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Teaching probability at secondary school level with technological tools: issues at stake for mathematics teacher education

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Little is known about what needs to be considered in probability teacher education. A qualitative exploratory research has been carried out concerning training of in-service secondary school teachers and mathematics consultants on teaching probability with technological tools (Thibault, 2021). Our goal was to describe and understand “training issues” in this context, and we adopted a research-training method articulating research and training. In this paper, we present two of the seventeen training issues that have been identified by our analysis, and briefly discuss their different forms and origins as well as the link they illustrate between training issues and teaching issues. The issues selected here are: 1) to highlight the powers of amplification or reorganization of a simulator for teaching probability; 2) to prepare to deal with exceptional results in the frequentist approach.

Keywords: Professional development, probability teaching, technological tools.

Research context

In French-speaking Quebec universities, there is a near absence of mathematics education courses devoted to probability for pre-service teachers and few initiatives seem to be offered for in-service teachers. This seems to be the case in other countries as well, and a review of research in the field (Thibault, 2021) reveals that little is known about what needs to be considered in probability teacher education. Simulation tools have potential for teaching probability (Koparan, 2019; Savard et al., 2013), such as dynamical software that can provide an opportunity to strengthen probability reasoning while studying experiments of chance. For example, a simulator has the potential to raise a need to understand probabilistic concepts after creating a conflict between what is predicted by the student and what actually occurs (Lim et al., 2016). But it seems like not many teachers use these tools, and little is found targeting these uses.

Secondary school teachers, in Quebec for example, report a significantly lower sense of competence for teaching probability than for teaching other mathematical domains (Martin et al., 2021). They could thus benefit from professional development in probability teaching. But what exactly should we highlight when we train teachers? Is it important, as suggested by Stohl (2005) for example, to draw attention to how technological tools allow carrying out a large number of trials quickly? To answer such questions, it is necessary to look into what is at stake in such teacher education: what must be insisted on? Might teachers resist some ideas? We designed and implemented a research aimed at describing and understanding “training issues” during a professional development program for secondary school teachers to help them teach probability with technological tools. In this paper, we present two training issues related to the frequentist approach, that help us better understand the work to be done, as a trainer, in relation to the preparation of teachers in such a context. We then

discuss their different forms and origins, and to the link they illustrate between *training* issues and *teaching* issues.

Conceptual framework

In Thibault (2021), the concept of training issue is defined as an important element to consider when organizing teacher training. Based on what the trainer notices and identifies as important for the professional development of the trainees, training issues are benchmarks that take into account, in various dimensions, constraints and margins of maneuver of training. It should be noted that, since this was an exploratory research, this conceptual framework is used from an idea-generating perspective to analyze emerging issues rather than from an observable verification perspective.

With regard to the training issues targeted for this paper, namely those related to the frequentist approach, it is necessary to explain what characterizes it and makes it important. In his Reference praxeological model of school probability, Otaki (2019) determines the particular types-of-tasks, techniques, technologies and theory related to the frequentist approach. This approach provides a link between the field of statistics and probability, by performing a random experiment and then observing a frequency of occurrence of an event to infer its probability (Chernoff and Sriraman, 2014), while the number of trials should be considered even if we can't obtain a precise, final value. Taking non-determinism into account through the frequentist approach allows for reasoning about uncertainty. The frequentist approach thus offers a path paved with unpredictability, variability and chance in the short run, but opens up the possibility of predicting the trend of results in the long run.

In probabilistic reasoning, well-known biases or misconceptions can be found in children as well as adults, and in experts as well as novices in probability (Tversky & Kahneman, 1971). It is therefore not so simple to develop students' probabilistic conceptions, which adds an additional challenge for teachers. For example, a learner could be satisfied with a small number of trials, which could be a manifestation of the Law of Small Numbers conception (Tversky and Kahneman, 1971), which leads to assigning a certain determinism for a small number of trials. The frequentist approach makes it possible to work with students' conceptions, such as this effect of sample size, which can hardly be done when the theoretical approach is used exclusively.

Research-training method

This study is part of the qualitative and exploratory research stream. The research-training method was relevant because our concerns at the origin of this study touch both training and research. Indeed, this method posits research and training as two poles in interrelation, each maintaining its specificity while enriching the other. Moreover, such research "embedded in the training action leads to a better understanding of the training situations, it allows to rethink these situations, to formulate new paths for the trainer's action, etc., thus feeding back into the training itself" (Bednarz & Proulx, 2010, p. 25, our translation). In research-training, the goal is to document professional content *by, for and among* teachers from within in-service teacher education (Bednarz & Proulx, 2010).

For the research-training we present here, the researcher-trainer is the primary author of this paper. He chose to play this dual role because, on the one hand, it was practically impossible to find active trainers intervening specifically on probability. On the other hand, taking on these two roles at the

same time allowed him to observe participants from inside the training. Being involved in the interventions of the training sessions, the researcher-trainer could be sensitive to the teachers' needs in relation to teaching secondary school probability with technological tools, while on the front line to describe and understand training issues as he adjusted the planning of the sessions as needed.

The research participants were recruited on a voluntary basis and chosen in order to bring together a variety of expertise, since this complementarity and richness of expertise could bring out various training issues. Five teachers and two secondary school mathematics consultants took part in the professional development sessions. Their expertise varied in terms of experience (3 with 8 to 10 years of experience, and 4 with 17 to 26 years of experience) and (self assessed) familiarity with probabilistic concepts or technological tools (ranging from not very comfortable to very comfortable). The participants also intervened at different grade levels of secondary school (with students aged 12 to 17). It is worth mentioning that the teaching of probability in Quebec is prescribed by ministerial documents in most grades of a student's elementary and secondary education. The frequentist approach is part of the probability curriculum, notably through the simulation of random experiments with or without technological tools and the link to the theoretical approach.

Five sessions took place at regular intervals over a period of seven months, addressing the three following themes: probabilistic notions related to the frequentist approach; the contributions and limitations of technological tools in probability; and the appropriation of technological tools in a probabilistic situation. Individual interviews before and after the sessions were conducted and recorded. We also recorded the five group sessions. Each session was also the subject of a detailed report written by the researcher-trainer to make an initial sorting of the data. A synthesis in the form of a "training narrative" was also written by the researcher-trainer for the participants, and used in the initial stages of the analysis.

For the analysis itself, we immersed ourselves in the data to better understand the participants' perspective (Maher et al., 2018). One way to analyze the additional data collected was through an analysis using conceptualizing categories. This is a perspective where the "analytical work is an inductive, incremental work, making very little use of concepts or categories that have already been clearly formulated" (Paillé & Mucchielli, 2016, p. 331, our translation). Wishing to "listen to the data" in order to properly analyze the phenomenon in all its complexity (Paillé & Mucchielli, 2016), we targeted 16 moments that were particularly interesting in terms of the richness of the remarks. These moments, identified through the detailed reports and training narratives, were transcribed and analyzed in greater detail using the conceptualizing categories being developed. This allowed us to both better reflect the data and better conceptualize the categories from which we formulated the training issues using the definition we presented in the previous section. As a result, we identified, described and explained 17 training issues. More specifically, beyond the wording of each training issue, it was the excerpts analyzed that made it possible to describe and understand those training issues by situating them in the process, through examples of exchanges that bring out the successes, challenges and needs that emerged with the participants. Figure 1 illustrates the data analysis process that led to the formulation of these issues.

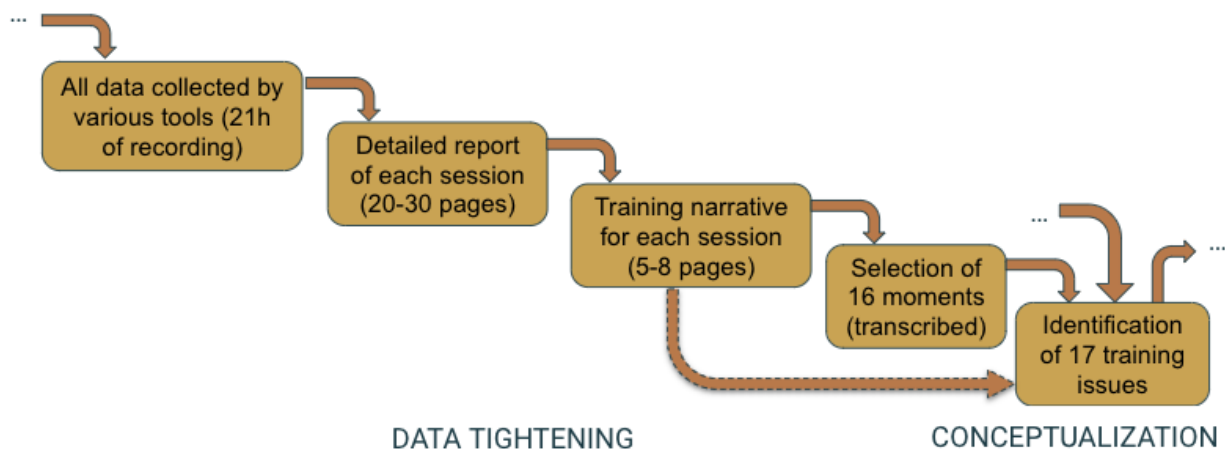


Figure 1. The approach to data analysis in Thibault (2021) research-training

Results

This paper presents two of these training issues, exemplified by using excerpts (translated from French). The first is to make teachers realize the amplifying or reorganizing powers¹ of a simulator for teaching probability. The second is to prepare teachers to deal with exceptional results in the frequentist approach. As we will also see, these issues give us a better understanding of the work to be done as a trainer.

To highlight the amplifying or reorganizing powers of a simulator for teaching probability

In the first professional development session, the researcher-trainer questions the "efficiency" of a simulator by asking the participants to compare two simulators, where one (which generates up to 1,000 trials in one click) is clearly more efficient than the other (where it takes a few clicks to complete a single trial). Initially, participants struggle to talk about what distinguishes the two, seeming not to recognize that efficiency is an important consideration when choosing a simulator. Grace (pseudonym) notes that she likes the first simulator because it helps with understanding (though she does not specify how this simulator promotes understanding), but also observes that this aspect is only part of what the simulator offers: "It helps with understanding, but it does not do calculations." The trainer then asks the group which of the two simulators generates the results of 100 simulated trials more quickly, wishing to bring out the amplification power the participants do not mention. In their response, the participants then seem to be aware that it is necessary to generate the results of a large number of trials to be able to observe a clear trend (thus avoiding the Law of Small Numbers conception from Tversky and Kahneman [1971]), and that the first simulator might then be preferable. The trainer adds that this simulator could compensate for the lack of time to teach probability the participants complained about earlier (they stated several times during the sessions that they lacked the time to properly work on probabilistic notions with their students).

The trainer then shows how it is possible to modify the parameters of a simulator in order to run tests on a virtual bottle containing for example 1000 balls of two different colors (which a real bottle of

¹ Concepts were developed by Pea (1985): a technological tool can offer a power of amplification by making it possible to do more and faster as well as a power of reorganization by making it possible to do something else, or to do it differently.

reasonable size would not allow). In an effort to harness the reorganizing power of the simulator to open up new possibilities, the trainer then runs an "extreme" simulation. To show how a simulator allows to ask new questions by offering conditions that surpass the physical constraints (power of reorganization), he creates a virtual bottle containing 3 red marbles and 10,000 black marbles, and asks "after how many trials (with replacement) will the first red marble be obtained?". While such a question would be much more complex to answer by the theoretical approach (e.g. using negative binomial law), the simulator made it possible to "get a sense" of expected frequency. This simulation elicits several different reactions from the participants: interest, excitement, new questions, but also disinterest. While participants become accustomed to conducting a large number of trials and then observing a frequency that estimates the probability of an event, there is a novelty here prompted by the nature of the question being asked. The varying reactions suggest that teachers might not have the same level of comfort in harnessing the power of reorganization of such a tool.

These powers of amplification and reorganization help to legitimize the use of a simulator, but also present challenges while opening for new possibilities. It thus appears as a training issue, something worth addressing while training teachers use technological tools to teach probability. If teachers are aware of such contributions, they could better appreciate the added value offered by a simulator and exploit it with their students. However, with the benefits of simulators also come limitations. For example, while the participants noted that the bottle simulator makes it possible to ask new questions (given that one can decide on the contents of the virtual bottle), it was clear from the exchanges that these new questions are not easy to formulate, and not all participants readily saw how these questions could support their teaching and contribute to the development of probabilistic notions.

To prepare to deal with exceptional results in the frequentist approach

Even though the exact outcome of a random experiment is obviously impossible to predict with certainty, some scenarios immediately seem more reasonable than others. Tossing a coin 20 times, we can accept without difficulty that there could be 10 tails or close to it (7-8-9 or 11-12-13), but it would be more surprising to get only 1 tail, even if it is not impossible. As a result of conducting the frequentist approach, some results are out of the ordinary, in the sense that somewhat unexpected exceptions are observed. During one of our professional development sessions, this question emerged as an issue from the point of view of preparing activities with students. The exchanges around this theme showed that it is an element that teachers did not seem to have considered and be ready to face before the sessions. Therefore, it clearly provides us with a training issue. Here is what was observed.

During the second professional development session, Florence points out the unusual character of certain results of drawing marbles (with replacement) obtained experimentally. The probability of drawing a red marble is 60%, but after 97 trials the observed frequency of red marbles is 49/97, nearly 50%. Since this is a result that is very different from what should normally happen, the participants seem to be quite unsettled by this result. They are not wrong to be surprised, since obtaining a frequency of 49/97 is indeed exceptional, with a probability of about 3%. At the next session, the trainer presents this low probability and explains to the participants that this was a rare case since, by convention, the significant threshold is usually set at 5%. He explains that this threshold is used to designate the boundary between expected and unexpected results, where variations are probably not

due to chance. He explains to the participants that for 100 samples of 97 trials, one would expect to get such an extreme pattern only 3 times, with the other 97 being closer to the expected pattern (i.e., a frequency near 60%). Brian then asks the trainer to say "after how many trials?" again, correctly implying that one should consider the number of trials (again avoiding the Law of Small Numbers conception from Tversky and Kahneman [1971]) in order to conclude whether these are exceptional results or not. Indeed, it is more likely to deviate from an expected pattern with 10 trials than with 100 trials (frequencies tend to converge to probability as the number of trials increases). When the trainer returns to the question and concludes "we have witnessed exceptional results", the participants seem to welcome it and no longer appear destabilized. Emilie even goes so far as to say that it was "fun" to observe this.

Other exceptional results occurred in subsequent sessions, and some participants seemed to develop a certain sensitivity towards them, wondering at various times whether some of the results they obtained in their simulations were out of the ordinary. Their initial discomfort with the marbles experimentation is understandable, however. The appearance of exceptional results requires a slightly different explanation than the expected result. If an exceptional result occurs in the classroom with students, the teachers must be able to respond. Also, by using a simulator, exceptional results can be obtained even more quickly (and not more frequently in a proportional sense), because one can generate results from many sample trials. Having actually discussed this in sessions seems to have made the participants comfortable with the fact that it is possible and even interesting for this to happen. The notion of a significant threshold and the associated calculations go beyond the content generally covered at the secondary level in the Quebec context and were not the subject of in-depth work with the teachers. Moreover, they seemed to have little interest in the technical details surrounding the explanation. Thus, even if the discussions during this session did not delve into this aspect, the professional development context seems to be a privileged moment to get teachers to prepare themselves to deal with exceptional results. Such preparation aims to get teachers to conceive of and recognize the occasional occurrence of exceptional results, and then to consider interventions to manage these occurrences if they ever happen in the classroom.

Concluding remarks

Our analysis reveals training issues of different origins and taking different forms. First, a training issue sometimes emerged in connection with an *important event* that occurred during the sessions and constituted a key moment in the research-training. In this case, the work of the trainer and the participants was analyzed in this sense. An event is important because it has a significant impact on the sessions, in the actions and words of the trainer and the participants. This is notably the case when the latter come to recognize and exploit the amplification power of the simulator or when certain participants take up the idea of exceptional results on some occasions. Secondly, a training issue sometimes takes the form of an *important element to understand or to do*, mentioned by the trainer or by the participants during the sessions. It can be something one would like the teacher to know or do in relation to her teaching. It is thus a question of intentions, or an ideal that one would like to achieve. We see this when the researcher-trainer wants the participants to become aware of the powers of amplification and reorganization of a simulator in order to be able to exploit them in their teaching, or when he suggests that it is important to be sensitive to the exceptional results that can occur. Third,

a training issue is sometimes related to a *problem* that arises in the sessions, which may or may not be explicitly discussed. An important teacher education consideration is sometimes perceived as a problem by participants or by the trainer. In this case, it may be something difficult for the trainer or the participants to do, because of an obstacle or constraint to overcome. We observe this when, for example, some of the participants could not see how to harness the reorganizing power of a simulator.

The results of this research show a strong link between *teaching issues* and *training issues*. The teachers' comments highlighted issues that can be taken into account in teacher education (pre-service and in-service), but these comments also often seem to indicate important things to do *in the classroom*, to get students to understand, etc. In other words, if something is important to consider in organizing instruction, it might be important to address it in teacher education... and vice versa. The professional development sessions can allow participants to work on teaching issues, by provoking discussions so that they can share their successes, challenges and needs as teachers. The sessions made this possible by a) asking the participants to work collectively on probabilistic situations and technological tools, b) making them work individually on assignments between sessions, then sharing their thoughts as a group, and c) prompting them on a disagreement or making a link with an idea mentioned previously. All of these opportunities for interaction allowed the participants to reflect on and discuss teaching issues, thus bringing out the training issues for this research.

The contributions and outcomes of this research must, however, be nuanced considering the limitations and challenges of the research-training undertaken here. For example, it was challenging to keep a good balance between the research and the training dimensions. Related to this challenge, a limitation of this research is that the training issues that emerged are constrained by the choices made for this research as well as by the particular context of in-service training, which brought together five teachers and two secondary school mathematics consultants for five professional development sessions. Had the choices and context been different, other issues might have emerged.

Our work also opens up other research perspectives. We observed that preparation in probability education is not sufficient to establish a strong link between the theoretical and frequentist approaches, which is a central piece of the reference theory presumed by Otaki (2019). Further studies could highlight the potential for articulation of probabilistic approaches, as suggested by Batanero et al. (2021). Moreover, as mentioned by Savard et al. (2013), teachers could benefit from specific coaching in the use of simulators in class and in the development of students' probabilistic conceptions. Our observations support this. Other research could further clarify the issues of such support by taking into account actual (and not only declared) teaching practices, which would be possible by alternating between professional development sessions and classroom observations.

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