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Exploring the components of the competence to construct a mental representation of mathematical problems in a fourth-grade student during mathematical logic problem solving

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Problem solving in arithmetic remains a challenge for students and a significant concern for teachers. The difficulty often lies in the students' inability to construct a meaningful representation of the problem, especially when problems involve written language and arithmetic concepts. This study aims to explore how environments presenting mathematical logic problems, initially devoid of arithmetic concepts and written language, but later incorporating short sentences, facilitate the utilization of components necessary for constructing a mental mathematical problem representation in 4th grade students. A qualitative analysis of an illustrative case demonstrates that these environments provide students with opportunities to utilize each component of this competence, often in conjunction with others. Furthermore, these environments appear to encourage students to employ these components strategically.

Keywords: Problem solving, representation, mathematical logic, elementary school.

Introduction

Most teachers and researchers agree that developing students' problem-solving skills is one of the primary goals of mathematics education (Lester, 2013). In Quebec, the learning program targets the development of three disciplinary competencies in mathematics. Two of them are mainly evaluated through the resolution of word problems. This formal evaluation places students before so-called complex situations (Gamo, 2007), i.e., presented in texts that are often quite long and elaborate, in ordinary language. Faced with such tasks, given the scope of the text and the written information to be processed, many students find themselves disorganized and, eventually, give up (Mary & Theis, 2007).

According to Montague and her colleagues (2014), one of the main difficulties associated with solving problems in written language is the inability to mentally represent the problem. According to Roegiers (1998), this inability is due to the fact that problems are in written language as well as the arithmetic concepts that are involved. How can we help students develop this competence, without having to overcome these obstacles? With this study, we wanted to explore whether mathematical logic problems, without arithmetic concepts and gradually increasing the level of complexity of written language, are environments that could solicit the components of the competence to construct a mental representation of a problem.

Various approaches to help students solve word problems and construct a representation of them have been explored, from the teaching of strategies to the construction of didactically rich environments aimed at developing students' potential. To the best of our knowledge, no study has looked at the potential of logic problems in the construction of the mental representation of a mathematical

problem. To explore this perspective, the concept of mental representation of a mathematical problem is theoretically explicated, which will lead to define what a problem-solving environment is and what mathematical logic is. Then, the goal of the research will lead to the methodology and the qualitative analysis of the results through an illustrative case.

Theoretical framework

Problem solving is a process that has been studied by several authors. They have sought to model its process (Hanin & Van Nieuwenhoven, 2016; Julio, 1995; Lester, 2013). All identify essentially the same phases that are not organized sequentially, but rather in a circular mode. For the present study, we will focus on the beginning of this process, i.e., the construction of the problem representation. We refer mainly to the work of Julio (1995, 2002), since he has produced a very well-documented synthesis of the work in this field, still in use today.

The representation that allows the problem to be understood is built up gradually from the information that is extracted. The first phase is the interpretation and selection of data. The individual must first interpret the semantic context of the problem to identify the object and the task. It is also a question of identifying the information relevant to the task. The second phase is structuration, and it concerns the way in which the initial representation is organized and how this structure is eventually questioned and adjusted. The individual builds a schema of the problem. It is a way to visualize the mathematical structure of the problem. This schema can be traced or mentally built. Finally, the third phase is operationalization. It is the process of getting started, either in gestures or mentally (Julio, 1995). In this study, the construction of the representation is considered as a competence, a "know how", and the phases are components of this competence. Difficulties or dysfunctions can be observed with each of these components.

Since the publication of *How to Solve it!* (Polya, 1957), many have dissected the strategies to be developed when solving problems (Montague et al., 2014; Xin et al., 2005; Zheng et al., 2013). Some have proposed to transform the student's relationship with errors in problem solving (Bisaillon, 2005; Sensevy, 1998). Many others have reflected on ways to smooth out the difficulties of reading the proposed texts in such situations (Voyer & Goulet, 2013). Finally, still others have thought about environments conducive to the development of useful problem-solving competence (Bisaillon, 2005; Julio, 1995). The present study is situated in this field.

According to Julio (1995, 2002), a problem-solving environment can be described in terms of mathematical content, didactic process, or cognitive functioning. The environment that most encourages the development of the competence of constructing a problem representation is a conditional environment. It concerns "the elements that have a helping role and that intervene only when certain conditions are met" (Julio, 1995, p. 149). It is hard to create because it is conditional on the student's mathematical progress. The precise characteristics of this type of environment are not yet documented and may never be given the complexity of such a task. However, Julio (1995), suggests that each situation should contain elements likely to stimulate the representational activity. In addition, it is important to allow students to accumulate various experiences. The proposed situations should be complex and allow students to experience success.

Environments offering mathematical logic problems seem to be consistent with these characteristics. Mathematical logic is a branch of mathematics that studies the formulation of valid mathematical reasoning (Baruk, 1992). Our hypothesis is that mathematical logic would enable the presentation of problems in a rich problem-solving context, while reducing the impact of written language and arithmetic concepts in the construction of the mental representation of the problem.

Research problem and goals

The goal of this paper is to explain a phenomenon, to understand it or to make sense of it (Thouin, 2014). Indeed, the main goal of the study is to explore how environments proposing mathematical logic problems, first without written language and then integrating short sentences, enable the mobilization of components of the competence to construct a representation of a mathematical problem in 4th grade students. In addition, there are no arithmetic concepts in the problems proposed by these environments. The originality of this project is the use of logical problems not in terms of the development of mathematical reasoning, but in terms of their potential for developing the ability to construct a representation of a problem.

Method

To meet the objective, a multi-case study was chosen. This methodology makes it possible to observe several factors that come into play during the realization of tasks to better understand their complexity (Mucchielli, 1996). Thus, it is possible to study the manifestations of the different components of the competence to construct a representation through the students' actions.

Participants

Six 4th graders were selected to participate in this study. The 4th grade class allows for a better exploration of the impact of reading and statement complexity, given the intermediate position occupied by these students in the elementary school academic pathway. We met with two students considered "slow" in mathematics, two "average", and two "fast", as judged by their teachers. In this paper, we use an illustrative case that emerged from data collected.

The mathematical environments

An analysis of the manifestation of the components of the construction of the representation of a mental mathematical problem in two environments has been carried out. The first (E1) is an existing environment developed by Lyons and Sabinin (2015). The second (E2) was adapted from existing problems created by Lyons and Lyons (1999). We have transformed some of the existing problems so that they better meet our research objectives. For example, we've adapted the presentation of the problems, and we've adjusted some of the sentences. The two environments are considered conditional environments (Julo, 1995), since they can offer different levels of complexity depending on the student's progression. All environments were presented on the interactive digital whiteboard. Students work in pairs. For the first environment (E1), students had the nine puzzle pieces and a grid to solve the task and for the second one (E2), they had a laminated grid and an erasable ink pen.

Description of the analysis

A preliminary analysis of each of the environments was carried out regarding their potential to solicit

or facilitate each of the components of the competence. An analysis of six students' activity was conducted for each of the environments. For this analysis, videos of the student interviews were reviewed with the transcript. The students' actions were analyzed using a code grid. Some codes were chosen beforehand based on the components associated with the construction of the representation: interpretation, selection, structuration, operationalization. During the analysis, the interpretation code was separated into two to distinguish between interpretations that showed understanding of the clue and interpretations that showed that the student knew where the clue went in the grid. The code "back-and-forth," when the student returned to certain clues, and the code "validation," when the student was checking the accuracy of the clues, were added. A color has been associated with each of the codes. The analysis of the illustrative case resolution process through the mobilization of the components of the competence to represent a problem, in relation to one clue is presented for the two environments. In the following paragraphs, we first present the analysis of the environments before the experimentation followed by the analysis of the student's mathematical activity.

Logic puzzles (E1)

The goal of this logic puzzle is to place all nine geometric forms in the grid, respecting all the visual clues. Each clue gives information about the piece and then about where it goes. In the example of Figure 1, the first clue indicates that a blue form goes to the top left box of the grid. The information needed to solve the whole puzzle is strictly visual. The hypothesis was that this could allow the student to learn what it means to select, interpret, structure, and operationalize data, without interpreting a semantic context and understand any arithmetic concept. This seemed particularly true in relation to the interpretation of data. Students can learn to "read" data and interpret it without the heaviness of ordinary language. In fact, not all the clues are "direct" clues. For example, some indicate where a form does not go, as the fifth clue in Figure 1 and others say that the piece could go to different places, as the second clue in Figure 1. As far as structuration is concerned, the hypothesis was since the logical environment already proposes a structure, the grid, the student would not really need to structure the data himself. Finally, the visual and logical environment should help the student get into action quickly, it should be seen as a game. Operationalization would therefore be facilitated.

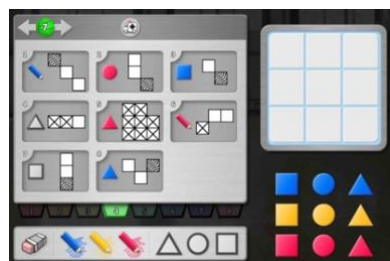


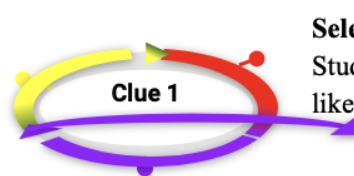
Figure 1: Example of a logic problem in the first environment

Figure 2 shows how one student activated some of the components of the problem representation competence to interpret the first clue. Each arrow represents one or two components. A gradation was used when it was difficult to identify when one component began and the other ended. The verbatim that allowed the interpretation of his mathematical activity is written next to the arrow as an example. First, it is possible to see that this student uses several components almost simultaneously. He first interprets (yellow arrow [comprehension] and green arrow [localization]), but while continuing his

reflection he selects the three blue forms and places them immediately in the grid (red arrow [selection] that turns into purple [operationalization]). It is also possible to observe that he quickly gets into action and continues his momentum towards clue two, without stopping (purple arrows that goes to the left [operationalization]). We could add that he uses the components strategically. He processes the entire clue by keeping track of all the possibilities (he places all three blue pieces in the grid) and considers the clues one after the other.

Interpretation (comprehension and localization)

Student: So we see that it's like we have to make a diagonal, but we can't put them here, we can just put here and it says a blue color.



Selection

Student: So, me, I would just do like one of each blue shape..

Operationalization

(Place the three blue pieces on the top left corner).



Figure 2: Example of the mobilization of the components for clue 1

Our analysis has shown that he mobilized the components throughout the whole problem-solving process and that he does not have a linear resolution process. We observe that he moves fluently from one clue to another and that several components are called upon for a single clue. It was also possible to note that he validated his answers on numerous occasions and that he went back and forth a lot. These are essential problem-solving strategies. For example, from clue 4, he returns to clue 2 to place a form; its operationalization is resulting in a return to the previous clue. It was also possible to observe that he had to structure his representation to know how to place his pieces, contrary to what was assumed in our initial analysis of the environment. Indeed, we had thought that, given the presence of the grid, students would have little opportunity to construct their representation of the problem.

Logic grids (E2)

The goal is to place nine letters (A to I) in the grid, respecting all the clues. This is the same type of problem as logic puzzles, except that this environment uses simple written language. Each clue provides information about the location of a letter or a related position of a letter to another one. Figure 3 shows an example of this type of problem. We made the hypothesis that presenting the clues through simple written sentences would make the construction of the representation more complex, especially regarding interpretation and selection of the data. A visual glossary is provided to help students visualize what "next to" means, for example. This should lessen the effect of the written language. The hypothesis regarding structuration and operationalization were the same as for E1.

- A is not above B.
- B touches E.
- C is on the bottom, not in a corner.
- D doesn't touch B.
- E is next to C on the left.
- F is in a corner, on the right.
- G touches A.
- H is at the bottom.
- I is between H and a corner on the left.

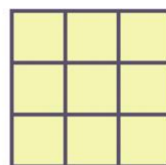


Figure 3: Example of a logic problem in the second environment

Figure 4 shows how the student processed the clue “I”. Small grids have been added to the Figure 4 to make the student's speech more explicit. Again, the student mobilizes several components of the competence almost simultaneously. He interprets the clue; he tries to understand it (yellow part of the arrow) and interpret its location (green part of the arrow). He places the "I" in the grid, but reinterprets the clue (blue arrow turns green). He returns to the data when he encounters a block (brown arrow [back-and-forth]).

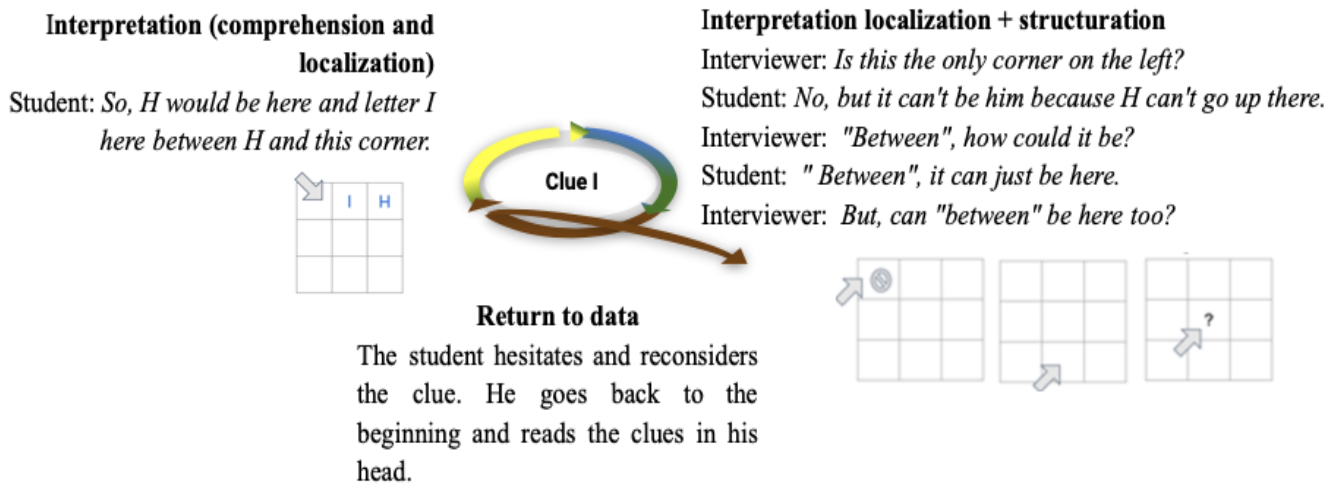


Figure 4: Example of the mobilization of the components for clue “I”

Here again, we were able to observe that the student mobilizes the components of the competence of constructing a representation of a problem almost simultaneously for each clue. The back-and-forth strategy and the validation are also present. Again, the student solves the entire problem in a non-linear way. In this environment, students had a little more difficulty interpreting the data than in the previous environment because of the vocabulary (for example "between" or "next to").

Discussion

Our analysis revealed that these two environments effectively engaged all the components of the competence, often in a nearly simultaneous manner. Moreover, they appeared to provide students with opportunities to strategically employ these components. As demonstrated by the illustrated case, the student interpreted clues, maintained a structured approach, utilized back-and-forth strategies, and validated their answers. In this way, these environments seemed to address some of the difficulties associated with constructing a mental representation of a problem (Julo, 1995). Notably, he demonstrated the ability to interpret and select relevant data, and when faced with difficulties, he returned to the provided clues and validated their solutions. Furthermore, he exhibited a consistent tendency to restructure his problem-solving approach, exemplified by his use of the back-and-forth strategy. Finally, he engaged quickly and actively (operationalization).

The analysis also highlighted that the student successfully carried out the entire problem-solving process. These environments could therefore enable students to learn to solve problems completely, something that some students with difficulties, should we recall, are not always able to do (Mary & Theis, 2007).

There were no significant differences observed among the six students in E1 and E2. This consistency across students and environments suggests that the initial training offered in E1 was effectively transferred to E2, emphasizing the value of incremental learning. This finding is in line with Rochelau (2009) recommendations for designing a sequence of activities. Indeed, the progression from one environment to the next should allow students to reinvest what they had learned in the previous environment while providing an additional challenge.

Conclusion

This study is part of a larger project to better understand how to support the development of students' word problem-solving competence. It focuses on a specific goal to explore how environments proposing mathematical logic problems, without arithmetic concepts and first without written language and then integrating short sentences, enable the mobilization of components of the competence to construct a mental representation of a mathematical problem in primary school pupils.

The analysis of two logical environments indicates that students engage all the components of constructing a representation of a mathematical problem. This analysis is important because it allows us to study the mathematical activity of students solving logic problems with a framework derived from problem solving to better understand how these environments support problem solving activity.

These findings emphasize the importance of gradually introducing challenges associated with mathematics concepts and written language to support students in constructing effective mental problem representations. Furthermore, the consistency observed across students in E1 and E2 suggests that the knowledge acquired in one environment can be successfully applied in subsequent ones, highlighting the benefits of incremental learning. To deepen our understanding, future research will look at the manifestation of each component in other environments integrating more and more language and possibly arithmetic concepts. The transfer of knowledge between them will be studied.

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