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# Identifying components of number sense in the teaching practices of first year of primary education: a case study

Elvira Fernández-Ahumada<sup>1</sup>, Natividad Adamuz-Povedano<sup>1</sup>, Jesús Montejo-Gámez<sup>2</sup> and Enrique Martínez-Jiménez<sup>1</sup>

<sup>1</sup>Universidad de Córdoba, Córdoba, Spain; [elvira@uco.es](mailto:elvira@uco.es)

<sup>2</sup>Universidad de Granada, Granada, Spain

*This research work presents a case study focused on an experienced teacher whose instruction in relation to number sense in the first year of primary school is studied and characterized. This characterization starts by identifying the components of number sense that become evident in her instruction, following the categories established by the Foundational Number Sense (FoNS) framework. Results were obtained on the basis of recordings of classroom sessions. The analysis reveals a comprehensive instruction of the teacher under study, given the large number of FoNS components she addresses in each session. The approach to these components is based on the proposal of tasks that make use of a series of well-structured and complementary teaching materials.*

*Keywords: Foundational number sense, teacher, year one students, primary education.*

## Introduction

The presence and dominance of the term number sense in the scientific literature related to children's basic numerical understanding and its role in later mathematical development is undisputed, as Sayers and Andrews (2015) state. Although there is no agreed definition, it is a key stepping stone in mathematical competence, and this importance, recognised by many authors, has been reflected in the increasing presence of the term number sense in curriculum reforms all over the world (Pitta-Pantazi, 2014).

In Spain, the term number sense was included for the first time in primary education curricula in 2006, as a result of Organic Law 2/2006 on Education (LOE, 2006); although at the international level, there were already previous references that highlighted the importance of considering it in the curricula (Cockcroft, 1985; NCTM, 2000).

In relation to the development of number sense and the way of approaching calculation and basic operations in the first years of learning, different methodologies have proliferated in the Spanish context. They seek to present mathematics in a more natural, transparent and useful way i.e., open number-based algorithms (ABN, acronym in Spanish) (Martínez-Montero, 2011) and tactic calculation (García Pérez & Adamuz-Povedano, 2019). However, in general, student performance in mathematics is far from the desired level, which invites us to investigate the reasons behind this (Ministry of Education, Science and Sport, 2015a, 2015b).

Inevitably, in this scenario, the role of teachers comes into focus. There are several studies documenting the relationship between the quality of mathematics teachers and the performance of their students (Baumert et al., 2010; Hill et al., 2008), a relationship that has been shown to be stronger the younger the students are. This quality has been characterised in research in different ways

(Baumert et al., 2010; Hill et al., 2008), but ultimately, it is reflected in teachers' practice, the result of their knowledge and beliefs and the educational context in which it is embedded.

Although there are numerous studies that provide knowledge about the performance of primary school students in relation to number sense (Aunio & Niemivirta, 2010; Chard et al., 2005; Geary et al., 2009), with a wide range of possibilities and instruments for assessing it (Fernandez-Ahumada et al., 2022), there are few studies that focus on teacher performance, as indicated by Andrews and Sayers (2015). This paper presents a case study of one teacher. This is an experienced teacher whose performance in relation with number sense development is to be studied and characterized. Concretely, the aim is to identify which components of number sense can be recognized in her practice.

In the literature, there is no clear definition of what an experienced teacher is (Gore et al., 2013). In this paper, we have considered an experienced teacher to be one who has a long professional career (more than 10 years) and who bases her teaching practice on scientific evidence both from other researchers and from the action research she carries out.

### **Conceptions and perspectives of number sense**

It is well known that there is no consensus on the definition of number sense. The definitions of the term that appear in the different works range from those that refer to very specific aspects to those that provide a broader and more holistic vision. For more details on this discussion, see Adamuz-Povedano and Bracho-López (2019).

Of particular importance to this work have been the contributions of Sayers and Andrews (2015) or Whitacre et al. (2020). Sayers and Andrews identify three distinct but related perspectives: preverbal number sense, which reflects number perceptions that are innate to all humans and involves the understanding of small quantities in ways that allow comparison; foundational number sense (FoNS), which encompasses those number-related skills that require instruction and typically occur during the early years of schooling; and applied number sense, which refers to a set of basic number-related skills that permeate all mathematical learning and is considered necessary for all adults, regardless of their profession, and whose acquisition by all students should be one of the main objectives of compulsory education. More recently, Whitacre et al. (2020) point out that the problem of defining the term number sense lies in the polysemy of the term, in such a way that it includes three different constructs that they call approximate number sense, early number sense and mature number sense. We consider that this proposal has many points in common with that of Sayers and Andrews (2015). In this paper, we adopt the view of these two works and focus on what they call foundational or early number sense. What is interesting about the perspective of authors Sayers and Andrews (2015) is that for foundational number sense they have developed an easily operationalised framework for analysing the opportunities related to foundational number sense that arise in classrooms, and thus have the potential to inform teacher education or facilitate the assessment of teaching practice. In the developed framework, they distinguish 8 dimensions or components of FoNS: 1) Number recognition, 2) Systematic counting, 3) Relationship between number and quantity, 4) Quantity discrimination, 5) Different representations of number, 6) Estimation, 7) Simple arithmetic competence, and 8) Awareness of number patterns (for more details, see Sayers and Andrews, 2015).

This framework has been used in the present study for the identification of the components of number sense that are evident in the teacher's instruction.

Therefore, the objectives of this work are:

O.1. To identify which components of number sense are addressed in the instruction carried out by the teacher.

O.2. To illustrate how these components are addressed from the point of view of the teaching materials used and the tasks proposed.

## **Methods**

This is an observational case study (Soto & Escribano, 2019) in which sessions taught by an experienced mathematics teacher in the first year of Primary Education were analysed. The choice of this teacher is based on her teaching background and her knowledge of number sense research. Although she is not specifically familiar with the FoNS framework, she has been aware of the importance of number sense in the teaching of numbers and arithmetic for more than ten years. In addition, she is the main author of a book that delves into the development of number sense in the early years of learning; its reference is García-Pérez and Adamuz-Povedano (2019).

Two researchers attended the teacher's mathematics classes to record the sessions for a period of three and a half months. Each session lasted around 1 hour, reserved in the class timetable for mathematics. During this period, sessions were observed and videotaped. The first sessions were discarded to include only 14 sessions for the study, randomly chosen, when teacher and children were already familiarized with the presence of researchers and cameras. The teacher had not received any indication from the researchers. She simply had to carry out her lesson as usual.

For the viewing of the recordings, an observation grid was drawn up taking into account the eight categories that make up the foundational number sense proposed by Sayers and Andrews (2015). This instrument results very useful because it allows the elements that make up the object of study to be organised in a systematic way. The results obtained were contrasted among the authors of the study through a process of triangulation.

## **Preliminary results**

As a first result, Figure 1 shows the components of the FoNS that were identified in each session. This figure reveals the following: 3 sessions addressed 5 components, 7 sessions addressed 6 components, 3 sessions addressed 7 components and 1 session addressed all 8 FoNS components. There were no sessions in which less than 5 components were addressed. These results show that the instruction of the teacher under study is quite comprehensive in relation to foundational number sense. Components such as number recognition, systematic counting or an understanding of different representations of number were present in all sessions, while the component estimation was addressed in only two sessions.

Session 1	X	X	X	X	X		X	
Session 2	X	X	X		X		X	
Session 3	X	X		X	X		X	
Session 4	X	X	X		X	X	X	X
Session 5	X	X	X	X	X			X
Session 6	X	X	X		X		X	X
Session 7	X	X	X	X	X		X	X
Session 8	X	X	X	X	X		X	
Session 9	X	X	X	X	X		X	
Session 10	X	X	X	X	X	X	X	X
Session 11	X	X	X		X		X	
Session 12	X	X	X		X		X	X
Session 13	X	X	X	X	X		X	X
Session 14	X	X	X		X		X	X
	Number recognition	Systematic counting	Awareness relationship number - quantity	Quantity discrimin.	Understand of different represent. of number	Estimation	Simple arithmetic competence	Awareness of number patterns

**Figure 1: Components of FoNS identified along the sessions analysed**

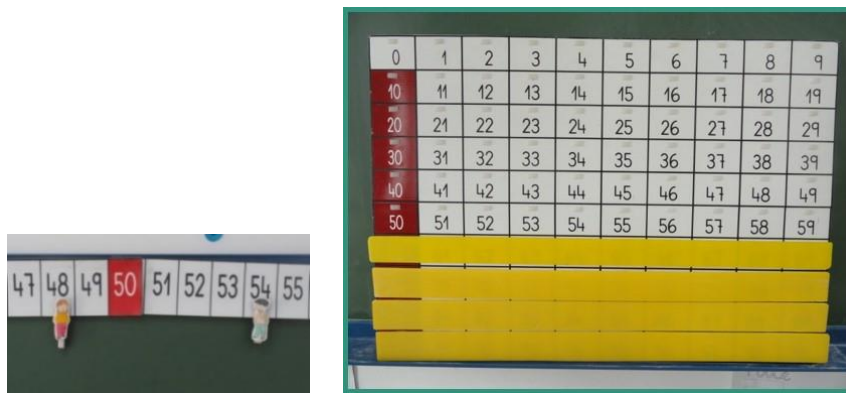
Addressing a large number of FoNS components in each session is made possible, in part, by a series of routines that the teacher performs at the beginning of each session. The recordings show that in these routines, each day, one child is responsible for carrying them out, while the rest of the class pays attention in case he/she makes a mistake. These activities make use of a calendar like the one shown in Figure 2. The teacher asks the child in charge of the activity the day of the week, the day of the month and the year they are in. The child has to turn the number of the day of the month and also answer questions such as “how many days have passed since the month started?” or “how many days are left until the end of the month?” (by counting). Then, the child has to recite the number sequence ascending up to number 20 and descending from number 20 (in other lessons, they recite other numerical sections). As a playful way, at the end of the countdown, the child launches a rocket that the teacher has tied to the wall. To end the routine activities, the child in charge names a number between 1 and 10, e.g., 7, and his/her classmates have to form this number with their hands. In addition, the teacher highlights it on the number tape. Then, the classmates intervene to express different ways of getting 7, at first by proposing only basic additions and later on, by proposing other ways involving subtractions. In this manner, with these routine activities, the components of number recognition, systematic counting, understanding of different representations of numbers and simple arithmetic competence are addressed in practically all sessions of the mathematics subject.

The teacher makes use of other teaching materials such as the number line and the number board (Figure 3). In both cases, the numbers from 0 to 100 are presented, but organised in different ways. In the first, as a linear and ordered sequence, and in the second, by families. These two materials serve as support for the approach of FoNS components such as quantity discrimination, estimation, simple arithmetic competence or awareness of number patterns.



**Figure 2: Calendar used in each day's routine activities**

The number board is especially useful for addressing estimation and awareness of number patterns. To work on estimation, the teacher uncovers the different number families as she goes through them in depth, but asks her students where they think a hidden number will be. The number patterns are approached with activities in which she shows the panel (or a section) practically empty (Figure 4). She names the missing numbers in no particular order and the students have to indicate where the numbers are.



**Figure 3: Extract of the number line and image of the number board with numbers organized by families**

It is important to note that a smaller scale version of the teaching materials is available for students to use in the activities on an individual basis. It is also worth noting that a textbook is not used, but that the students' work material is created by this teacher together with the teaching team of the rest of the first-year groups.

0									
10									

**Figure 4: Example of an activity with a part of an empty number board to be completed**

To illustrate how other components are addressed, such as awareness of the relationship between number and quantity, quantity discrimination or the different representations of number, the use of the number box is essential (Figure 5). It facilitates the exploration and manipulation of numbers, favouring a correct understanding of the decimal number system. Working with the number box produces a qualitative leap in the understanding of numbers and their size, as it provides a concrete model that is faithful to the visible reality, which gives meaning to the use of written symbols and concepts related to positional value.



**Figure 5: Number box and sticks used to set up units, tens and hundreds**

## **Discussion**

The preliminary results presented provide insights into how a year one teacher develops learning opportunities that foster the development of foundational number sense. The analysis reveals that the instruction of the teacher under study is quite complete, given the large number of FoNS components she addresses in each session. The approach to these components is based on the proposal of tasks that make use of a series of well-structured and complementary teaching materials.

The results, in terms of components recognised in instruction, are in line with other studies where estimation, quantity discrimination and awareness of number patterns are the least addressed components (Sunde & Sayers, 2022).

In terms of how the FoNS components are approached, the number line is widely used as in previous works (Booth et al., 2008; Sunde & Sayers, 2022) but also with other variations. Other teaching materials such as the number board, in its full and empty version, or the number box provide opportunities to work on components such as estimation, quantity discrimination and awareness of number patterns, which, as we have seen, are not the most commonly addressed.

This work is a starting point for further research. Following the analysis of the teacher's role, the next step would be to explore her students' performance in relation to number sense and to compare this performance with that of students of other teachers whose practice includes a different distribution and a different approach to the FoNS components addressed. In this way, it is expected to be able to identify which practices are most effective for the development of number sense in the early years of learning. Other future line of work is to analyse the workbooks produced by the teacher for the children, following the methodology of analysis presented in Sayers et al. (2021).

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