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An introduction to TWG6: Applications and modelling

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This paper presents and gives an overview of the papers and posters discussed in the Thematic Working Group 6 (TWG6) at the CERME 13 conference taking place in Budapest, July 11–14, 2023.

Keywords: Applications, learning, mathematical modelling, models, teaching.

Overview and introduction of the CERME TWG6 on application and modelling

The thematic working group TWG6 on applications and modelling acknowledges the significance of investigating the importance of mathematics when it is used and applied in the study of real-world phenomena in educational contexts. TWG6 examines the relationship between empirical research and theoretical approaches to the teaching and learning of applications and mathematical modelling, in mathematics and other subjects, at primary, secondary and tertiary levels, as well as in teacher education. Since the fourth ERME Congress (CERME 4) in 2005, when TWG6 was first established, it has remained an active thematic working group throughout the ten ERME congresses leading up to CERME 13. At CERME 13, TWG6 received contributions discussing theoretical, methodological, and empirical research on various topics, including the application, modelling, and simulations in connection with mathematics and other subjects; sustainability; the impact and use of technology and resources in modelling; in- and pre-teacher education for modelling and applications, among others. The contributions collectively demonstrate a fruitful diversity in research questions, school levels, theoretical approaches, and methodologies in the educational research field on mathematical application and mathematical modelling. For CERME 13 TWG6 received a total of 46 submitted contributions, out of which 33 papers and 10 posters were accepted and presented at the congress.

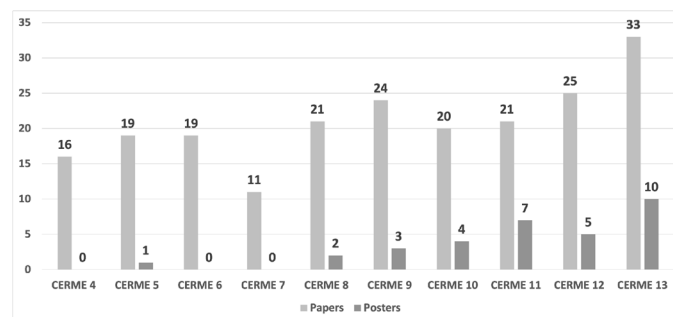


Figure 1: Overview of the presented contributions in TWG6 in CERME 4 – CERME 13

A total of 52 participants from 20 countries attended the TWG6 at CERME 13. The majority of participants (67%) were from Europe, but there were also participants from North and South America, Asia and Australia. Figure 1 gives an overview of the TWG6 papers and posters presented at the different CERME meetings. At CERME 13, the TWG6 sessions discussed the various papers and posters, which were organised around seven main leading themes. Although some of the papers fit into more than one of the identified leading themes, their proximity to other contributions and the constraints of the assigned congress schedule resulted in the distribution shown in Table 1 below.

<i>Leading themes</i>	<i>Papers</i>	<i>Posters</i>
LT1: Implementation of mathematical modelling and classroom practices	6	2
LT2: Aspects of task design and analysis of modelling tasks	5	3
LT3: Digital tools, technology and other resources in mathematical modelling	4	2
LT4: The role, function and impact of context	5	-
LT5: Students' mathematical modelling work and skills	5	-
LT6: Modelling for sustainability	4	2
LT7: Teacher education and professional development	4	1

Table 1: Leading themes (LT) defined and number of contributions in the proceedings

The first theme focuses on issues, investigations and explorations of the implementation of mathematical modelling in schools and classroom practices, such as the use of shared drawings and retelling strategies. Related to the first theme, the second one focuses on different aspects of task design and analysis of modelling tasks from different sources and the use of different frameworks. The third strand concerns the use of digital tools, technologies and other resources in mathematical modelling, and this strand discusses, for example, the role and function of simulations in different educational settings. The fourth strand looks at different perspectives on the role, function and impact of context, for example when pre-service primary teachers pose problems, or the influence of personal interest in the context of the real-world problem being tackled by focusing on students' assumptions in the modelling process. The fifth strand focuses on students' mathematical modelling work and skills and includes, for example, papers that analyse the models that students have produced using different mathematical content such as geometry and proportionality. The sixth theme concerns a relatively new topic in the models and modelling educational discourse that came to the fore in the TWG6 contributions, namely how models and modelling are used in problems and situations related to aspects of sustainability and a sustainable world. Finally, the seventh theme focuses on applications and modelling in teacher education and professional development. Table 1 gives an overview of the distribution of papers and posters in the seven main themes that can be found in the CERME 13 proceedings and that we briefly summarise in this introductory paper to the work of TGW6.

Leading themes and overarching questions

LT1: Implementation of mathematical modelling and classroom practices

When considering the implementation of mathematical modelling, several challenges need to be addressed. First, institutional conditions and constraints play an important role in how and whether mathematical modelling becomes part of classroom practice. The Anthropological Theory of Didactics (ATD) provides a solid theoretical framework to conduct an institutional analysis of the conditions and constraints that might promote or hinder the implementation of mathematical modelling. Institutions could create favourable conditions for mathematical modelling by allocating explicit hours to modelling and applications in the curriculum and through teacher training. Indeed, teachers are the main facilitators and their previous experience with modelling activities is crucial. It is therefore important to include modelling activities in teacher training for both pre-service and in-service teachers. The empirical studies show an urgent need to negotiate a new didactic contract, which seems to be a necessary condition to promote the implementation of mathematical modelling.

In addition to the conditions and constraints, some difficulties that students experience when faced with mathematical modelling were reported, such as getting involved in the situation of the task (or the so-called devolution in the sense of Brousseau), metacognition and validation seemed to be quite complicated for students. From the empirical studies, some elements of response can be drawn on how to support students in modelling processes. For example, retelling or hands-on activities that allow for experimentation could be used to help students enter the task, and question-answer logs seem to be a good tool to accompany the modelling process steps. Another important aspect of the implementation of mathematical modelling is the social organisation of students' work. It was agreed that modelling activities should not be carried out as individual work, as this leads to isolation and blockages. Peer interaction and collaboration was presented as an appropriate social way of experiencing mathematical modelling because it enriches the milieu, provides opportunities for exchange and confrontation of ideas and contributes to the validation process. It is also important to address the needs of metacognition and to raise awareness of the strategies involved in modelling processes. As these strategies do not come naturally, they need to be the subject of explicit communication and teaching. In this sense, qualitative empirical studies are complementary to quantitative studies and are necessary to describe and better understand the processes involved in modelling.

There are a number of issues that need to be further discussed for future developments in this area:

- How to negotiate a new didactic contract that would create favourable conditions for the implementation of mathematical modelling?
- How to address the specific needs of meta-cognitive strategies when implementing more open modelling tasks?
- How to bring educational modelling tasks closer to real life applications?
- What is the role of the teacher when engaging students in modelling tasks?

LT2: Aspects of task design and analysis of modelling tasks

When we engage in task design for mathematical modelling and application, we need to consider the transposition work done in terms of problem context and problem formulation as part of the process. That is, we need to consider transpositions that are targeted at specific school levels, but also how these should relate and be linked to each other. For example, in tertiary education, more attention

should be paid to whether we are producing modelling skills or competences that are relevant and linked to this level of schooling. That is, we need to ask in what ways we are imitating and presenting the real-world practices of modelling in higher education (engineering, data science, physics, etc.), or whether we are actually preparing autonomous modellers for these fields of expertise.

On the other hand, we need to rethink the purpose of teaching and learning mathematical modelling: do we really need to teach students to become model constructors? Perhaps we need to find ways of teaching students to become citizens who are aware of how and why they are 'model consumers'. This may change the focus of teaching towards model analysis, the study of existing models. If this is indeed the case, then some kind of coherence across educational boundaries and an awareness of what kind of modelling activity we are proposing and why – model construction, model analysis, empirical or theoretical, negotiating epistemic value, etc. – is needed.

For more appropriate task design with respect to issues related to transpositions aspect, we may need to collaborate more closely with experts from other fields, and we need to embrace the fact that we are doing research in an interdisciplinary field - we (researchers, teachers and student teachers) need to learn more about the disciplines and domains that can inform the modelling process, with the aim of developing task design for modelling. This could promote students' engagement with all aspects (sub-competencies or holistic) of the modelling process.

We need to emphasise the need for pre-service and in-service teachers to experience designing modelling tasks throughout their training and careers, and we need to think about what we can do to support their implementation of this in real classrooms, to develop some kind of infrastructure. This could further support teachers' interpretations of curriculum documents.

A number of questions for discussion arise for future developments in this area:

- What resources are needed for the development of task design (in the broad sense) within modelling? This relates to: other experts, contexts, design of milieu, support for teacher design, open-ended problems, etc.
- How can we develop our knowledge of task design in mathematical modelling taking into account other domains and theories? These could be fields such as engineering, data science, stochastics, and theoretical frameworks such as problem solving, theory of didactic situations, etc.?

LT3: Digital tools, technology and other resources in mathematical modelling

The focused aspects related to learning and teaching with technology concentrate on three areas. These are the role of simulations, the relationship between technology and modelling, and concerns about the use of technology in modelling.

One area concerns the role of simulations in the learning process. This involves discussing how the model created interacts with the data generated by that model. In addition, the importance of trial and error as a heuristic within the modelling process is brought into focus. This raises the question of how trial and error contribute to effective learning and problem solving.

A second area focuses on the relationship between technology and mathematical modelling. This involves the selection of a theoretical framework in the development and analysis of modelling tasks

and student processes. A major consideration is whether mathematics and technology should be treated as separate entities or whether they should be more integrated. This is related to how technology can enhance mathematical understanding and how it affects the modelling process.

The third area addresses concerns and limitations in the use of technology in mathematical modelling. These include identifying the additional competencies or skills that both teachers and students need to use technology effectively. For example, teachers may need to increase their knowledge of technological tools. In addition, the problem of critical thinking and a questioning attitude toward technological outcomes is highlighted, emphasising the importance of not blindly relying on the authority of technology.

These three areas inspire discussions about effective learning strategies, the integration of mathematics and technology, and the need for both technological skills and critical thinking in the learning process. Some questions for discussion emerge for future developments on this topic:

- What is the connection between the nature of the task, the characteristics of the students and the technological tools used for the modelling activity?
- How can curricula adapt to the rapid development of technology in reality-based lessons?
- What is the connection between the nature of the task, the characteristics of the students and the technological tools used for the modelling activity?
- How can curricula adapt to the rapid development of technology in reality-based lessons?

LT4: The role, function and impact of context

Based on the view that mathematical modelling is a multifaceted process, one of the main issues highlighted and discussed in this theme was the variety of aspects that can be explored from different perspectives when modelling with so-called Fermi problems. Although generally more limited in scope, complexity, and size, the discussion of this theme illustrates that Fermi problems can be used to explore the role and impact of context, culture, interest, self-confidence, adaptability of strategies, use of assumptions and the role of estimation in the modelling process.

In particular, one area of discussion was about the way to gain knowledge of the structure of the solutions of Fermi problems and other modelling problems. Here, the development and exploration of tools for identifying students' strategies for developing solutions and characterising their models were at the core of the discourse. The classification of the strategies of developing models has implications for task design and for teacher training, especially with respect to the appropriateness, efficiency, adaptability, and flexibility of the models developed by the students.

Another central area of discussion in this leading theme was the role of assumptions made in the modelling process. The nature of Fermi problems and other modelling problems that require making assumptions brings about cultural, motivational (e.g., interest), and cognitive (e.g., strategic) issues to consider and discuss in the modelling process. The presented papers, in addition, clearly illustrate that the structure of a solution to a problem may be due to the interest of the problem-solvers in the context of the problem and their interest-based assumptions.

With respect to mathematical modelling teacher education, the discussion of the papers revealed that differences in teacher education programs may produce differences in preservice teachers'

understanding of models and in motivation towards modelling processes. This possible lack of perspective in the teachers' knowledge about modelling (e.g., use of context and role of different types of quantities as the knowledge provided by expert teachers) can be identified through problem posing activities with teachers and prospective teachers.

Looking ahead, some of the questions that could be interesting to discuss in the future with respect to this leading theme are:

- In which ways students' interest, close context, and culture can be considered for/contributed to the design of the modelling tasks?
- What are tools that can be used to analyse the structures of models?
- How to develop teacher training activities promoting effective modelling competences?

LT5: Students' mathematical modelling work and skills

The focused aspects related to students' mathematical modelling work and skills concentrate on four areas: modelling skills and capacities, the modelling process, educational policies and curricular regulations, and the teaching and learning of mathematical modelling.

Central to modelling skills and capacities is the ability to estimate, especially in real-world contexts and applications. The use of benchmarks as a strategy to promote mathematical and measurement sense is important, providing students with a practical framework for understanding and applying mathematical concepts.

Multiple problem-solving strategies are critical in modelling processes. It is important to develop multiple problem-solving approaches and connect these strategies for validation. Validation activities in scenarios such as Fermi problems are highlighted, emphasising the need for thorough mathematical a priori analysis in preparation for validation situations. The nature of the problem and contexts are seen as influential factors in effective validation and mathematization.

On educational policy and curricular arrangements, it is a challenge to integrate modelling into the training of pre-service teachers, especially in countries where modelling is not part of the original curriculum. In Europe, where this integration is already relatively advanced, ecological analysis based on the Anthropological Theory of the Didactic framework (ATD) is considered a valuable tool for assessing the situation. The role of student assessment and higher education entrance exams regulations within the education system is highlighted.

For teaching and learning, the integration of specific experiments in the classroom is beneficial, drawing on the prior experiences of students and teachers in terms of both contextual understanding and mathematical modelling. Overcoming barriers to learning, such as conceptual blocks, can be achieved by designing activities such as group work, retelling, and using engaging contexts. These strategies are suggested to provide students with productive ways to overcome challenges. Some questions for discussion emerge for future developments on this topic:

- What are the indispensable mathematical competencies for modelling at different levels?
- How can we create contexts for engaging in both the modelling activities as well as learning other relevant disciplinary content and knowledge?

- How to improve the perception of students and teachers about Mathematics as a living field of knowledge?

LT6: Modelling for sustainability

Education for Sustainable Development (ESD) is an important topic in many countries or is about to become one. At present, there is still an enormous need for research, as previous scientific discussions in the field of mathematics didactics and mathematical modelling are primarily theoretical/conceptual in nature and empirical contributions work with small samples. At the same time, it became clear that addressing sustainability contexts in the classroom can make sense of learning mathematics and acquiring modelling competences and thus provide an answer to the question of why the acquisition of modelling competencies should be the content of mathematics education. On the other hand, sustainability measures should be evidence-based, which in turn highlights the importance of mathematics as a tool for sustainability contexts and the need to question assumptions made and models chosen to model sustainability problems. The latter point also represents one of the three aspects that were discussed more intensively on the basis of the contributions presented:

- Framework for ESD in different countries: ESD is anchored in the curricular standards in different ways. It became clear that a framework for ESD is also necessary not only for mathematics education, but for different subjects, and that the mathematical procedures used in this framework are not very comprehensive.
- Aim of the tasks and task design: The aim should be to create tasks that students care about and are able to comprehend, and that provide solutions for different stakeholders (local and global challenges). The tasks should make clear that social issues also have costs and that there is not always hard data for social issues on sustainability.
- The role of mathematics in sustainability contexts: In addition to the aspects already mentioned above, the role of other disciplines in ESD was highlighted, as well as the fact that decisions are not only fact-based but often emotionally charged. Highlighting this and allowing students to make their own decisions based on mathematical reasoning was stressed as significant.

Concluding from the discussion, it can be stated that sustainability is an emergent topic, which is why it seems appropriate to create communities for sharing experiences and research results of sustainability contexts, to explore the role of critical and statistical thinking in the context of ESD and to support teachers to create tasks, teaching materials and learning environments.

LT7: Teacher education and professional development

Teacher education and professional development for teaching mathematical modelling have gained increasing attention in recent years. The various issues debated within this topic reveal that the pedagogical content knowledge for teaching mathematical modelling is complex and requires effective and continuing actions. The importance of the bi-directional relationship between mathematics and the real world must be emphasised, particularly to avoid the narrow view of “applicationism”, in which mathematics assumes the role of a service subject. Related to this point, the views of mathematical modelling held by teachers have an influence on the way they choose modelling activities for their students, accept and value reasonable ideas from the students (including

ideas in which the basis is not mathematical) or show a greater or lesser tolerance towards uncertainty when working on a real modelling problem.

Several possibilities for networking theories (e.g. Realistic Mathematics Education interlinked with Models and Modelling) could be promising in devising strategies and frameworks aiming to contribute to the development of specialised knowledge for teaching mathematical modelling. In this regard, there is still work to be done on the production of teacher education materials (pre-service or in-service) that allow for an effective analysis and discussion of the factors that can favour or hinder the students' modelling activity in the class (videos, vignettes, class observation, etc.). The issue of helping and supporting students while solving a modelling task is one of the most relevant in teacher education. Noticing, scaffolding, and thoughtful interventions are decisive skills and competencies. It should also be noted that different ages and educational levels of students entail different ways of acting and thinking about mathematical modelling in the school context. The case of primary education seems particularly interesting when it comes to specific modelling teaching competencies.

- Which framework(s) may better explain the development of teacher's pedagogical content knowledge for the teaching of mathematical modelling at different educational levels?
- How can the activity of mathematical modelling in mathematics education connect to a broader view of mathematics, including the role of modelling in developing mathematics?
- How might teachers be empowered and feel confident in teaching mathematical modelling?
- Which resources/materials may be effective in advancing teachers' knowledge of and about mathematical modelling in mathematics education?

Concluding remarks

The seven leading themes illustrate the diversity of research frameworks, methodological approaches and research questions that were explored and discussed in TWG6 at CERME 13. Looking back at the themes that came to the fore at CERME 12 (e.g., Barquero et al. (2022)), as well as the perspectives and approaches considered in the evolution of the TWG6 since CERME 4, as described by Carreira et al. (2019), it is reassuring that the TWG6 at CERME 13 showed a mix of continuity as well as new elements and foci. This bodes well for future research in TWG6 at future CERMEs.

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