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Design of a semi-structured interview to capture flexibility in mental calculation

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Among educators and researchers there is a wide consensus that flexibility in mental calculation is a key topic of elementary mathematics education. Furthermore, the research interest in students' individual competencies of flexibility in mental calculation increased in the last decades. Our study focuses on flexibility in mental calculation in elementary students and examines the connection between the cognitive elements which sustain the solution process and the solution tools. Therefore, we developed a semi-structured guided interview and conducted 155 interviews with German third graders. In this paper, we focus on a detailed description of our approach for data collection and the coding of the interviews. Finally, we report descriptive results of the first 20 double-coded interviews of the main study. These results reveal that students justify about 60 percent of the addition and subtraction problems by problem characteristics, number patterns, and relationships.

Keywords: Flexibility in mental calculation, addition and subtraction, number patterns, problem characteristics, elementary arithmetic.

Introduction

There is a broad agreement on the important role of fostering individual competencies in flexibility in mental calculation in elementary school students. This is shown and emphasized by many researchers (e.g., Green & Rathgeb-Schnierer, 2020; Hickendorff, 2022; Torbeyns & Verschaffel, 2016), but also by national curricula (for an overview see e.g., Torbeyns & Verschaffel, 2016). The important role of flexibility in mental calculation is also shown by the high number of studies in this field. For instance, different studies investigated the practical flexibility of students after they learned the standard written algorithm. They report that children often solve multi-digit addition and subtraction problems with the standard algorithm (e.g., Hickendorff, 2022; Selter, 2001; Torbeyns et al., 2017) and do not refer to number patterns and relationships as well as specific problem features. In the last years, some studies distinguished practical and potential flexibility. These studies show that the students know more and different strategies as they exhibit when solving a problem (for a current overview of this research area see e.g., Hickendorff, 2022).

In summary, the presented studies have shown that students do not use their full potential of flexibility skills in practical applications (in traditional lessons) – especially after learning the standard algorithm. There are also studies which show that it is possible to support and foster students' flexibility in mental calculation by appropriate instructional approaches. At the beginning of the millennium Schütte (2004) emphasized the importance of the approach “*Zahlenblickschulung*” (p. 142; see also Rechtsteiner & Rathgeb-Schnierer, 2017) for the development of perceiving and using number patterns and relationships as well as flexibility in mental calculation. The studies of Rathgeb-Schnierer (2010) as well as Rechtsteiner and Rathgeb-Schnierer (2017) integrated elements

of the *Zahlenblickschulung* in the mathematic classes. Both studies showed that the students' flexibility in mental calculation was supported and fostered by this approach. There are also different further approaches, which can promote flexibility in mental calculation (for a wide overview of the studies on this topic look for example at Green & Rathgeb-Schnierer (2020) and Heinze et al. (2020)). Overall, it is possible to support the development of flexibility in mental calculation with appropriate teaching approaches.

Theoretical framework

There is not only a broad body of research regarding flexibility in mental calculation but also various terms and definitions. Thus, the definition of *flexibility in mental calculation* varies between different studies. However, there is a consensus that flexibility in mental calculation can be understood “as an appropriate way of acting when faced with a problem based on a repertoire of available strategies, which is to say that flexible strategies are adapted dynamically to problem situations” (Green & Rathgeb-Schnierer, 2020, p. 3). According to Rechtsteiner & Rathgeb-Schnierer (2017), two types of operationalizations of flexibility in mental calculation can be distinguished. On one hand, it is possible to define on a normative level which solution tools are (not) fitting to problem characteristics, number patterns, and number relationships. In this vein, the correct and fast solving of different problems are often additional indicators for flexibility in mental calculation. But in the last years there started a discussion of a rigid version of this approach (e.g., Torbeyns & Verschaffel, 2016), also in the context of the difference between potential and practical flexibility (Hickendorff, 2022). On the other hand, there is an approach for operationalizing flexibility in mental calculation by *cognitive elements that sustain the solution process* (e.g., Rathgeb-Schnierer & Green, 2013). This approach emphasizes the recognition of number patterns and problem characteristics in the process of solving a problem: "Only if the tools of solution are linked in a dynamic way to problem characteristics, number patterns, and relationships would we consider as evidence of flexibility in mental calculation" (Rathgeb-Schnierer & Green, 2013, p. 357). In this paper, we define flexibility in mental calculation according to these authors. In this sense, flexibility in mental calculation is manifested in the combination of perceived problem characteristics, number patterns, and relationships of a given problem and the use of several solution tools. The process of solving a computational problem can be illustrated by the model “domains of calculation process” (Figure 1) that includes three areas inherent to the process: *types of calculation, cognitive elements, and solution tools*. Furthermore, this model underlies our definition of flexibility in mental calculation.

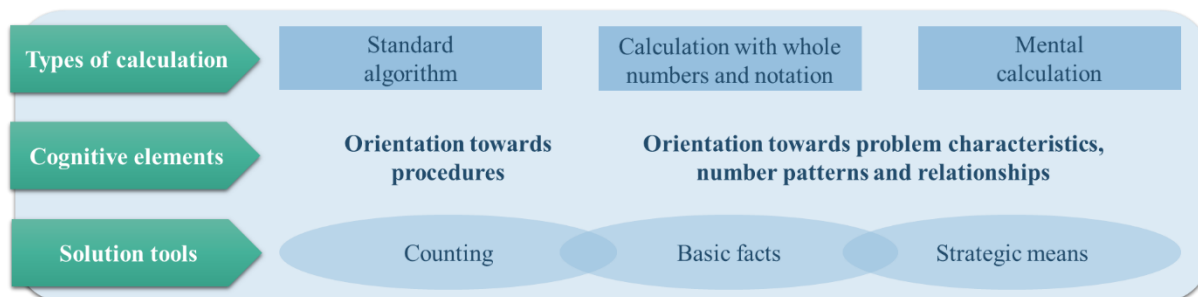


Figure 1: Domains of calculation process (own representation based on Rathgeb-Schnierer & Green, 2013, p. 353)

The domain *types of calculation* comprise three general types such as *standard algorithm*, *calculation with whole numbers and notation*, or *mental calculation* which can be directly observed during the process. The domain *cognitive elements* addresses “specific mental actions that sustain a solution process [...]. These can be learned *procedures* (such as computing algorithms) or recognition of *number characteristics* (such as number patterns and relations)” (Rathgeb-Schnierer & Green, 2019, p. 5, emphasis in original). This domain cannot be observed directly. The result of a calculation problem is determined at the level of the solution tools. In the solution process different tools can be used and combined, such as *counting*, using *basic facts* (e.g., the addition up to 20), and *strategic means*. Strategic means can rather be understood as flexible combinable tools of a toolbox than as a holistic strategy. Rathgeb-Schnierer & Green (2019) describe two main categories of strategic means: decomposing and composing as well as modifying a problem to make it easier. It is only semi-manifestly possible to capture the solution tools (for a detailed description of the domains of calculation look at Rathgeb-Schnierer & Green, 2013; 2019).

As described above, this model underlies our approach to capturing flexibility in mental calculation by revealing the connection between the cognitive elements that sustain the solution process and the solution tools. Although there are studies that focus on the cognitive elements that sustain the solution process while investigating flexibility in mental calculation (e.g., Rathgeb-Schnierer & Green, 2013; Rechtsteiner & Rathgeb-Schnierer, 2017), the instruments of capturing flexibility mental calculation this way are not sufficiently validated. According to this, there is a lack of research that investigates, how to capture the connection between the cognitive elements that sustain the solution process and the solution tools in a semi-structured way. Furthermore, the already published research results only allow limited generalizable conclusions. Our study faces that desideratum and aims to capture flexibility in mental calculation by revealing the connection between the cognitive elements that sustain the solution process and the solution tools in a semi-structured way. In this paper, we focus on two research questions (RQ):

RQ1: How to reveal in a semi-structured interview if students refer to procedures or problem characteristics, number patterns, and relationships?

RQ2: How to identify in a semi-structured interview if students recognize different problem characteristics, number patterns, and relationships and use various solution tools?

To pursue this aim and answer the research questions, we developed and evaluated a semi-structured guided interview. Additionally, further constructs were captured such as different person characteristics, math achievement as well as vocabulary and word-finding competencies. Since we focus on the development and evaluation of the interview, we subsequently report only this part of the study.

Methods

Sample and procedure

The study was designed for capturing flexibility in mental calculation of students in the third grade. Altogether, 215 German third graders from 5 schools and 13 classes took part in at least one part of the study. 155 students out of the whole sample got consent to take part in the videotaped interview.

The sample consists of nearly half female students ($n = 77$) and nearly half male students ($n = 78$). The mean age of all 155 students is 9.53 years ($SD = 0.37$ years), of the 77 girls, it is 9.46 years ($SD = 0.33$ years), and of the 78 boys, it is 9.60 years ($SD = 0.40$ years). The data collection started in May 2022 and ended in September 2022. Before the main study started, we completed two phases of piloting (in July 2021 and May 2022 with 14 and 11 interviews). The pilots targeted the training of both interviewers, the evaluation of the guideline, and the development of a coding manual.

Interview

With our focus on cognitive elements that sustain the solution process, we had to face the challenge that they cannot be directly observed. For valid conclusions, it is necessary to ask the students for their explanations and justifications (e.g., Rathgeb-Schnierer & Green, 2013; Rechtsteiner & Rathgeb-Schnierer, 2017). Therefore, we developed a semi-structured, guided interview. This interview consists of three parts with different foci. The problem selection of the first part is based on Rathgeb-Schnierer and Green (2013). We adjusted the problem selection and arrangement as well as the way of asking several times during and after the pilot runs. Finally, the interview contains 12 problems that are displayed in Figure 2. Each problem includes at least one specific feature that allows to act flexibly when solving the problem. For instance, the problem $31 - 29$ has a small range between numbers, and therefore the difference could be found by adding up. The subtrahend of the problem $46 - 19$ is close to 20, and therefore it would be possible to modify the problem to $47 - 20$ which is easier to solve. Besides the two mentioned features, we also included double-half-relationship ($66 - 33$), crossing the ten boundary ($47 + 28$) or numbers at the ones place adding up to ten ($64 + 36$) (detailed information in Green & Rathgeb-Schnierer 2020).

$56 + 29$	$63 - 25$	$71 - 36$	$31 - 29$
$88 - 34$	$33 + 33$	$94 - 14$	$47 + 28$
$64 + 36$	$46 - 19$	$73 + 26$	$66 - 33$

Figure 2: Overview of the problems

In the first part of the interview, the students are asked to sort the problems in categories “easy” and “hard” and give reasons for their sorting. Concretely, the interviewer asks the students at first for the two easiest problems. After choosing two problems the students were encouraged to tell why these problems are considered easy. Then, the students have the task to find two hard problems. Here, the interviewer also asks for the reasons. After that, all remaining problems are sorted (prompt: “Is this problem rather easy or hard for you?”) and the children articulate their reasons for sorting. The communication of the reason is the important part of this interview task. In this vein, the students can justify their sorting for example via problem characteristics, number patterns as well as relationships, or the way of calculating the problem. Via this approach, we try to capture the individual cognitive elements (e.g., Rathgeb-Schnierer & Green, 2013). For the easy problems, the interviewer asks the students, if the solution to the problem is known. If this is the case, the interviewer asks for the way of solution.

The second part of the interview follows a new design. Here, 5 problems of the first part are taken up again. Every known problem is compared with a new problem. Each pair of problems is characterized by the fact that the numbers are from the same number range. The prompt from the interviewer is: “Which problem is easier for you?” After sorting the problem, the students are asked why the problem is easy or hard. Here, the students can argue via problem characteristics, number patterns as well as relationships, or, for example, via the size of the numbers. Comparing and contrasting can support learning (e.g., Alfieri et al., 2013), but it was never used in an interview setting for capturing flexibility in mental calculation by revealing the cognitive elements that sustain the solution process. The last interview part contains conversations about how the students solve the problems. This question was only asked for problems that were not already solved in the first part. The conversation about the way of solution enables a conclusion about the solution tools. All in all, we capture the connection between the cognitive elements that sustain the solution process and the solution tools in the first and third part of the interview. Furthermore, we integrated a second approach of revealing the cognitive elements via comparing and contrasting problems.

Data analysis

In this paper, we report the first analyses of the videotaped interviews and therefore describe the data analysis in detail. The videotaped interviews are analysed with the method of *Qualitative Content Analysis* by Mayring (2014). The deductive base of the category system is from Rathgeb-Schnierer and Green (2013). The coding manual was further developed, supplemented, and optimized with codes from the coding process of the data from the two pilot studies. Table 1 displays a little part of the detailed manual (with a few examples for sub-codes) which allows coding on independent levels: sorting of problems (easy, medium, hard), correctness of the solution (correct, incorrect), justifying the sorting (e.g., by procedures or problem characteristics, number patterns, and relationships with sub-codes), and used solution tools (e.g., basic facts and strategic means with sub-codes).

Table 1: Excerpt from sub-codes for reasoning by problem characteristics, number patterns, and relationships

Sub-codes (1 st level)	Sub-codes (2 nd level)
Numbers at the ones place	The ones are equal One of the ones is twice the other one The sum is less than ten
Number at the tens place	The tens are equal
Feature of one number	The second number is close to the next ten
Relationship of the numbers	Distance between the two numbers

The interviews are coded by event sampling. The coding manual includes sub-codes, descriptions and anchor items for each code. The procedure of coding is presented below using two interviews. At first, a (translated) excerpt from the first interview part which refers to the problem 31 – 29:

Interviewer: Is this minus problem rather easy or rather difficult?
 Student: Rather easy.

Interviewer: Why rather easy?

Student: Because the two numbers are very close together and [...] if you calculate 29 plus 2, then you have [...] the result 2.

The student sorted the subtraction problem to the category easy (code: *easy*) and justified the sorting by the distance between the two numbers – the subtrahend 29 and the minuend 31 are very close together (code: *problem characteristics, number patterns, and relationships*; sub-code (1st): *relationship of the numbers*; sub-code (2nd): *distance between the two numbers*). Further, the connection between the perceived relationship of the numbers and the use of an appropriate solution tool (code: *indirect addition*) can be seen. Besides, the student got a correct result (code: *correct*). The next excerpt is from another student and the second interview part. It is about comparing and contrasting the subtraction problems 31 – 29 and 34 – 26:

Student: The (points to the number 29) is closer to the (points to the number 31) and therefore the problem is easier [...] than that problem (points to the problem 34 – 26).

Here the student stated that the problem 31 – 29 is easier than the problem 34 – 26 (code: *the first problem is easier*) because the subtrahend 29 and the minuend 31 are very close together (code: *problem characteristics number patterns, and relationships of both problems*; sub-code (1st): *relationship of the numbers of both problems*; sub-code (2nd): *distance between the two numbers of both problems*). The videos are coded independently by two trained university members (the first author of this paper and another collaborator). Up to now, the first 20 interviews were double-coded, and we can report the value of Cohen's kappa for each coding level (based on the conscious thresholds, e.g., McHugh, 2012). For the first and third interview parts: problem sorting: $\kappa = .93$ (almost perfect), reasoning: $\kappa = .79$ (moderate), solution tools: $\kappa = .69$ (moderate), correctness of solution: $\kappa = .80$ (strong). And for the second interview part: problem sorting: $\kappa = .99$ (almost perfect), reasoning: $\kappa = .60$ (moderate)¹. With the coded data, various analyses are possible such as the variety of mentioned problem characteristics number patterns, and relationships, the variety of utilized solution tools, and the connection of mentioned characteristics and utilized solution tools. Thereby, it is possible to focus on all problems or only on correctly solved problems.

First results

Currently, 20 of 155 interviews are double-coded. Table 2 reports some descriptive results. For each category, the mean and standard deviation is indicated. In the rows, where the reasoning by (different) problem characteristics, number patterns, and relationships (in the table named "reasoning by problem characteristics"), as well as the solution tools, are given, only one reason respectively solution tool per problem is considered. Furthermore, also incorrectly solved problems are included in these results. Regarding the variety, we capture how many different problem characteristics, number patterns, and relationships were mentioned, respectively how many solution tools were named. In the first part of the interview, students justified their sorting of an average of 7.25 out of 12 problems by problem characteristics, number patterns, and relationships (60.4%), and 3.60 different characteristics were mentioned. In comparison, 2.85 of the 5 pairs of problems of the second

¹ In the second part we coded also to which problem each reason of the child referred (to the left or the right or both problems). This approach caused a lower interrater reliability in the first interviews.

interview part were reasoned by problem characteristics, number patterns, and relationships (57.0%), and 1.85 different characteristics were named on average. Further, on average 1.85 of the 12 problems from interview part one were solved incorrectly, and 2.75 different solution tools were named.

Table 2: Descriptive results of the first 20 interviews

Categories	Mean (<i>M</i>)	Standard deviation (<i>SD</i>)
Correctness of solution (part 1; 12 problems)	10.15	1.90
Reasoning by problem characteristics (part 1; 12 problems)	7.25	3.60
Variety of reasoning by problem characteristics (part 1; 12 problems)	3.60	1.60
Variety of solution tools (part 1; 12 problems)	2.75	1.07
Reasoning by problem characteristics (part 2; 5 problems)	2.85	1.81
Variety of reasoning by problem characteristics (part 2; 5 problems)	1.85	1.27

Discussion and outlook

Our first results suggest that our approach of sorting and reasoning in a semi-structured interview is promising to reveal if students refer to procedures or problem characteristics, number patterns, and relationships when they are confronted with an addition or subtraction problem (RQ1). Additionally, our interview method allows identifying if students recognize different problem characteristics, number patterns, and relationships and use various solution tools (RQ2). But in this paper, we reported just the first descriptive analyses based on the first 20 double-coded interviews. After all videos will be coded, more comprehensive analyses will be possible. For example, analysing the correlations between the correctness of solution, the reasons for sorting, and the solution tools. Furthermore, there will be the option to compare students' reasons during the first and the second interview part. In addition, our data allows us to analyse if the cognitive elements that sustain the solution process match the solution tools. Beyond, further constructs such as math competencies, vocabulary, and word-finding competencies as well as different individual characteristics (math interest, math self-concept, and need for cognition) were collected. This allows us to investigate the connection between these constructs and the competencies in flexibility in mental calculation. Finally, in addition to the various quantitative analyses, it is possible to look qualitatively at the competencies in flexibility in mental calculation (following e.g., Rechtsteiner & Rathgeb-Schnierer, 2017).

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