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► **To cite this version:**

Jonas Bergman Ärlebäck, Lluís Albarracín. Characterising pre-service primary teachers' posed Fermi problems. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04413794

**HAL Id: hal-04413794**

**<https://hal.science/hal-04413794>**

Submitted on 24 Jan 2024

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# Characterising pre-service primary teachers' posed Fermi problems

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*In this paper 58 Fermi problems posed by grade 4–6 pre-service primary teachers are analysed with respect to (i) what extent the posed Fermi problems are clearly formulated and authentic for grade 4–6 students; (ii) if the Fermi problems are inviting to, or setting up an introduction to, a relevant mathematical concept or content from the curriculum, or some relevant aspect of the real world; and (iii) what situations and contexts was used in the posed Fermi problems. Both pre-defined coding categories as well as open coding is used to analyse the 58 Fermi problems. The results provide a snapshot of the competence of pre-service primary teachers to pose problems that promote mathematical learning and learning about the proposed and diverse contexts in the problems.*

*Keywords: Fermi problems, estimation, mathematical modelling, problem posing, teachers training.*

## Introduction and research questions

*Problem posing* is a topic within the research on teaching and learning of mathematical modelling that recently have gained more interest (Hartmann et al., 2021). In a way this is not surprising, given that the trend within the general area of research on problem-solving has evolved and diversified into problem-posing. In this paper, we understand the term problem posing as an activity of generating new problems or reformulating given problems before, during, or after a problem-solving sequence (Silver, 1994). This activity is relevant because the skill or ability that students show in solving problems is directly related to the skill or ability to pose new problems (Baumanns & Rott, 2019).

Research has identified several aspects to consider when problem posing activities are implemented in classrooms. One aspect is the typology of problems that are presented prior to the problem posing activity. Ayllón et al. (2011) pointed out the influence of the contents that students have worked on prior to engaging in a problem posing activity and how these contents affect the difficulty of the posed problem. The authors point out the complexity that the posing of new problems entails for students, especially when asked to generate "difficult" problems to solve. Baumanns and Rott (2019) found a close relationship between the type of tasks students had been working on and solving, and the type of problems they posed. When students solved routine exercises, a large proportion of solvers generated new exercises that directly applied algorithms and similar logic. On the other hand, after solving non-routine problems, students tended to generate new non-routine problems with greater diversity of contexts and mathematical richness. The authors stress the importance of proposing non-routine problems, since these require creativity and imagination, given that, faced with an initial situation, the solver does not know a priori a direct path to achieve his or her objective.

The real-world situations involved in modelling tasks can offer rich contexts that inspire students and pre-service primary teachers to pose problems, fostering modelling (Hartmann et al., 2021). When a problem raises a crucial question pertinent to the real-life situation being depicted and accurately reflects relevant elements of the situation, it has an authentic link to reality. If not, it is deemed to

have an artificial connection. In modelling research, open problems are called problems with missing information or vague conditions, and they connect with the formulation of Fermi problems as open problems requiring estimations. Students should identify the knowledge that is lacking from these issues and make reasonable assumptions to solve them (Carlson, 1997)

This paper aims to characterize so-called Fermi problems (elaborated and discussed below) that pre-service primary teachers were asked to formulate as part of a workshop on the nature and use of this type of problems in the teaching and learning of mathematics in grades 4–6. The research questions investigated are: (RQ1) *To what extent are the posed FPs clearly formulated and authentic for grade 4–6 students?*; (RQ2) *To what extent are the posed FPs inviting to, or setting up an introduction to, a curriculum relevant mathematical concept or content, or some relevant aspect of the real world?* and (RQ3) *What situations and contexts are used in the posed FPs?*

### **Mathematical modelling and Fermi problems**

In this paper we adapt a broad and inclusive understanding of mathematical modelling as a general problem-solving process that involves mathematizing a real-world phenomenon and devising a mathematical model to describe, understand, or predict the behaviour or properties of the phenomenon under study. Following Lesh and Harel (2003), mathematical models are conceptual systems that describe other systems that are built from a set of concepts to describe or explain the mathematical objects relevant to the phenomenon. Such models include the procedures used to create useful constructs, manipulations, or predictions to achieve clearly identified goals. Robinson (2008) has argued that a particular type of problems called *Fermi Problems* can be understood and used as small modelling problems that capture the essence of full modelling problems. Hence, Fermi problems have the advantage of being less difficult and more controllable for teachers and students because they are more focused and delimited in terms of the real-world situations they address.

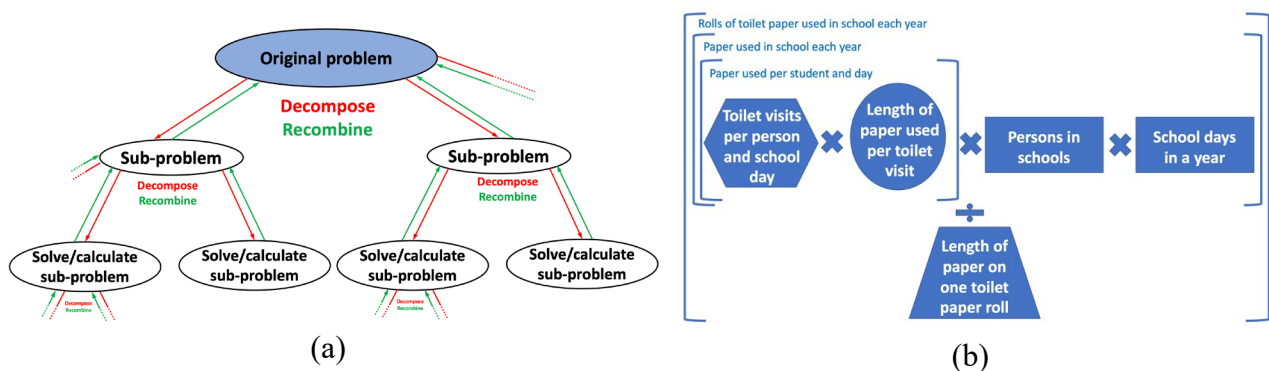
Originally, Fermi problems (FPs) were developed, promoted, and exploited by Enrico Fermi (1901–1954) to demonstrate the value of deductive reasoning and the need for planning before conducting experiments (Efthimiou & Llewellyn, 2007). To give a universally accepted and well-defined definition of FPs is however challenging since different aspects are stressed by different authors (Ärlebäck & Albarracín, 2019). Nevertheless, FPs are first and primarily characterized by how their solutions are obtained, even though they may initially appear to be intractable. This approach, known as the *Fermi (estimates) method* in the literature, comprises the *decomposing* of the problem into a series of interconnected sub-problems, which are then solved using straightforward computations and reasonable estimates, informed guesses, and common-sense assumptions, before being *recombined* to arrive to an answer to the original problem (Carlson, 1997); see Figure 1a below.

Even though FPs frequently use terse language and offer little to no explicit information needed to solve the problem (Efthimiou & Llewellyn, 2007), research has demonstrated that students can successfully be taught the Fermi method and appreciate it (Barahmeh et al., 2017). Notably, FPs have been applied at all educational levels because of the adaptability and flexibility they provide considering the students' abilities and knowledge (Ärlebäck & Albarracín, 2019).

Focusing at the elementary school level, Peter-Koop (2009) discovered that students working on a FP to estimate the number of vehicles in a traffic jam can create their own models and utilize the

Fermi method. To come up with their solutions, the students employed a range of techniques and gained new mathematical understanding. Additionally, it has been demonstrated that working with FPs helps primary students acquire modelling skills by educating them about the stages of modelling and helping them build sub-competences in modelling such as simplification, mathematization, interpretation, and explanation of real phenomena.

The division of the original problem into sub-problems in the Fermi method can be connected to Thompson’s framework of quantities and quantitative reasoning (Thompson, 1994). This framework categorizes quantities into *extensive-* and *intensive quantities*. Extensive quantities can be measured in absolute units such as lengths, areas and, volumes using cardinalities and ordinalities. Intensive quantities can only be measured by units that are constructed using a comparison of quantities, often describing a multiplicative comparison (ratio) of two quantities, such as speeds, velocities, densities, temperatures, and pressures. In this paper, we add a third intermediate category called *pre-intensive quantities*, which we define as an extensive quantity that is calculated given an interval of time. An example of a pre-intensive quantity is *the number of cars passing the schoolyard-crossing*. This quantity can also be thought of as car passing per hour, which is an intensive quantity, and in this sense a pre-intensive quantity can be understood as a precursor for an intensive quantity. Quantities also come to the fore in the so-called *Fermi Problem Activity Templates*.



**Figure 1: (a) A graphical representation of the Fermi estimate method; (b) A F<sub>p</sub>AT representation of a solution to The Toilet Paper Problem**

## F<sub>p</sub>ATs – Fermi problem Activity Templates

*Fermi Problem Activity Templates, F<sub>p</sub>ATs* (Albarracín & Ärleback, 2019) is a framework for design of, and analysis of solutions to, FP structured around the following four mathematical activities to get the needed quantities to solve a FP: (1) *Guesstimation*, represented by an *ellipse*, is the process of providing an answer to a (sub-) problem using educated guesses and estimates of the relevant unknown quantities; (2) *Experimentation*, represented by a *trapezoid*, is the process of conducting a (physical) experiment to find suitable values for the need quantities required to solve the given problem; (3) *Looking for data*, represented by a *rectangle*, is the process of leveraging external records and sources to look for the pertinent quantitative data required to solve a FP; and (4) *Statistical data collection*, represented by a *hexagon*, means engaging in data collection and statistical analysis to obtain the values of the relevant quantities needed to solve a FP. Figure 1b shows a solution to The Toilet Paper Problem, *How many rolls of toilet paper are used in your school every year?* It outlines

the sub-problems, the activities to engage in to get the needed quantities, and the algebraic structure of the calculation required to provide a solution.

## Setting, method and data collected

The data analysed in this paper comes from a 3-hour teacher training workshop on FPs for grade 4–6 pre-service primary teachers (PSPTs) in Sweden. The workshop was organized around groupwork activities carried out in groups of 4–6 students and was structured in the following three main phases: (i) a general introduction to FPs, beginning with students solving a ‘starter FP’ (How many SMS are sent in Sweden in a year?) followed by a discussion about what FPs are and how this type of problem has been used in science- and mathematics education; (ii) students working in three stages on The Toilet Paper Problem, including the introduction of the F<sub>p</sub>AT framework; and (iii) formulating two FPs per group on their own and constructing a potential solution of their FPs in terms of F<sub>p</sub>ATs.

With respect to the students’ work on The Toilet Paper Problem in the second phase of the workshop the groups: (a) discussed and wrote down their proposals of a solution strategy of how to solve the problem; (b) implemented their solution strategy; and (c) after receiving a 20-minute introduction to the F<sub>p</sub>AT framework, created and wrote down F<sub>p</sub>ATs representing their solutions of the problem. The third phase of the workshop, formulating two FPs of their own, was mainly carried out as homework.

The participating PSPTs in the workshop studied their fourth semester out of total eight semesters in the primary teacher training degree, and had previously studied one combined course in mathematics and mathematics education (not including the topic of modelling or real-life problem solving). Data was primarily collected in phases (ii) and (iii) of the workshop, and consisted of the various documents generated by the PSPTs (their written solution strategies, solutions and F<sub>p</sub>ATs of The Toilet Paper Problem from phase (ii) and the students’ formulated FPs as well as their F<sub>p</sub>AT for solving these from phase (iii)). Due to space constraints, in this paper we only focus on the student generated FPs in phase (iii), which under three consecutive years amounted to a total of 58 FPs.

## Analysis

The unit of analysis adapted in this paper is a single PSPT formulated FP, and after having read the 58 FPs the students formulated and translated these into English, we decided to apply both some pre-defined coding categories as well as an open-coding approach. The five pre-defined coding categories were: (1) *Is the Fermi method natural or artificial to use for the intended problem-solver in solving the problem?*; (2) *Is the question clearly formulated?*; (3) *Is the situation and context in the question artificial for the intended group of problem-solvers?*; (4) *Is the question inviting to, or setting up an introduction to, a curriculum relevant mathematical concept or content?*; and (5) *Has the question the potential for the problem-solver to learn some relevant aspect of the real world?* The aim of these categories was to address RQ1 and RQ2.

To address RQ3 regarding what situations and context the PSPT used in their posed FPs, we applied a two-step grounded theory inspired coding strategy (Strauss & Corbin, 1998). First, the FPs were iteratively openly coded and organised into themes or open categories using the guiding question: *What situation and context is used in the posed problem?* Then, the open codes were subjected to axial coding, which identified relations and patterns among the open coding categories.

## Results

Of the 58 PSPT formulated FPs 52 (90%) were considered to be suitable questions to solve by applying the Fermi method. Examples of FPs that was categorized not suitable questions to solve by applying the Fermi method either asked for a quantity which almost by necessity only could be arrived at by students in grades 4–6 through directly provide a guesstimate, such as for example: *How much does an ice-cream cost on average in Sweden?*; *How many elephants are there in Africa?*; and *How large proportion of all in Linköping eligible to vote did vote in this year's election? Answer in percent.*

Ten of the 58 FPs (17%) were categorized as not being clearly enough formulated. All ten of these FPs displayed an ambiguity regarding what the asked about quantity actually was. The ambiguity could either be seen to be derived from the use bad grammar or sentence structure such as in the example *How many times does a student go down the slide per break per semester?*, or from the formulation of the question just not being enough clear. Two examples of the latter are:

- *How many of the school's students eat tacos every Friday in a year?* In this question there is an ambiguity if the asked about quantity is the number of students who eat tacos on Fridays 52 weeks a year (to which the answer likely is zero, making the problem rather uninteresting), or about the frequency of taco dinners on Fridays among the students in the school.
- *How many artistic paintings are painted in Linköping in a decade?* In this question, it is not clear what counts as an *artistic painting*.

Among the 58 posed problems by the PSPTs, there were an almost perfectly even split between problems asking for extensive quantities ( $n=28$ , 48%) and pre-intensive quantities ( $n=30$ , 52%). In the extensive category the type of quantities asked for could broadly be characterized as either a discrete number or as coming from a continuous quantity (an amount). Similarly, the pre-intensive quantities asked for could be categorized as a *discrete number per a given time unit* or an *amount of a continuous quantity per a given time unit*, with an additional third *Other* category added. Table 1 shows the breakdown for the distribution of the quantities in the posed questions.

**Table 1: The distribution of quantities in the problems the PSPTs posed**

Extensive quantity (total 28)		Pre-intensive quantity (total 30)		
Number	Amount	Number per time	Amount per time	Other
20 (71%)	8 (29%)	15 (50%)	13 (43%)	2 (7%)

The majority ( $n=20$ , 71%) of the 28 extensive quantities asked for was a specific number for the involved quantity, such as in: *How many pizza places are there in Linköping municipality?* and *How many households in Sweden keep pets?* Problems asking for continuous quantities constituted 29% ( $n=8$ ) of the 28 problems, and two examples are *What is the total length of the students in the class?* and *How long does it take to go by elevator to the top of the Eifel tower?*

In the posed FPs asking for pre-intensive quantities, the distribution was more even compared to the extensive quantities (see Table 1). Fifty percent ( $n=15$ ) asked for a quantity given by a (discrete) number per time unit such as *How many garbage bags are thrown away in a household containing 4 persons each year?* and *How many meals are consumed during one's lifetime?* FPs asking for a

quantity of an amount per time unit constituted 43% ( $n=13$ ) of the FPs in this category and two examples are *How much food is thrown away in the school cafeteria each year?* and *How many minutes are middle school students spending on TIKTOK each week?* Of the two posed FPs in the Other-category none were considered suitable questions to solve by applying the Fermi method: one asked for a proportion (*How large proportion of all in Linköping eligible to vote did vote in this year's election? Answer in percent.*); and the other an average (*How much does an ice-cream cost on average in Sweden?*).

With respect to the used situations and contexts the in the posed questions, multiple societal levels of increasing proximity distance to the students intended to solve the problems were identified. In addition, the problems could be seen to be either asking about the problem-solvers *own* life and context on the one hand, or a *generic* person or group on the other. Table 2 shows the breakdown of the coding of the posed questions in terms of own/generic as well as the various levels for proximity: *student; generic person; family; class; school; city; municipality and country.*

**Table 2: The proximity of the situations and contexts proposed in the FPs posed by the PSPTs**

Context	Student	Generic person	Family	Class	School	City	Municipality	Country	Other
<b>Own</b>	0	0	0	9	14	5	1	9	0
<b>Generic</b>	2	4	2	3	2	0	0	0	0
<b>Total</b>	2	4	2	12	16	5	1	9	7

Of the 58 posed FPs, 18 were categorised as artificial for the intended group of problem-solvers (e.g. *How many coffee filters are used in the teachers' room in a school year?* and *How many erasers fit in a moving box?*). In this research, we consider an artificial context to be one that is outside the background and/or interests of the students intended to solve the problem. Seventeen of these could however be considered to invite, or setting up an introduction to, a curriculum relevant mathematical concept or content. With respect to the posed FPs having the potential for the problem-solver to learn some relevant aspect of the real world, 36 such FPs were identified; for example *How much food is wasted in the school cafeteria each year?* eliciting ideas about sustainability and *How much time in average a day does a student in the class spend watching a screen (smartphone, computer, TV, iPad, etc.)?* regarding taking a critical look at ones consumption of media.

## Discussion and conclusions

In this research we investigated in the FPs proposed by PSPTs after a workshop on Fermi problems. We are interested in the competence of PSPTs to create and develop problems that promote mathematical modelling, mathematical learning, and learning about the proposed context. We acknowledge that the requirement for PSPTs to generate a F<sub>p</sub>AT for their proposed FPs does impose constraints on the possible FPs to generate, as it forces PSPTs to know a priori how to formulate a solution in terms of the F<sub>p</sub>AT and to have control over the procedures necessary to solve the FP. However, the analysis shows that most of the proposed problems centred around estimations are suitable for working with and learning mathematical modelling, mathematical content, and the

context in the posed problems. Nevertheless, some details and aspects of the posed FPs especially draw our attention.

First, some PSPTs proposed problems that were ambiguous in their formulation. Here, it is key for the PSPTs to be able to distinguish between problems in which the only possible solution is a (direct) guesstimate, which implies that the result is not precise (Carlson, 1997), and those problems that have an imprecise and flawed formulation making the problems practically insolvable. Another relevant aspect that emerges from the analysis are the types of quantities involved in the posed problems. On the one hand we observed problems that require working with an extensive quantity, and on the other hand problems that require working towards an intensive quantity due to the problem statement most likely prompting the problem-solvers to answers with an extensive quantity when considering the specified period of time in the statement. We suspect the large number of pre-intensive problems might be due to the problem the PSPTs worked during the preceding workshop (The Toilet Paper Problem) (c.f. Ayllón et al., 2011). Showing different typologies of FPs with respect to the quantities involved and making this distinction explicit is important in this type of training. This aspect is relevant because the work with intensive quantities is demanding for primary school students and the PSPTs must be able to control the difficulty of the work proposed in their formulated problems.

Another important aspect is the closeness of the context used in the posed FPs. Here, we observe that an important part of the proposed problems connects to contexts close to students (classroom, school, city). This fact is relevant in the sense that it allows elementary school students to use their knowledge about the real world accessible to them and allows them to further learn about their environment by generating meaning. Some of the contexts have been classified as artificial and do not easily allow the development of knowledge about the real world to be generated during the solution processes. These problems however have the potential to introduce concrete mathematical knowledge that can be used later in a sequence of problems that do connect with the real world (Albarracín et al., 2022). More research is needed to understand the relationship between the contexts close to primary students in which FPs can be proposed to enrich the modelling activity. This is a key factor for PSPTs to consider in order to be able to identify mathematical elements in such contexts.

The results of this study show that it is possible for PSPTs to learn how to propose FPs that can be used in Primary Education classrooms. Hence, we suggest that proposing FPs can be a viable and beneficial approach to identify recent or close contexts that allow PSPTs and teachers to connect reality with mathematics. At the same time, we identified some relevant aspects to improve in this type training, such as clarifying the concrete nature of Fermi problems, the role of different types of quantities and the complexity of choosing real contexts. A next step for this research is to study the proposed problems alongside with the generated  $F_pAT$  representing the solutions to the FPs, which should allow us to understand the type of classroom work that PSPT anticipate, and which is an important part of their teaching competence.

## **Acknowledgment**

Serra-Hünter Fellow at the Universitat Autònoma de Barcelona, Lluís Albarracín, is part of GIPEAM research group (2021 SGR 159). This work was supported by grant PID2021-126707NB-I00 funded by MCIN/AEI/10.13039/ 501100011033 and by the project ‘ERDF A way of making Europe’.



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