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A framework for exploring the use of Fermi problems to elicit students' and pre-service teachers' intercultural awareness in mathematics classrooms

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This theoretical paper outlines and discusses our first attempt to formulate the foundation and selected components of an emerging framework for using novel modelling problems (Fermi problem) as a tool in the teaching and learning of mathematics to make different cultural aspects of the world visible and accessible for students. We present (i) an argument relating mathematical modelling, aspects of culture and intercultural awareness; and (ii) an analysis investigating to what extent the six design principles of models eliciting activities aligns with, or need to be developed and modified in order to align with, the three levels of intercultural awareness discussed by Will Baker in a paper from 2011. The focus of the analysis is to evaluate the potential of the (potentially modified) six design principles of model eliciting activities to effectively also be applicable for designing Fermi problems that make different cultural aspects of the world visible and accessible for students.

Keywords: Culture, Fermi problems, intercultural awareness, mathematical modelling, models and modeling perspective.

Introduction

Mathematics in various degrees permeate many, if not all, aspects of society as well as the private and professional lives in our modern day and age (Geiger et al., 2015; Niss & Jablonka, 2020; Quadling, 1982). As such, mathematics has different functions and play different roles, but not seldom in a secretive and invisible way (Devlin, 1998). In this context, mathematical modelling, understood as a generic and general problem-solving process involving the mathematization of real-world phenomena (Niss & Blum, 2020), provides an approach to make mathematics more tangible. Expressed in the notion of Niss (2015), mathematical modelling facilitates using mathematics to control and influence aspects of the real world (*prescriptive modelling*) as well as to describe, understand, or predict the behaviour or properties of phenomenon in the real world (*descriptive modelling*). Often prescriptive- and descriptive modelling in educational settings focus on problems set in concrete contexts. However, in this paper we want to focus on the potential of mathematical modelling as an approach to elicit the more silent *cultural aspect of the world* present in the world around us in the general direction pointed out by the so-called socio-critical perspective on

mathematical modelling (cf. Barbosa, 2006) as well as the research tradition of ethnomathematics (D'Ambrosio & Rosa, 2017; Rosa & Orey, 2011) and ethnomodelling (Rosa & Orey, 2013; Rosa et al., 2020). Our goal is to formulate a theoretical scaffold (c.f. Lester, 2005) and design principles for how to formulate and use novel mathematical modelling problems (i.e. Fermi problems) in the teaching and learning of mathematics aimed at making cultural plurality visible and conscious for students.

Culture and intercultural awareness in a mathematical modelling context

In the Merriam-Webster (n.d.) dictionary the first entry on *culture* is:

a: the customary beliefs, social forms, and material traits of a racial, religious, or social group *also:* the characteristic features of everyday existence (such as diversions or a way of life) shared by people in a place or time...

b: the set of shared attitudes, values, goals, and practices that characterizes an institution or organization...

c: the set of values, conventions, or social practices associated with a particular field, activity, or societal characteristic...

d: the integrated pattern of human knowledge, belief, and behavior that depends upon the capacity for learning and transmitting knowledge to succeeding generations.

Drawing on these four related facets of culture, we tentatively define our understanding of the notion of culture to be *shared customary beliefs, attitudes, values, goals, and practices that are shared by people (or an institution or organization) in a (given) place and/or (given) time.*

Intercultural awareness

Due to the mobility of people, companies and ideas in the modern society, students need to develop a capacity, sensitivity and consciousness to see, understand and act adequately to the changes in their local communities and society at large related to differences in cultures across the globe (OECD, 2020). Cultural aspects can be expressed in many different ways ranging from artifacts, personal expressions, behaviours and actions to larger-scale media outlets (Becker et al., 2017). As noted in reports by UNESCO (2009, 2011) cultural plurality is expressed in multiple ways in terms such as ethical-, cultural-, social- religious- and language diversity. It is in the interaction between an individual and the environment, with other people or various media outlets that differences in culture get contrasted and come to the fore. In other words, it is in *interactions* and *communication* (both in a broad sense) that cultural diversities potentially are made noticeable. In this sense, the need of students to develop a capacity, sensitivity and consciousness to see, understand and act adequately to the changes related to cultural aspects can be operationalized as gaining “a conscious understanding of the role culturally based forms, practices, and frames of understanding can have in intercultural communication, and an ability to put these conceptions into practice in a flexible and context specific manner in real time communication” (Baker, 2011, p. 66). This conscious understanding that Baker alludes to is what he defines as *intercultural awareness* (ICA); see Table 1.

Table 1: Levels of cultural awareness (Baker, 2011, p. 66)

Level 1: basic cultural awareness – An awareness of:
<ol style="list-style-type: none"> 1. culture as a set of shared behaviours, beliefs, and values; 2. the role culture and context play in any interpretation of meaning; 3. our own culturally induced behaviour, values, and beliefs and the ability to articulate this; 4. others’ culturally induced behaviour, values, and beliefs and the ability to compare this with our own culturally induced behaviour, values, and beliefs.
Level 2: advanced cultural awareness – An awareness of:
<ol style="list-style-type: none"> 5. the relative nature of cultural norms; 6. cultural understanding as provisional and open to revision; 7. multiple voices or perspectives within any cultural grouping; 8. individuals as members of many social groupings including cultural ones; 9. common ground between specific cultures as well as an awareness of possibilities for mismatch and miscommunication between specific cultures.
Level 3: Intercultural awareness – An awareness of:
<ol style="list-style-type: none"> 10. culturally based frames of reference, forms, and communicative practices as being related both to specific cultures and also as emergent and hybrid in intercultural communication; 11. initial interaction in intercultural communication as possibly based on cultural stereotypes or generalizations but an ability to move beyond these through: 12. a capacity to negotiate and mediate between different emergent socioculturally grounded communication modes and frames of reference based on the above understanding of culture in intercultural communication.

Here it should be noted that the origin of the 12 components of Baker’s (2011) ICA in Table 1 grouped into the three levels of awareness – *basic cultural awareness*, *advanced cultural awareness*, and *intercultural awareness* – originates within the strand of English Language Educational research.

Connecting mathematical modelling, culture and intercultural awareness

In many discourses the notions of *culture* and the notion of *context* are often used interchangeably. However, as noted by Savard and Mizoguchi (2019), it is not uncommon to conceptualize the context as a substrate where expressions of culture come to the fore. This observation is a key aspect in we how we connect aspect of culture as defied above to our generic perspective and understand of mathematical modelling. As we have argued elsewhere (Ärlebäck at al., 2022), in most perspectives on mathematical modelling the contexts of the situations in modelling problems can be viewed as substrate in which the modelling process enfolds, and the mathematics and models are developed and applied. In this sense, we see the potential of the contexts of the modelling problems worked on as the substrate for both students’ mathematical (modelling) work as well as an opportunity for students to become aware about cultural aspects that explicitly or implicitly surface and become visible.

One perspective on mathematical modelling that especially stress the contextual aspects in the problems worked on is *the models and modeling perspective* (Doerr & Lesh, 2003). According to this view of mathematical modelling, mathematical models are conceived as conceptual systems that describe other systems, and which are constructed from a set of concepts aiming to describe or explain the mathematical objects relevant to the phenomenon in question. In addition, these models include

the procedures used to create useful constructs, predictions, or to do manipulations, in order to achieve clearly identified goals (Lesh & Harel 2003). *Model Eliciting Activities* (MEAs) are the generic activities used with in the models and modeling perspective that support students' learning and development. MEAs focus on and use real-life and for students meaningful situations that they engage in to develop mathematical models to solve problems (Lesh et al., 2000). To support the design of MEAs to elicit problem-solvers' models and support their learning, the following six research-based-developed design-principles are available: (1) the reality (personal meaningfulness) principle; (2) the model-construction principle; (3) the construct documentation principle; (4) the construct shareability and reusability principle; (5) the self-assessment principle; and (6) the effective prototype principle (Lesh et al., 2000).

The similarities and parallels between these six design-principles for MEAs – or rather the similarities and parallels between problems designed based on these principles – and the characteristic of so-called Fermi problems, have been pointed out in Ärlebäck (2009). One characterisation of a *Fermi problem* (FP) is as “open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations” (Ärlebäck, 2009b, pp. 331–332). Two examples of FPs are: “*How many rolls of toilet paper are used in the schools every year?*” and “*How many bikes there are in my city?*”. Given the similarities between FPs and MEAs, we apply the MEA-design-principles also as principles for developing FPs. In addition, Robinson (2008) has argued that FP for all practical purposes can function and be used as miniature modelling problems that capture the essence of full modelling problems. In this way, by working on the basis of estimates, the FPs make it possible to deal with real and complex situations without the need for specific mathematical knowledge. Thus, students can use their own knowledge of the problem context to build the mathematical models that will allow them to achieve a solution (Albarracín & Gorgorió, 2013). This fact opens the door for cultural differences to become evident in the models and results when solving FPs in different cultural settings.

Given the background presented above, we now turn to explore particular aspects of our ambition to formulate a theoretical scaffold (c.f. Lester, 2005) for a framework how to formulate, develop and use novel mathematical modelling problems (i.e. Fermi problems) in the teaching and learning of mathematics aimed at making cultural plurality visible and conscious for students. We do this by comparing the design principles for MEAs (Lesh et al., 2000) and the 12 components of intercultural awareness (Baker, 2011), to see how these relate and to what extent the MEA design principles can be applied to also elicit aspect of intercultural awareness in students.

Design principles eliciting intercultural awareness

Having the six MEA design principles as a guide, we sought for indicators of the levels of intercultural awareness, particularly, whether these would be possible to observed in problem-solvers' solutions. Our initial examination produced a matrix of relations between the six principles and the levels of intercultural awareness (see Figure 1 below). This relation matrix indicates possible relations and connections between the design principles of MEAs and the levels of intercultural awareness. As seen, not all principles were possible to relate, match up or connect to the indicators of levels of

cultural and/or intercultural awareness. Therefore, the matrix involves different codes or markings indicating the extent to which relations and connections could be identified.

Six Design Principles	Levels of Intercultural Awareness											
	Level 1: basic cultural awareness				Level 2: advanced cultural awareness				Level 3: Intercultural awareness			
	1	2	3	4	5	6	7	8	9	10	11	12
1. Reality (Personal Meaningfulness)	*	*	x	*	~	*	x/~	~	~	~	~	~
2. Model-Construction	*	*	x	*	~	*	x/~	~	~	~	~	~
3. Construct Documentation	*	*	x	*	~	*	x/~	~	~	~	*	~
4. Construct Shareability and Reusability	*	*	x	*	~	*	x/~	~	~	~	*	~
5. Self-assessment	*	*	x	*	~	*	x/~	~	~	~	~	~
6. Effective Prototype	*	*	x	*	*	*	?	~	~	~	~	~

Figure 1: Matrix of six principle versus indicators of levels of awareness

In the matrix, a star sign (*) indicates that this indicator of the level of intercultural awareness is possible to be addressed in relation to a particular design principles, while a question mark (?) shows that an indicator may not be possible to be explicitly addressed in solutions to FPs. A cross mark (x) and an approximate sign (~) indicates that the indicator may be possible to be addressed in solutions to FPs, respectively, depending on the implementation of the FP (individual vs. group work and homogeneous vs. heterogeneous groups for example) and depending on the problem context (whether the problem requires comparative perspectives related to one or more cultures).

As seen in the matrix, as the level of intercultural awareness increase from Level 1 to Level 3, the possible match between the MEA design principles and the indicators of intercultural awareness becomes less direct and less explicit. More specifically, while the relation was marked more often with a star sign (*) in Level 1 across the six principles compared to other codes, it is sparsely present in Level 3 where the approximate sign (~) is dominating. This shows that addressing intercultural awareness requires the FPs to be designed carefully considering the purpose of eliciting intercultural awareness. Hence, this preliminary examination led us to re-think the six design principles of MEAs and re-visit them in relation to (inter)cultural awareness. Hence, we produced a list of modified principles that can be used to design FPs that have a potential to elicit cultural and/or intercultural awareness; see Table 2.

Table 2: Design principles for Fermi Problems with potential to elicit (inter)cultural awareness

Six Design Principles	Design principles of Fermi Problems eliciting (inter)cultural awareness
Reality (Personal Meaningfulness) Principle	Interpreting of the problem based on cultural and contextual meanings, norms, and practices
Model-Construction Principle	Developing ways of solving and producing a solution that reflect certain cultural assumptions
Construct Documentation Principle	Selecting ways of expression and representation appropriate for cultural assumptions that are considered in model-construction and model-documentation

Construct Shareability and Reusability Principle	Selecting ways of expression and representation to communicate with the community of interest, and finding alternative ways of expression and representation that are relevant to other cultures
Self-assessment Principle	Setting cultural norms and practices as reference for evaluation and revision
Effective Prototype Principle	Developing ways of solution that have culturally or interculturally prototypical nature

It is important to note that the design principles presented in Table 2 are at the early stage of their development and open to revisions. We acknowledge that the modified design principles need to be tested and evaluated through empirical data. Still, this gives us an initial draft of the revised principles that we may draw upon when designing new FPs to investigate their possibility of eliciting intercultural awareness in problem-solvers' solutions to FPs.

Discussion and conclusions

In this theoretical paper we have outlined particular aspects of an initial scaffold (c.f. Lester, 2005) for an emerging framework for how to formulate, develop and use novel mathematical modelling problems (i.e. Fermi problems) in the teaching and learning of mathematics aimed at making cultural plurality visible and conscious for students. We have done this by comparing the design principles for MEAs (Lesh et al., 2000) and the 12 components of intercultural awareness (Baker, 2011). The result is summarized in Figure 1 and Table 2 respectively.

In order to promote cultural awareness in mathematics classrooms from the work with FPs, it is necessary to meet a number of requirements. First, the problem statement that evokes a particular context must be directed at socially and culturally relevant contexts. Some FPs are oriented to promote the use of concrete mathematical models from artificial or over-constructed situations, and in such cases no level of cultural awareness would be reasonably elicited. However, dealing with realistic and relevant situations related to the daily life of students (such as related to different types of food, forms of transportation, healthy habits, etc) allow reaching Level 1 of intercultural awareness.

If it is the case, that the solution of FPs promotes students to make comparisons between different situations that show differences between ways of life in different places, then students arguably are offered opportunities to reach Level 2 of intercultural awareness. One of the problems we have used in our pilot work is to estimate the number of bicycles in a city by comparing two specific cities from different countries and using images of bicycle parking spaces from these two contexts (Ärlebäck et al., 2022). The difference in the numerical results between the two cities and countries is a clear indication of the differences in the way of using bicycles, and allows us to consider new related problems, such as the use of public or private transport, or the distance people live from their schools or workplaces.

To reach Level 3 of intercultural awareness, a different type of activity and reflection is necessary, since normally the results obtained when working with FPs in classrooms do not allow us to delve into the form that cultural differences take as described at this level of intercultural awareness. To do reach Level 3 of intercultural awareness, our proposal is to take advantage of the potential of FPs, by confronting students with the models and methods used by other students from other cultural settings to solve the same problem. We hypothesize that the deeply rooted aspects of cultural differences will permeate the models that students built, evidencing a different way of understanding the phenomenon addressed in the FP at hand. However, the specific formulation of FPs that aspire to elicit intercultural awareness at Level 3 are strongly oriented to communicative aspects. The knowledge generated from

mathematical modelling, by its nature, makes it possible to explain the complexity of the real world in particular ways, and has its own forms of communication and representation, but it is clearly aimed at understanding the phenomena embedded in the PF at hand. In this way, we observe the need to reformulate the description and categorisation of Level 3 cultural awareness, to adapt these to better align with the way of working mathematically. This future work lay ahead of us and would imply refining Baker's (2011) theoretical framework towards a reformulation of what we potentially might call *mathematical-based cultural awareness* or *mathematical modelling-based cultural awareness*.

Next step in this theory building endeavour is to engage in small-scale piloting by designing and implementing FPs based on the emerging framework and its accompanying reformulated FP design principles. The evaluation of these experiences will be crucial for further developing and clarifying this approach of the teaching and learning of mathematics aimed at making cultural plurality visible and conscious for students in terms of *mathematical modelling-based cultural awareness*.

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