

One task for assessing students' ability to order a list of outcomes

Miriama Kmeciková, Nikoleta Ratvajová and Veronika Hubeňáková

Pavol Jozef Šafárik University in Košice, Faculty of Science, Košice, Slovakia;

miriama.kmecikova@student.upjs.sk

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Introduction

Systematic ordering is an important ability when solving math problems. To solve counting problems correctly, one has to list every desirable outcome exactly once. Mostly, we connect systematic listing with solving combinatorial tasks. Yet, it is also an important problem-solving strategy. Lockwood and Gibson state that “when students are capable of systematic listing, it suggests that they are intentional about the listing they do” (2015, p.250). Students can create a complete list when they are solving a counting problem or create a partial list and then see patterns, which can help them to find a solution. A list of outcomes is an important combinatorial tool. The model of combinatorial thinking (Lockwood, 2013) consists of three components – sets of outcomes, counting processes, and formulas/expressions – and relationships between them. The three components can be understood as levels of combinatorial thinking. The first level of combinatorial thinking – sets of outcomes – requires students' ability to list outcomes in systematic order. In the beginning, the list outcomes might be chaotic, which is often the reason for missing outcomes and also for listing outcomes twice. However, there is not only systematic or non-systematic order – there are also some “in-betweens”, which can tell us a lot about students' skill to order a list of outcomes.

Methods

Presented poster is part of a bigger study that aims for an instrument tracking development of students from socially excluded environment, mostly with the language barrier. Here, we analysed ten 15-minute video-recorded interviews where Slovak students from grades 5, 7, and 9 solved ordering tasks (Mathematics, n.d.). In the task, they were given “snakes” consisting of several tiles. The tiles were of two to five different colours. Number of colours, tiles and snakes differs according to level of activity. In Figure 1 we can see level 1 of this task with two colours, two tiles, and four snakes.



Figure 1: Task – level 1

Students were asked to order these snakes and explain their system. Our interest was to see (1) whether the levels are organized with the increasing difficulty, (2) what medium steps between chaotic and systematic order for this particular task are, (3) what hints are useful to help students finish the level and continue on higher levels of this task.

Results

In Table 1, we can see the highest level each student achieved. The levels of this activity increase according to the number of snakes. For example, st_7.2 achieved level 4 (4 tiles, 2 colours, 16 snakes),

and during his work, he received three hints. In level 1 example of listing snakes was shown to him, in level 3, he was asked if he see snakes with similar properties, and in level 4, he was asked to look at the number of tiles with the same colours. In students’ solutions, we observed problems with Level 4. It was easier for them to order two tile snakes with more colours than two colours snakes with more tiles. Therefore, we suggest inserting Level 4 between Levels 6 and 7.

Table 1: Achieved level and hints

grade	student	achieved level	hints
5	st_5.1	4 (4 tiles, 2 colours)	level 1 – showing example of listing snakes; level 2 – asking, if there are others snakes, which have similar properties; asking student to look at the number of tiles with same colour
	st_5.2	4 (4 tiles, 2 colours)	level 2 – asking, if there are others snakes, which have similar properties; level 4 – asking student to look at the number of tiles with same colour
	st_5.3	3 (2 tiles, 3 colours)	level 1 – showing example of listing snakes; level 2 – asking, if there are others snakes, which have similar properties
7	st_7.1	6 (2 tiles, 5 colours)	level 5 – asking, if there are others snakes, which have similar properties;
	st_7.2	4 (4 tiles, 2 colours)	level 1 – showing example of listing snakes; level 3 – asking, if there are others snakes, which have similar properties; level 4 – asking student to look at the number of tiles with same colour
9	st_9.1	7 (3 tiles, 3 colours)	level 2 – asking, if there are others snakes, which have similar properties; level 4 – asking student to look at the number of tiles with same colour
	st_9.2	4 (4 tiles, 2 colours)	level 1 – showing example of listing snakes; level 4 – asking student to look at the number of tiles with same colour
	st_9.3	7 (3 tiles, 3 colours)	level 4 – asking student to start over
	st_9.4	5 (2 tiles, 4 colours)	level 4 – asking what rule can help us to order snakes; asking, if there are others snakes, which have similar properties
	st_9.5	5 (2 tiles, 4 colours)	no hints

In students’ solutions, we observed three main systems of ordering. Firstly, pairing snakes with inverse colours, where they got stuck when the level contained an odd number of snakes. Secondly, the “domino” system. We observed two types of this system – st_9.5 arranged snakes so that the last tile of the snake has the same colour as the first tile of the next snake; st_5.3 collected the same colours in one area – this system does not work for all levels. Thirdly, the developed system, in which student starts with monochrome snakes, then chose one colour of the first tile, and list all variations.

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