Two situations to deal with the subjective meaning of probability in the elementary school: students' productions and teachers' knowledge

Luis J Rodriguez-Muñiz, Carmela Suárez, Laura Muñiz-Rodríguez

Two situations to deal with the subjective meaning of probability in the elementary school: students' productions and teachers' knowledge

Luis J. Rodríguez-Muñiz, Carmela Suárez[1] and Laura Muñiz-Rodríguez[1]

[1]Universidad de Oviedo, Spain; luisj@uniovi.es

There is an ongoing demand to emphasize the use of probability as a subjective measure of uncertainty in elementary schools at the international level. We propose two situations to teach students the subjective meaning of probability and how to incorporate new experimental information in their probability assessments. These activities were implemented with 5th-grade students (10-11 years old). Our results align with previous research and highlight the ability of 5th-graders to process contextual information and integrate it with their understanding of chance and the context, even if they may struggle with the mathematical expression of their reasoning. Additionally, we examine the specialized knowledge and beliefs that teachers need to mobilize in these situations. By combining an analysis of student reasoning and teacher knowledge, our goal is to design effective lessons on subjective probability that can be successfully implemented in the classroom.

Keywords: Elementary school, probabilistic reasoning, subjective probability.

Introduction

The current work can be seen as a continuation of our previous work in TWG5 at CERME12 (Rodríguez-Muñiz et al., 2022), where we examined the implementation of a situation involving subjective probability in a chess game. It still holds true what we stated in our previous work regarding the polysemy of the notion of probability and the debates surrounding its interpretation in the classroom (Batanero et al., 2005). What was previously just an academic discussion became a real educational challenge when the new Spanish curricular guidelines, introduced in March 2022, included the concept of "probability as a subjective measure of uncertainty". The scarcity of resources about the subjective aspect of probability in textbooks and teaching resources (e.g., Gómez-Torres et al., 2014) presents a significant challenge for elementary school teachers in teaching the new curricular guidelines, and this issue is not limited to Spain or Spanish-speaking countries.

With the purpose of addressing this challenge, the first goal of this current work is to design and implement two situations involving subjective probability for the upper elementary school students. To do this, we have stated two research questions: (RQ1) How do 5th-graders (10-11 years old) reason in situations where classical or experimental definitions of probability cannot be applied? and (RQ2) How do they integrate contextual and experimental information with their understanding of probability? A secondary goal is to determine the specialized knowledge required of teachers to handle these situations in the classroom. Our third research question is, therefore: (RQ3) What specialized knowledge should teachers mobilize and how do their beliefs may impact for teaching these situations?

Theoretical framework

The implementation of subjective probability in elementary education relies heavily on the language of chance (Kazak & Leavy, 2018), the intuitive meaning of probability (Batanero et al., 2005), and the use of verbal quantifiers (Blanco-Fernández et al., 2016). These elements are closely tied to the concept of probability as a degree of belief (De Finetti, 1974), and therefore, as a subjective construction influenced by the amount and quality of information available and the individual's personal conceptions and understanding, including biases (Konold, 1989), that enables the elicitation of subjective probability estimates for certain random events.

Previous studies (Kazak, 2015; Vásquez & Alsina, 2019) demonstrated that students can work with probability language and generate qualitative probability estimates. When faced with experimental information, they may draw upon their personal knowledge and beliefs, under certain circumstances. These studies focused on situations that could be analyzed using combinatorial methods, such as counting cases, even if students did not use that approach. In Rodríguez-Muñiz et al. (2022), we studied a chess-based situation where determining probabilities through counting cases was not possible. We found that students, without being prompted, assigned probabilities as fractions or percentages, and they were able to adjust their initial probability assignments by incorporating experimental information and considering the quantity and quality of the information provided by others, performing an informal Bayesian reasoning.

The second aspect of the theoretical framework consists of a model for describing teachers' knowledge in teaching the subjective meaning of probability. Given its focus on teachers' beliefs and how these influence mathematics teaching and learning, the *Mathematics Teachers' Specialized Knowledge* (MTSK) model by Carrillo-Yáñez et al. (2018) was selected for our theoretical framework. The role of probability in models that describe teachers' mathematical knowledge (Franco & Alsina, 2022a) is commonly viewed within the framework of the Statistical Knowledge for Teaching (SKT) (Growth, 2007). However, the extent to which this SKT is distinct from mathematical knowledge is a topic of ongoing research. As proposed by Franco and Alsina (2022b), we consider a hybrid approach in which the SKT is not treated as a separate knowledge, but rather incorporated into the MTSK model, with a focus on the unique aspects of teaching probability. Besides, the scientific literature provides evidence of the role of beliefs in the MTSK model and their relationship with the other two domains, as seen in Aguilar-González et al. (2018, 2022). However, there is limited research in this relationship specifically regarding probability.

Methods

Context

The situations took place in April-May 2022 at two elementary schools in Oviedo, Spain (one in the urban center and another in the surrounding rural area). Twenty-nine (18 from one school, 11 from another) and 20 (16 from one, 4 from another) 5^{th}-grade students participated (10-11 years old) in each of the situations, respectively. In previous years, the students had studied probability language (for ranking events), the concepts of chance and random experiments (mostly with a classical approach but also including observing patterns), but they had not properly studied any definition of probability or how to determine estimates. The second author was the teacher who taught the sessions corresponding to these situations and conducted the student observation.

Instruments

The situations, as in Rodríguez-Muñiz et al. (2022), were designed to prevent the use of the experimental and classical interpretations of probability, characterized by unequal outcomes and the inability to repeat the experiment under identical conditions. In both cases, the data was collected through students' written answers to a set of questions and oral discussions in Spanish.

First situation: MasterChef TV show

Stage 1: There are 3 contestants for the final MasterChef test. Inés has never competed in an elimination challenge; Samuel has been in 4 and Diego in 7 in the previous programs. Samuel was eliminated in program 4 and rescued later. Diego has participated in two summers in the MasterChef camps. Order the contestants according to the position you think they will have and explain your choice. Do you think your choice will be certain? Represent the probability each contestant has of winning (use a circular model if you want). What probability do you think Inés has of winning? Stage 2: A survey of 500,000 spectators says that Inés is the favorite, with 50% of the votes. Would you change your previous answer? What probability do you think Inés has of winning? Stage 3: A dish of codfish must be prepared, which none of them has used in the program. Diego's grandfather is a fisherman, Samuel was rescued for cooking an exquisite hake, and Inés is a vegetarian. Would you change your previous answer? What probability does of each one have of winning?

Second situation: Planning an excursion

Stage 1: Can you estimate now in April the probability of good weather in June? What are you basing this on? Do you think your answer is reliable? Stage 2: The data of temperature and daily precipitation for the past five years in June is given. What do you think is the probability of good weather in June? And on June 8th, 9th, or 10th? Stage 3: The weather forecast for June predicts an average of 16.4ºC and 10 rainy days with 94 mm of precipitation. Would you change your answer? Stage 4: The day before the excursion was very cloudy and it rained all morning was sunny in the afternoon. Would you change your answer regarding the probability of good weather the day of the excursion?

Teachers' specialized knowledge and beliefs

The specialized knowledge and beliefs of the teacher to teach these situations were analyzed by the other two researchers prior to and after the implementation of both situations in the classroom by conducting an interview with the second author. The researchers identified and classified the knowledge and beliefs into the subdomains of MTSK by using deductive categorization.

Results

First situation: MasterChef TV show

In stage 1, only three different answers were proposed. More than half of the students (18 out of 29) chose Diego, Inés, and Samuel as their preferred order, providing 15 answers based on data from the situation and 2 based on their beliefs (one answer was not justified). In the remaining two possible orders, most of the answers were also based on information from the problem. Some students chose Diego first based on his experience in cooking camps, while others considered his experience in 7 elimination challenges as either a positive (competitiveness) or negative (challenge stress) factor. It was much more difficult for students to argue about the reliability of their own predictions: just a few were able to acknowledge randomness. For instance, some gave a probability of their prediction being true or pointed out there were other not mentioned factors that could affect their performance. When asked about representing probabilities on a circular model, Figure 1 shows three different types of answer, going from a standard sector distribution to nonstandard ones.

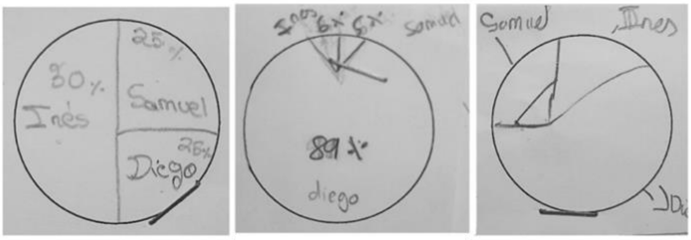

Figure 1: Students' productions for representing probabilities of winning of each contestant

When estimating the probability of Inés winning, most students (25 out of 29) answered with a percentage, even when not being told to do that, using previous pie charts as a support. One student initially stated he could not give a probability for each one due to the many factors to consider, but later wrote 50% for Inés. Only one student gave a combined representation of verbal language and a percentage (12%). In stage 2 students were informed about an online survey and they were asked whether they keep or change their initial estimate. The results showed that the majority of students did not change their answer. Some students modified their answer, citing that they now trusted the audience and believed Inés had a 50% chance of winning. Meanwhile, other students who did not change their answer cited reasons such as not trusting the audience's opinions and using personal beliefs to mistrust the data from the survey ('I don't change my answer because what other people vote is not relevant, you don't change your opinion just because other people say so'). Other students justified their answer by assuming that winning does not depend on the audience.

The stage 3 introduced more information about the final challenge, and students were asked if they would change their answer again. Most students (16 out of 25) modified their answers, influenced by the new data provided. Two thirds mainly focused on Inés being a vegetarian as the factor affecting her probability of winning (e.g., 'I changed my answer to 15% because if she is a vegetarian, it is likely she hasn't cooked fish before'), while the others considered all the new information including Diego's and Samuel's contexts ('Inés is a vegetarian, but she still has probabilities of winning. Diego's grandfather is a fishmonger and could have taught him about fish. Samuel knows how to cook fish well, so he has more probabilities of winning than before'). Students who did not change their answers argued that the data was not relevant ('It's nothing to do with her being a vegetarian, she can cook fish just as well') or relied on previous information or personal beliefs ('Diego is, for me, the best cook and Samuel is the worst'). New probabilities were expressed using verbal language, percentages, or graphical representations. Some students used integer numbers while others used decimals. Even some students attempted innovative graphical representations, but accuracy was not always ensured (see Figure 2).

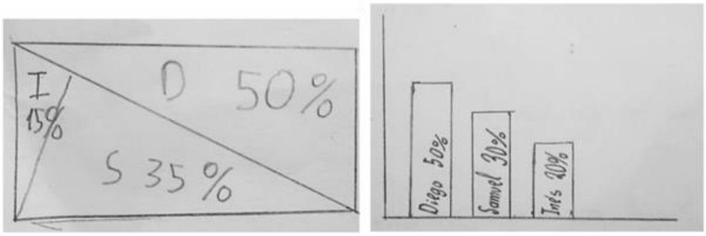

Figure 2: Students' productions for representing probabilities of winning in stage 3

Second situation: Planning an excursion

The first question of stage 1 asked students to predict the probability of good weather in June. The vast majority of the students (19 out of 20) answered using percentages, with only 1 verbal answer ('Good weather is very likely'), 4 of them included also a graphical explanatory pie or bar chart. When asked to justify their answers, the reasons could be classified into different categories: basing on observation of the weather that week or that month ('I base my answer on the weather in April'), using their previous knowledge about weather ('It is summer and in summer there is good weather') or making a scientific argument ('Climate change is getting worse and June will be a very hot month'). Again, the question about the reliability of their answer was very difficult and most students did not reply. Those who did expressed different arguments showing variable self-confidence ('I am 70% certain of my answer', 'I am pretty sure of my answer', or 'I am an insecure person').

In stages 2, 3, and 4, different data about the weather in June in the last 5 years, the forecast for June 2022 and the forecast for the 3 days of the excursion was respectively given. Again, students predominantly provided a percentage estimate of the probability, with some verbal approaches. When we focus on students' justifications for changing or not their initial probability estimate, we find that most of the students dealt with the provided data. In stage 2 data on temperature and rainfall justified the answers. Temperature was seen as stable throughout the month, leading students to use rainfall as their primary source of information. Most students analyzed data from 2-4 years. The previous year was used frequently because it was seen as the most reliable. In stage 3, most students used just part of the data. Students who modified their answer justified it based on the data provided ('It rains more than I thought it would and it will be colder than I thought'), most of which showed the weather being worse than expected. Students who did not change their answer also used the data provided to justify their response, but this time the data matched their expectations of good weather ('I thought there would be between 16ºC and 18ºC and the forecast says 16.4ºC'). Finally, in stage 4, most students decided to maintain their previous answer and supported their justification on the data, for instance, by estimating the probability of good weather to be 50% based on the assumption that it would rain in the morning and be sunny in the afternoon. Others kept their previous answer by citing their knowledge of the extremely variable microclimate in the region as their reason for not changing.

Teachers' specialized knowledge and beliefs

Following Franco and Alsina (2022b), the specialized knowledge mobilized by teachers to deal with these two situations was classified into the different subdomains of the MTSK model. Within the MK domain, the subdomain Knowledge of Topics (KoT) is characterized here by the intuitive and subjective meanings of probability, which are explicitly used, but also by the connection with other possible meanings that could arise (classical or experimental). The Knowledge of the Structure of Mathematics (KSM) includes the verbalization, order, and quantification of chances, that is, the use of verbal, tabular, graphic, and numerical representations of chance, together with their exchanges. The Knowledge of Practice in Mathematics (KPM) refers to the use of daily-life contexts involving, handling, and interpreting uncertainty in random phenomena which are not suitable to be dealt with Laplace's rule or experimental meaning of probability. Additionally, within the KPM teachers must mobilize their understanding of the level of certainty or uncertainty that one can have regarding the occurrence of future events or their degree of predictability, that is, a meta reflection on the reliability of the probability estimate.

Within the PCK subdomains, the Knowledge of Mathematics Teaching (KMT) requires to provide students a wide range of experiences that allow observing random phenomena and differentiate them from deterministic ones, to stimulate the expression of predictions about the behavior of these phenomena and the outcomes, as well as their probability in different representations, and to handle experimental data so that students can contrast their predictions with the results produced and review their beliefs based on the results. The Knowledge of Features of Learning Mathematics (KFLM) includes here the knowledge of the process of students understanding the language associated with each probability meaning, as well as being aware of the different students' errors and difficulties in this field and biases in probabilistic reasoning (in this case, wrong conceptions about chance, or inferences based on one single case). The relevance of the Knowledge of Mathematical Learning Standards (KMLS) is emphasized in these situations due to the recent addition of the subjective meaning of probability in the Spanish curriculum. However, KMLS encompasses more than just this aspect, and includes identifying the required probability content in a particular course, identifying, and making guesses, estimations, and revisions about random situations, solving problems that involve mastery of probability concepts, and reflecting on the problem-solving process.

Finally, beliefs on probability and its teaching and learning constitute a very relevant domain in these situations due to the nature of the subjective meaning of probability. Teacher's beliefs about what probability is are involved when dealing with that meaning, and he/she must be aware about the influence of rationalist beliefs on mathematical notions (e.g., the existence of a *right* solution), as well as his/her own's biases in probabilistic reasoning. Also, beliefs on the nature of mathematical learning are permeating teacher's role in these situations since they are based on the dialogic learning.

Discussion and conclusions

This work contributes to creating learning situations for the current curricular demands by analyzing the knowledge and beliefs mobilized by students and teachers when dealing with the subjective meaning of probability. Concerning (RQ1), we found that 5th-graders were capable of dealing with chance situations that only rely on subjective probability with ease, seeming a rather intuitive approach for children of this age (Kazak & Leavy, 2018). They produced various probability estimates, seeking anchorage in the data, but percentages were the most common, to the extent that there was sometimes an identification between percentages and probabilities (also experienced in Rodríguez-Muñiz et al., 2022). However, there were evident difficulties in mathematizing their intuition (as shown in Figures 1 and 2). Additionally, we must underline there was much difficulty in judging the reliability of their own predictions. Concerning (RQ2), students integrated context knowledge (TV show, weather) and beliefs about chance and probability in diverse ways (e.g., by weighing them or applying the principle of maximum uncertainty). It is noteworthy to see that beliefs, a dispositional element in Gal (2005), play a significant role in information integration, even surpassing evidence-based reasoning (e.g., ‘I won't change my answer because all contestants have the same winning probability’). Personal experiences hold great importance to students when integrating new experimental data (e.g., as the area is very rainy, many students associated good weather with no rain). Therefore, students went into informal Bayesian reasoning by different ways, and experimental information did not always change the initial belief (Vásquez & Alsina, 2019). In short, our study corroborates the findings of Kazak and Leavy (2018) regarding the use of subjectivity in the estimation of probabilities, specifically through the incorporation of previous knowledge and additional information, as well as personal experiences and beliefs. Lastly, regarding (RQ3), it is evident that the knowledge of the subjective approach to probability, the understanding of biases in probabilistic reasoning (Konold, 1989), and the awareness of how personal conceptions about probability mediate the teaching are specialized knowledge necessary to effectively teach these situations. Additionally, adopting a social-constructivist approach can assist in negotiating the integration of experimental information among different students’ views. We agree with Franco and Alsina (2022b) about the necessity to further investigate and deepen about teachers’ specialized knowledge and beliefs when dealing with subjective probability in the classroom.

Acknowledgements

LJRM and LMR are partially funded by Grant PID2021-122180OB-100 from the Spanish Ministry of Science and Innovation, and they both belong to the MTSK Network (https://redmtsk.net/).

References

Aguilar-González, Á., Muñoz-Catalán, C., Carrillo-Yáñez, J., & Rodríguez-Muñiz, L. J. (2018). ¿Cómo establecer relaciones entre conocimiento especializado y concepciones del profesorado de matemáticas? [How to establish connections between specialized knowledge and mathematics teachers’ beliefs?] *PNA*, *13*(1), 41–61. https://dx.doi.org/10.30827/pna.v13i1.7944

Aguilar-González, Rodríguez-Muñiz, L. J., & Muñiz-Rodríguez, L. (2022). Las creencias y su papel en la determinación de relaciones entre elementos del conocimiento especializado del profesor de matemáticas [Beliefs and their role in determining relationships among elements of a mathematics teacher’s specialized knowledge]. In J. Carrillo-Yáñez, M. A. Montes, & N. Climent (Eds.), *Investigación sobre conocimiento especializado del profesor de matemáticas (MTSK): 10 años de camino* (pp. 109–120). Dykinson.

Batanero, C., Henry, M., & Parzysz, B. (2005). The nature of chance and probability. In G. A. Jones (Ed.), *Exploring Probability in School: Challenges for Teaching and Learning* (pp. 15–37). Springer. https://dx.doi.org/10.1007/0-387-24530-8_2

Blanco-Fernández, Á., Díaz-Díaz, P., García-Honrado, I., Ramos-Guajardo, A. B., & Rodríguez-Muñiz, L. J. (2016). A proposal for assessing imprecise concepts in Spanish primary and secondary schools. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 24*(suppl. 2), 71–91. https://dx.doi.org/10.1142/S0218488516400110

Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, Á., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, *20*(3), 236–253. https://dx.doi.org/10.1080/14794802.2018.1479981

De Finetti, B. (1974). *Theory of Probability*. John Wiley.

Franco, J., & Alsina, Á. (2022a). El conocimiento del profesorado de Educación Primaria para enseñar estadística y probabilidad: una revisión sistemática [The knowledge of primary school to teach statistics and probability: a systematic review]. *Aula Abierta*, *51*(1), 7–16. https://dx.doi.org/10.17811/rifie.51.1.2022.7-16

Franco, J., & Alsina, Á. (2022b). Conocimiento especializado del profesorado de Educación Primaria para enseñar estadística y probabilidad [Primary school teachers' specialised knowledge to teach statistics and probability]. *Educación Matemática*, *34*(3), 65–96. https://dx.doi.org/10.24844/EM3403.03

Gal, I. (2005). Towards "probability literacy" for all citizens: Building blocks and instructional dilemmas. In G. A. Jones (Ed.), *Exploring probability in school* (pp. 39–63). Springer.

Groth, R. E. (2017). Developing statistical knowledge for teaching during design-based research. *Statistics Education Research Journal*, *16*(2), 376–396. https://dx.doi.org/10.52041/serj.v16i2.197

Kazak, S. (2015). A Bayesian inspired approach to reasoning about uncertainty: 'How confident are you?' In K. Krainer & N. Vondrová (Eds.), *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education* (pp. 700–706). Charles Univ. & ERME.

Kazak, S., & Leavy, A. M. (2018). Emergent reasoning about uncertainty in primary school children with a focus on subjective probability. In A. M. Leavy, M. Meletiou-Mavrotheris, & E. Paparistodemou (Eds.), *Statistics in Early Childhood and Primary Education. Supporting Early Statistical and Probabilistic Thinking* (pp. 37–54). Springer. https://dx.doi.org/10.1007/978-981-13-1044-7_3

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction*, *6*(1), 59–98. https://dx.doi.org/10.1207/s1532690xci0601_3

Rodríguez-Muñiz, L. J., Rodríguez-Suárez, L., & Muñiz-Rodríguez, L. (2022). Subjective probability in use: reasoning about a chess game. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the 12th Congress of the European Society for Research in Mathematics Education* (pp. 964–971). Libera Università di Bolzano & ERME.

Vásquez, C., & Alsina, Á. (2019). Intuitive ideas about chance and probability in children from 4 to 6 years old. *Acta Scientiae*, *21*(3), 131–154. https://dx.doi.org/10.17648/acta.scientiae.v21iss3id5215