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An introduction to TWG2: Arithmetic and number systems

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Introduction

Working Group 2 focuses on discussing theoretical and empirical research on the teaching and learning of arithmetic and number systems. Our work has aimed to acquire and enhance knowledge about students' understanding and meaningful learning in this content area regarding different ages and achievement levels. The scope of the TWG comprises kindergarten to 12th grade and emphasizes research-based specifications of domain specific frameworks, concepts and goals, analysis of learning processes and learning outcomes in different classroom cultures, as well as innovative teaching and diagnostic approaches that attend to both procedural and conceptual knowledge.

Learning arithmetic and number systems from kindergarten and throughout school is a very broad field with a great variety of themes and research approaches. Research in arithmetic and number systems focusses not only on the mathematical content, but also on models for teaching and learning, approaches for heterogeneous and inclusive classrooms, analogue and digital tools to support understanding, and teachers' competencies and cultural practices. As for previous years, this was clearly evident in the contributions for our TWG. The group intensively discussed the contributions in the plenary whole group as well as in small groups, and 20 papers and one poster were accepted for the proceedings. The key themes were conceptual understanding of natural numbers and their operations (six papers), teacher's practices on teaching number systems (two papers), conceptual understanding of fractions and their operations (five papers), problem solving and problem posing (four papers and one poster), and assessment tools (three papers).

Papers and poster

Conceptual understanding of natural numbers and their operations

Timo Flückiger and Elisabeth Rathgeb-Schnierer, investigate the flexibility in mental calculation in primary school students, examining the relationship between the cognitive elements that support the solution process and the solution tools. Preliminary results showed that students justify around 60 percent of their addition and subtraction solutions based on problem characteristics, numerical patterns, and relationships.

Lóa Björk Jóelsdóttir, Pernille Bødtker Sunde and Paul Andrews investigate the influence of gender and age on students' strategies for solving addition tasks designed to elicit number-based strategies. The results showed an increase in the use of the standard algorithm as education levels advance and that girls favoured the standard algorithm, while boys opted for number-based strategies.

Using eye-tracking, *Jingyi Lai, Lukas Baumanns, Anna Lisa Simon, Achim J. Lilienthal and Maike Schindler* investigate 163 grade 5 students' strategies in multiplication tasks with array representations. Overall, the study found students used three different strategies. In two strategies students used the rectangular arrangement indicating multiplicative understanding. In one strategy, used in a fifth of the tasks, students counted all dots, indicating lack of understanding of multiplication in array representations.

In an interview study, *Robert Gunnarsson and Catherine Englund Eriksson* investigate understanding of associativity in students of the age 8 to 12. Based on the interview-data from sixteen students, they qualitatively reconstructed three different types of understanding: result-focused associativity, spatial associativity, and temporal associativity. The results suggest the usefulness of associativity as a tool for arithmetic depends on the type of understanding.

Ioannis Papadopoulos addresses sixth-grade students' use of unnecessary brackets in solving arithmetic expressions. Data were collected using a paper-pencil test which includes four different tasks requiring grouping processes. The analysis of 103 worksheets revealed different categories of using unnecessary brackets and the frequency of usage. The identified categories are described and visualized by an example of a students' solution.

Myriam Burtscher and Michael Gaidoschik report on a longitudinal study investigating 11 children's development of mental models of division from second to fourth grade. The study suggests that many children had problems linking their interpretations of division when using material (wooden cubes), which very often corresponded to measuring, to viable prototypes of everyday situations in the form of word problems.

Teacher's practices on teaching number systems

Mónica Arnal-Palacián and Nuria Begué examine the work of preservice teachers as they create a teaching proposal to enhance the comprehension of the base-10 number system. This served as a means for the participants to exhibit their solid understanding of the number system (although there were some challenges with zero's place value) and using the students' answers to improve their proposal.

Elvira Fernández-Ahumada, Natividad Adamuz-Povedano, Jesús Montejo-Gámez and Enrique Martínez-Jiménez, investigate a teachers' practice in relation to number sense. The teaching methods of an experienced teacher were analyzed, focusing mainly on her approach to number sense. The analysis reveals a comprehensive instruction of the teacher, as she addresses several components of number sense in each session using well-structured tasks and complementary resources.

Conceptual understanding of fractions and their operations

Renata Carvalho highlights the importance of establishing connections across various representations about fractions. In her study with prospective primary school teachers, she examined the kinds of fraction representations they used highlighting the diversity in the combined use of symbolic, visual and verbal representations as an indication of the participants' conceptual understanding of fractions.

Maria T. Sanz, Carlos Valenzuela, Emilia López-Iñesta, and Guillermo M. Luengo, explore the impact of an academic music intervention on seventh graders' understanding of musical notation and the representations of fractions. The results showed that, after the intervention, the students improved the use of the discrete model in representing fractions and were able to relate this to musical symbols.

Leonie Johann and Marja van den Heuvel-Panhuizen utilize video documentation to investigate the use of metaphors by primary school students, to gain insight on how certain notions (e.g., fraction) are collectively conceptualized by the students. The paper presented both, the spoken and non-verbal communication which correspond to three conceptual metaphors.

Sofia Graça, João Pedro da Ponte and António Guerreiro investigate grade 5 students' understanding of fraction division when participating in a teaching experiment based on the use of models. Analysing four students' work on classroom tasks, individual tests and interviews, the authors found, that by the end of the teaching experiment, the students demonstrated conceptual understanding of fraction division, associating the common denominator algorithm with the modelling process, and demonstrating operation sense.

Sinem Sozen-Ozdogan, Didem Akyuz, and Michelle Stephan investigate grade 7 students' taken-as-shared ideas on transitions from ratio table tool to verbal and written symbolic representations of ratio and proportion concepts. From analyzing observations of six lessons, activity sheets, and teacher-researcher interviews they found indications of a transition to the ratio and proportion verbal and symbolic representation after intensive use of the ratio table.

Problem solving and problem posing

Raúl Fernández-Cobos, Irene Polo-Blanco and Juncal Goñi-Cervera examine multiplicative problem-solving strategies of seventeen students from first to fourth grade with autism spectrum disorder. Therefore, a paper-pencil test was used, which incorporates different problem types. The results revealed that students predominately exhibit incorrect or low-level strategies. Furthermore, they suggested that measurement-division problems were challenging for the students.

Ioannis Papadopoulos and Nafsika Patsiala present a one-year intervention on problem posing involving primary school students and its impact on the students' problem-posing skills and the quality of the problems posed. After the intervention, students began to adopt more powerful strategies, improving their ability to perceive the problem structures and improving the quality of problems.

Nathalie Bisailon and Michel Lyons, hypothesise that exposing students to mathematical environments, initially devoid of written language and arithmetic concepts, but based on logic, can favour the development of their problem representation skills. The results showed that these

environments facilitate the use of different components of problem representation competence, often in a combined and strategic way.

Jørgen Sjaastad discusses the contribution of rich tasks (a type of inquire-based tasks) to the problem-solving (start-up and adjusted) strategies used by very young students. Furthermore, the paper provided evidence on how dealing with rich tasks might contribute to the enhancement of the students' number sense.

In a poster *Sofia Peregrina Fuentes, Carlos Valenzuela García, Maria T. Sanz and Emilia López-Iñesta* explore the effect of different illustrations on word problem solving involving fractions. In a qualitative study, 42 Mexican 14- and 15-years old students answered a fraction task with or without different types of illustrations. In general, the presence of illustrations supported the students in solving the fraction task.

Assessment tools

Laura Graewert, Daniel Thurm, and Bärbel Barzel contribute to the scarcity of research on the impact of automated feedback within digital assessment environments on students' learning outcomes. They presented a relevant tool and explored its influence on both the students' understanding of multiplication and the activation of metacognition.

The work of *Leonie Brumm and Elisabeth Rathgeb-Schnierer* addresses the lack of standardized tests that reliably and validly measures the accuracy of numerosity estimation, a skill that is important for everyday activity as well as development of mathematical skills. They presented the development of a digital test which includes seven different types of tasks in three number ranges. Construct validity and reliability was investigated in a sample of 196 German grade 3 students from thirteen classes.

In their paper, *Elisabeth Rathgeb-Schnierer and Silke Friedrich* delve into the concept of differentiation within the context of mathematics education in heterogeneous classrooms. To achieve this, they used an open-ended task affording solutions at different levels of difficulty, grouping together students who worked at the same processing level. They also bring forward the link between proficiency on mathematics and problem-solving approach.

Summary

The diversity in theoretical approaches and measurements across a large variety of topics we experienced this year, highlights the role of number system and arithmetic in the teaching and learning of mathematics. Even though the discussed projects varied a lot, they were characterized by common ideas and goals such as understanding students' thinking and reasoning, improving the teaching-learning processes, and supporting students to develop a deep understanding of arithmetic concepts.