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Arguing from different perspectives on quantities and change

Luisa Gunia¹ and Christine Knipping¹

¹University of Bremen, Germany; gunia@uni-bremen.de

Keywords: Argumentation, functions, calculus, change, differential equations.

Motivation and aim of the research project

Functions and functional relationships are central elements of mathematics teaching and learning. Describing quantities and change in different contexts is a promising basis for working with functions. In school, change is often considered as a function of time. However, change can also be thought of as a function of a respective quantity. The aim of this research project is to take into account different possibilities of representing functional dependencies in the context of change in order to enable the development of a flexible concept of change. In this context, argumentation is seen as a format that is particularly conducive to learning and therefore a desirable form of communication in the classroom. For the related tasks, a qualitative-graphical approach is chosen in order to foster conceptual reasoning that goes beyond a mere application of procedures and computational routines.

Theoretical framework: Qualitative calculus and argumentative learning

Qualitative calculus as a means to foster conceptual reasoning

In mathematics education, a qualitative-graphical approach has been suggested for the teaching and learning of calculus, with the intent to foster central concepts and conceptual reasoning instead of calculations (e.g., Hußmann, 2010). In this approach, which has been labelled as qualitative calculus, concepts and situations are mainly represented graphically, verbally and according to their quality. Without an emphasis on computational routines, such tasks are meant to stimulate conceptual and imaginative reasoning, to support the development of content-related ideas and thus to encourage a deeper understanding of basic concepts and notions of calculus (Hußmann, 2010, p. 4). This way, a concept-oriented perspective on calculus is intended, laying the foundation for meaningful learning.

Argumentation in a learning-theoretical dimension

Argumentation and reasoning are fundamental components of mathematics teaching and learning and are often named as central learning goals. In the last decades, the importance of argumentation not only as a learning goal but also for the learning process itself has repeatedly been emphasised (e.g., Knipping & Reid, 2019, p. 28). The associated assumption that social and communicative processes not only shape learning processes, but even constitute them, has gained increasing acceptance over the last years and led to a growing scientific interest in argumentation-theoretical approaches in mathematics education research (e.g., Knipping & Reid, 2019, p. 28). Following this line of research, reconstructing argumentation processes, as a method of analysis, allows us to get deeper insights into the processes by which students establish relationships between already existing knowledge elements in order to come to new concepts. In this sense, combining argumentation analyses with an epistemological view (e.g., Tabach et al., 2014), allows deeper insights into students' processes of learning and understanding concepts.

Method and Materials: The learning environment

The study is carried out with 16-year-old students in different schools in Bremen (Germany). During five 90-minute lessons, the students work in groups on various tasks, and the lessons and group works are filmed. The students' work is analysed by argumentation analyses with an epistemological focus.

The learning environment is divided into three sections, each focusing on a different perspective on quantities and change. In the first perspective, the focus is on the development of a quantity over time ('perspective of quantity'). In the second perspective, statements about the change of a given quantity are the starting point for the students' investigation ('perspective of change'). The third perspective focuses on the interdependence of a quantity and its instantaneous rate of change (as a function of quantity) ('differential equation perspective'). Figure 1 illustrates these perspectives for one example.

Starting from different perspectives, the students have to find the correct filling graph. While the first perspective starts from a real-life context and requires the interpretation of a filling graph in terms of a given situation, the second perspective starts from information about a filling rate as a function of time. While the first two perspectives establish well-known functional relationships between quantity, time and change, the third one focuses a completely new relationship between a quantity and its instantaneous rate of change. Change has to be interpreted here as a function of quantity (not time).

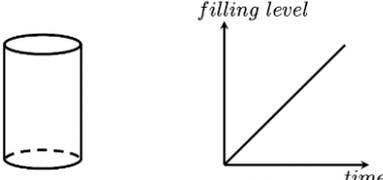
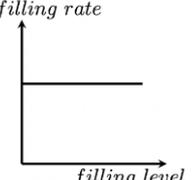
	<p>"The filling level changes at a constant rate."</p>	 <p>"The instantaneous filling rate is the same for every filling level."</p>
<p>"Perspective of quantity"</p>	<p>"Perspective of change"</p>	<p>"Differential equation perspective"</p>
<p>Q: Which is the correct filling graph?</p>	<p>Q: How does the corresponding filling graph look like?</p>	

Figure 1: Different perspectives on quantities and change

All tasks provide an opportunity for mathematical argumentation and require a constant transition between arguing in a real-life context and arguing with a graphical or verbal representation of the given situation. Mathematical argumentations require here different bases, which allow conceptual reasoning and support the learning of central concepts of calculus in a meaningful way. What this means for the conceptual learning of students is the subject of the presented research project.

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