solution in the aquitard**

517 To solve Equation 14 with the boundary conditions in Equations 7 and 8, we define:

518 
$$\widehat{h}'_0 = h'_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A1})$$

519 Thus, the equation system become:

520 
$$\left(\frac{\partial^2 \widehat{h}'_0}{\partial z^2}\right) = \frac{i\omega \widehat{h}'_0}{D'} \quad (\text{A2})$$

521 
$$\widehat{h}'_0(z = z_i) = h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A3})$$

522 
$$\widehat{h}'_0(z = 0) = h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A4})$$

523 The solution  $\widehat{h}'_0$  is of the form:

524 
$$\widehat{h}'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} \quad (\text{A5})$$

525 It yields

526 
$$A_1 = \frac{e^{\frac{(1+i)}{\delta}z_i} \left( h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \right) - \left( h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \right)}{2 \sinh \left( \frac{(1+i)}{\delta} z_i \right)} \quad (\text{A6})$$

527 
$$A_2 = \frac{-e^{-\frac{(1+i)}{\delta}z_i} \left( h_0 - \frac{B'K'_u}{\rho g} \varepsilon_0 \right) + \left( h'_j - \frac{B'K'_u}{\rho g} \varepsilon_0 \right)}{2 \sinh \left( \frac{(1+i)}{\delta} z_i \right)} \quad (\text{A7})$$

528

529 Thus:

530 
$$h'_0 = A_1 e^{\frac{(1+i)}{\delta}(z-z_i)} + A_2 e^{-\frac{(1+i)}{\delta}(z-z_i)} + \frac{B'K'_u}{\rho g} \varepsilon_0 \quad (\text{A8})$$

531

532 **Appendix B: Additional information on parameter exploration**

533

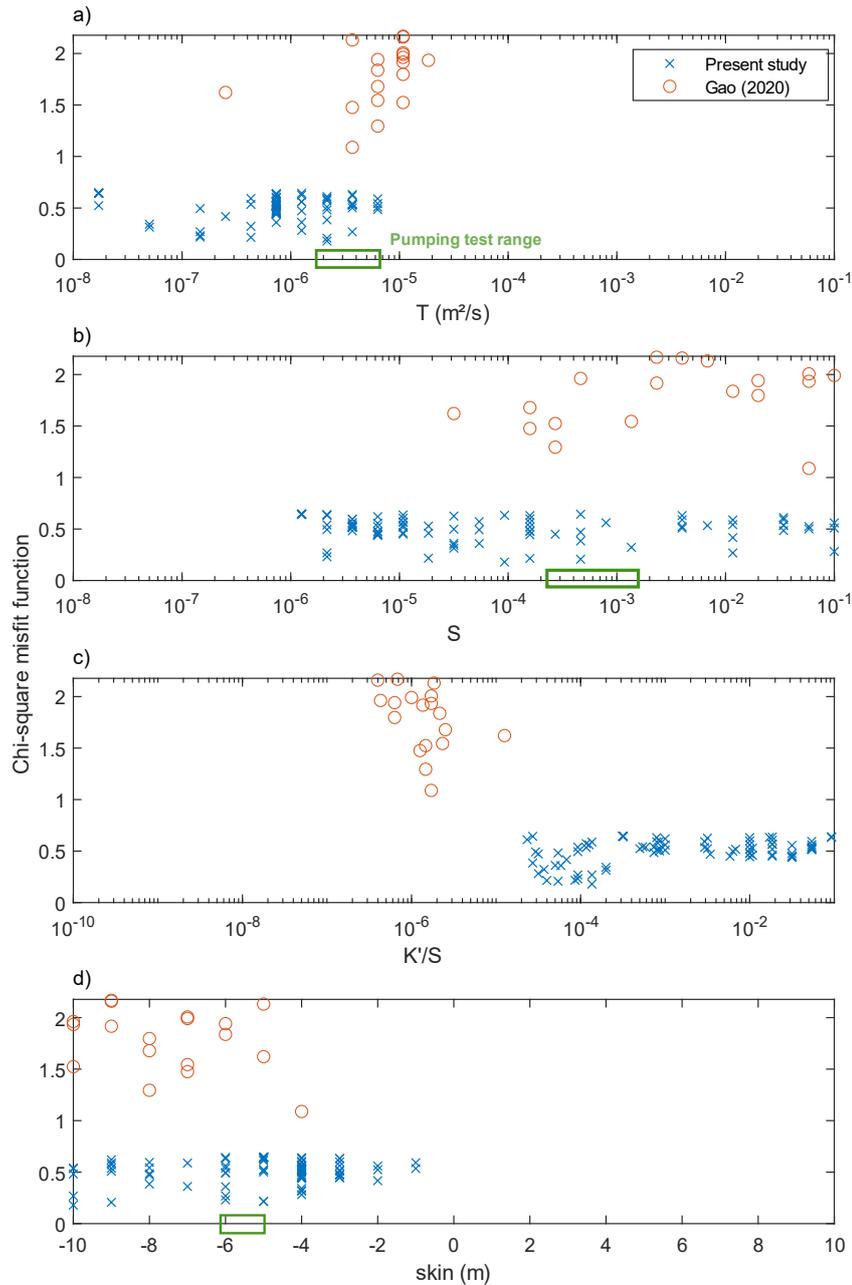
534 The figure A1 shows the misfit functions using the model developed in this study and the model of  
 535 Gao et al . (2020). Misfits are clearly higher for the older model that do not consider storage and  
 536 tidal response in the aquitard. Storativity best-fit using the model of Gao et al (2020) failed to  
 537 reproduce pumping test values. Nevertheless, transmissivity and skin estimates fall within  
 538 pumping test range.

539

540

541

542



543

544 *Figure A1: Comparison of misfit function for the present model and the one of Gao et al. (2020) for the Cambodian case study.*  
545 *Only the first hundred best-fit were plotted.*

546