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Teachers' orientations of noticing and its underlying mechanisms in the context of Lakatos-style proving activity

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Despite the educational benefits of Lakatos-style proving activity in school mathematics, there is still limited understanding of teachers' perception and cognition in this context. To gain insights into teachers' underlying decision-making processes, we identified teachers' orientations of noticing of students' thinking in Lakatos-style proving activity and tried to understand their orientations using their framing of professional obligations. Data was collected from semi-structured, vignette-based interviews with twelve Chinese pre-service and in-service secondary mathematics teachers. Findings show that teachers can be classified into different groups based on the extent of consistency or balance in their orientations of noticing. Three potential mechanisms for how teachers' framing of their professional obligations drove their orientations of noticing are hypothesised and discussed.

Keywords: Proof, Lakatos, professional noticing, framing, professional obligation.

Introduction

Lakatos (1976) described how mathematicians developed and utilised mathematical knowledge. Some elements of his approach (e.g., conscious guessing and the zig-zag path of reasoning) have been suggested to be incorporated into school mathematical activities to enhance students' engagement with authentic mathematics (Lampert, 1990). Yet, the field's understanding of teachers' perception and cognition in the context of the Lakatos-style proving activity remains limited (Yang et al., 2022). To take a step towards addressing this research gap, we explored teachers' professional noticing and sought to answer two main research questions: What orientations are reflected (if any) from teachers' ways of noticing of students' thinking in the context of Lakatos-style proving activity? To what extent can we use teachers' framing of their professional obligations to understand their orientations?

Theoretical framework

Lakatos-style proving activity

Building upon Lakatos' (1976) approach and mathematics education studies that explored its implementation in the school setting (e.g., Deslis et al., 2021; Komatsu, 2016; Reid, 2002), we conceptualise Lakatos-style proving activity as consisting of five phases to capture some elements of Lakatos-style reasoning: First, a conjecture is constructed through conscious guessing (Phase 1). The conjecture is then tested using supportive examples (Phase 2). A proof may be constructed to further validate the conjecture (Phase 3). It is also likely that counterexamples emerge that refute the conjecture or invalidate the respective proof (Phase 4), leading to the refinement of the conjecture or proof (Phase 5). Still, it is important to note that Lakatos' philosophy is more complex than this five-phase activity, highlighting also other important aspects, such as the crucial role of the interplay between defining and proving.

Professional noticing

In this study, we focused on teachers' noticing of student thinking to explore how they pay attention to and make sense of students' mathematical thinking. Based on Jacobs et al.'s (2010) framework, we define teacher professional noticing as an integrated set of three interrelated processes: (1) selectively *attending* to noteworthy students' strategies in specific instructional events; (2) *interpreting* students' understanding as reflected in these strategies; and (3) *deciding intended responses* to students (as opposed to actually implementing them).

Framing of professional obligations

Teachers' sense of what is going on in a situation, known as their framing, impacts what and how they notice in this situation (Russ & Luna, 2013). In this study, we adapted Erickson and Herbst's (2018) notion of teachers' professional obligation to conceptualise one type of teachers' framing – *teachers' framing of professional obligations* – which describes teachers' sense of their professional obligations in a situation. According to Erickson and Herbst (2018), teachers may frame their obligations towards the discipline of mathematics (e.g., presenting mathematical knowledge), individual students (e.g., addressing their learning needs), a class of students (e.g., fostering classroom interaction), and the institution of schooling (e.g., following curriculum requirements).

Research methods

In this study, we used semi-structured interviews to collect data from twelve Chinese mathematics teachers. These were recruited through convenience sampling: they were convenient for the first author to contact through personal networks (e.g., friends and their colleagues). To obtain a diversity of teacher profiles, the participants included four pre-service teachers, four novice teachers with an average of 2.75 years of teaching experience, and four experienced teachers with an average of 17.75 years of teaching experience in junior high school for students aged 12-15 (the age group of interest).

The interviews were conducted online with individual teachers. Each interview lasted about an hour. Teachers were presented with a comic-style scenario depicting a Lakatos-style proving activity, which was designed to capture the five phases of Lakatos-style proving and various student understandings (Yang et al., 2022). The scenario was based on real-world classroom episodes, as reported in Komatsu et al.'s (2014) research on Lakatos-style proving, so as to make the description of events sufficiently realistic. Also, the scenario was presented in comic style, so as to draw teachers' attention to students' thinking in the proving activity (rather than any extraneous information such as a student's gender) and capture their interpretations in this context (Herbst et al., 2011).

The scenario consisted of eleven episodes, each illustrating a student's argument during the Lakatos-style proving activity. For example, in an episode, a student found one counterexample and then argued that the conjecture was incorrect. After being presented with each episode, the teachers were asked to describe (i) what they paid attention to, (ii) how they interpreted the students' understandings, and (iii) in what ways they would respond to the students. These questions corresponded to the three components of teacher noticing: attending, interpreting, and deciding, respectively. They were also asked a follow-up question, "Why would you respond to the students in this way?", from which we inferred what framing of professional obligations drove their decisions in that situation.

The unit of analysis constituted each teacher response to a student argument, resulting in a total of 132 units. To analyse the data, we utilised a set of frameworks to code the teachers' responses into different categories. With regards to teachers' ways of attending, we identified three types of focus: (1) on the *outcomes* (e.g., conjectures), (2) on a mathematically *superficial process* (e.g., the student examined more examples), and (3) on the mathematically *essential process* (e.g., these examples were strategically identified with rationales) of students' reasoning. The latter two ways were differentiated based on whether teachers mentioned the mathematically important aspects of the students' reasoning process as suggested by our theoretical framework (see Yang et al., 2022, for more details). We identified three ways of interpreting among teachers based on Sherin and van Es's (2009) framework of three general stances teachers used to interpret: (1) the *descriptive* way to designate what teachers observed, (2) the *evaluative* way to assess whether students' reasoning was valid, and (3) the *interpretive* way to make inferences about students' reasoning (e.g., why students made a certain argument). Regarding teachers' ways of deciding, we categorised them into three ways: (1) the *general pedagogical way* which was primarily related to pedagogy (e.g., organising group discussion), (2) the *product-oriented way* that focused on students generating products of proving (e.g., a proof), and (3) the *process-oriented responses* that focused on students experiencing the process of proving (e.g., analysing why the conjecture was refuted). The first way was distinguished from the latter two based on Lee's (2021) framework about the general pedagogical versus content-specific focus of teachers' decision. Also, the latter two ways were distinguished based on Furinghetti and Morselli's (2011) framework of teachers' process versus product ways of dealing with proof. Finally, based on Erickson and Herbst's (2018) framework, we coded teachers' framing of professional obligations as follows: *individual obligation*, *interpersonal obligation*, *disciplinary obligation*, and *institutional obligation*. For responses that reflected more than one category, multiple coding was applied. Frequency patterns of teachers' noticing and their framing were identified based on the coding results.

Findings

Teachers' orientations of noticing

Figure 1 shows the frequency of each teacher using each way of noticing the eleven student arguments, with a range from 0 to 11. Teachers most frequently paid attention to the superficial process of students' reasoning, interpreted their reasoning in an evaluative way, and decided to respond in a product-oriented way, with an average frequency of 10.2, 9.9, and 8.2 times, respectively. Looking closer to the frequency patterns, we found some similarities and variations in teachers' orientation of noticing. Some teachers tended to primarily use one specific way of noticing, while some others tended to use multiple ways of noticing in a more balanced way. By grouping teachers with similar frequency patterns, we identified some potential profiles of teachers' orientations of noticing.

Regarding the Attending aspect, two groups were identified, A1 and A2. Both groups often focused on the superficial process of students' reasoning, but they tended to have different orientations of attending. Despite their predominant focus on the superficial process (averaging 10.3 times), Group A1 paid limited attention to the outcome and essential process, averaging 6.7 and 4.9 times, respectively. By contrast, Group A2 showed a more balanced focus on different aspects of students' reasoning. They often attended to two or three aspects of reasoning simultaneously – pairing

superficial process with the outcome and/or essential process – with an average frequency of 8.8, 10.0, and 8.2 times, respectively. This suggests Group A1 had a consistent orientation towards the superficial process of students’ reasoning, while Group A2 had a more holistic orientation, attending to multiple aspects of students’ reasoning in a more balanced and integrated manner.

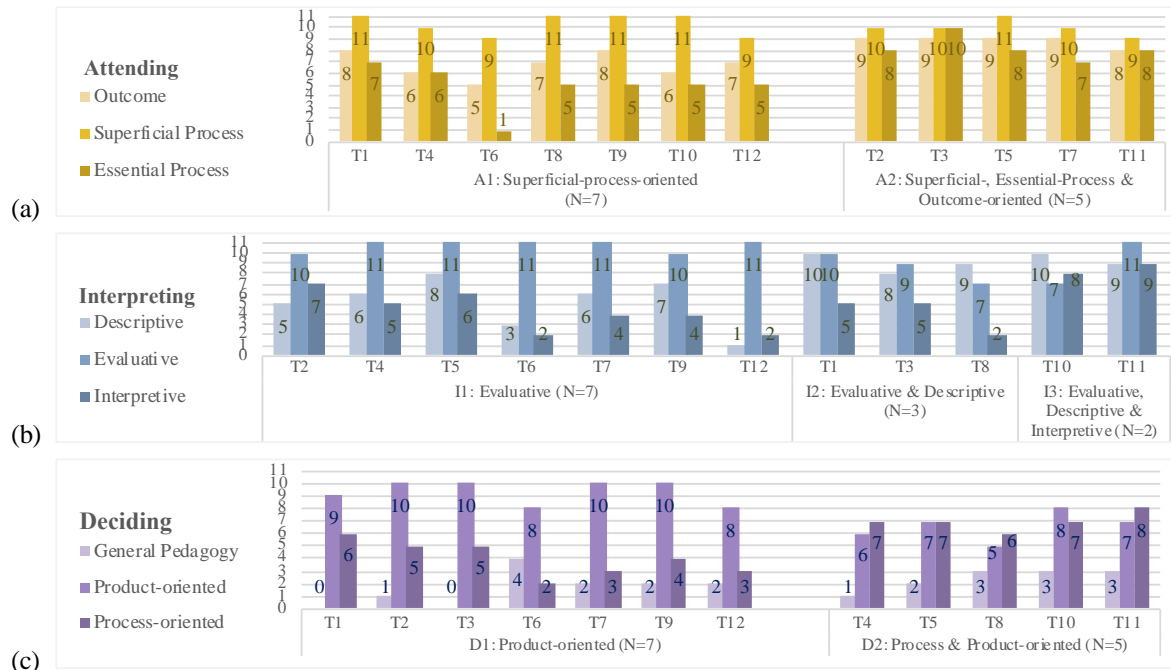


Figure 1: Grouping teachers based on the frequency that each teacher used each way of (a) attending, (b) interpreting, and (c) deciding

Concerning the Interpreting aspect, teachers were divided into three groups. Group I1 primarily utilised an evaluative way (averaging 10.7 times) as opposed to their lower use of descriptive and interpretive ways (averaging 5.1 and 4.3 times, respectively), which were often used in conjunction with the evaluative way. This suggests that Group I1 had a consistent orientation of evaluating the validity of students’ reasoning. In contrast, the other two groups of teachers showed a more balanced orientation when interpreting students’ understandings. Group I2 frequently used both descriptive and evaluative ways, with an average frequency of 9.0 and 8.7 times respectively, while they used the interpretive way relatively seldom, averaging 4.0 times. Group I3 showed the most balanced pattern, using all three ways of interpreting – descriptive, evaluative, and interpretive – at a high average frequency of 9.5, 9.0, and 8.5 times, respectively.

Regarding the Deciding aspect, two groups of teachers were identified: D1 and D2. Both groups rarely used a general pedagogical way, with low frequencies of 1.6 and 2.4 times, respectively. However, it was found that their orientations differed concerning their usage of the process- and product-orientated ways. Group D1 showed a consistent orientation towards the product-oriented way, with an average frequency of 9.3 times, but they infrequently utilised the process-oriented way, averaging 4.0 times. In contrast, Group D2 exhibited a more balanced pattern, often deciding to respond in a product-oriented and/or process-oriented way, with average frequencies of 6.6 and 7.0 times respectively.

Framing of professional obligations by different groups of teachers

Our analysis revealed a range of teachers' framing of professional obligations. In terms of their obligations to *individual students*, teachers stated four main reasons driving their decisions: to (1) provide the student with opportunities to explore how to prove, (2) facilitate the student's thinking, (3) point out limitations of the student's reasoning/methods, and (4) solve the task. For their obligations to *mathematics discipline*, they mentioned as their main obligations to (1) demonstrate rigour and the logic of mathematics and (2) cultivate students' mathematical competence. We categorised these specific obligations into different types, which enabled us to better capture the nuances and complexity of teachers' framing of professional obligations. For their obligations to *students' interpersonal interaction*, they reported reasons like providing students with opportunities to discuss with peers. Teachers also mentioned their obligations to the *institutions*, including fulfilling departmental (e.g., curriculum requirements) and school (e.g., class time) requirements. After grouping teachers in different groups, we calculated the average frequency that each group reported each type of framing of professional obligations (see Figure 2). Given that our identification of teachers' framing was based on their explanations for their decisions (i.e., a critical component of teacher noticing), we particularly focused on the variations in the frequency patterns between different groups, as these may cast some light on why different groups of teachers have distinct orientations in terms of attending, interpreting, and deciding.

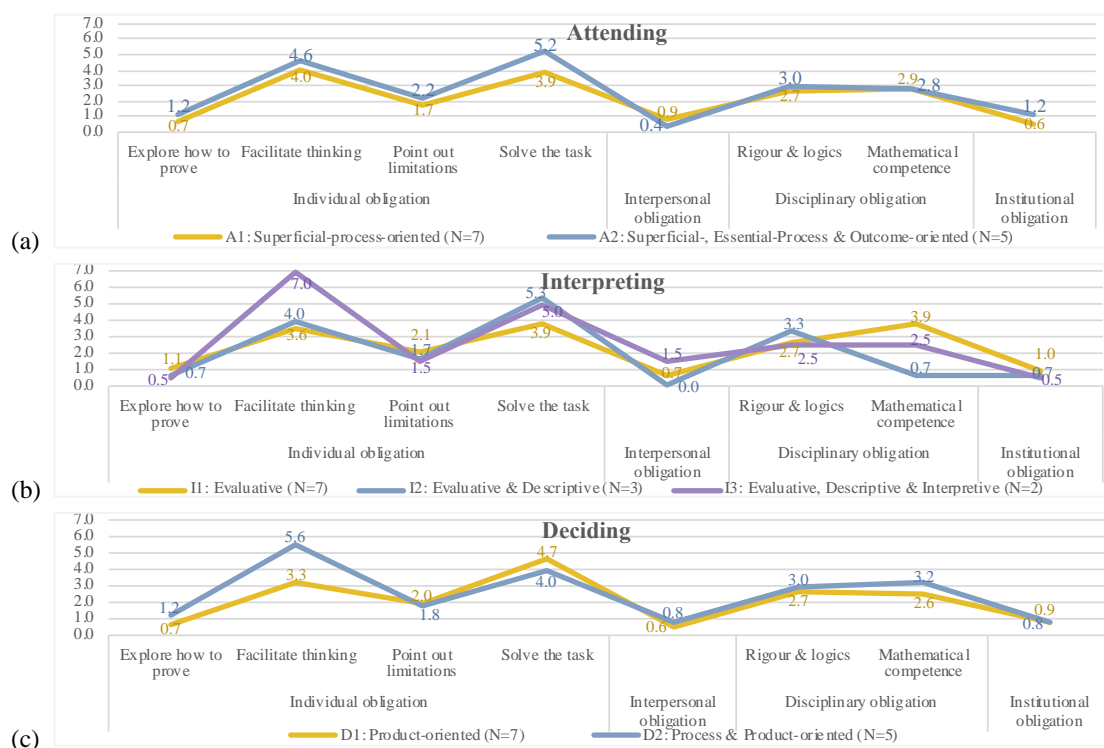


Figure 2: Average frequency of each type of framing of professional obligations by different groups

Group A1 and Group A2 had a similar frequency pattern in their framing of professional obligations. Among several reasons for their decisions, both groups most often mentioned the same reasons – their framing *to facilitate the student's thinking* (averaging 4.0 and 4.6 times respectively) and *to solve the task* (averaging 3.9 and 5.2 times respectively). Still, Group A2, with a more balanced focus

on all three aspects of students' reasoning, showed a slightly higher average frequency in mentioning both types of framing compared to Group A1. It is possible to form a hypothesis (H₁) that even a slightly increased tendency towards the framing *to facilitate the student's thinking* and *to solve the task* may prompt teachers to extend their focus beyond the superficial process of students' reasoning.

The frequency patterns of Groups I1, I2 and I3 were more varied. Groups I1 and I2 most often mentioned the same framing *to solve the task*, but Group I1, which consistently used the evaluative way, also prioritised the framing *to cultivate students' mathematical competence* (averaging 3.9 times), while Group I2, which balanced descriptive and evaluative ways, did so rarely (averaging 0.7 times). Such differences suggest a new hypothesis (H₂) that teachers' stronger tendency towards the framing *to cultivate students' mathematical competence* may drive them to consistently use the evaluative way instead of the descriptive way. Compared to the former two groups, Group I3, with a more balanced orientation of using multiple ways of interpreting, most often mentioned the framing *to facilitate the student's thinking*, averaging 7.0 times. This suggests another hypothesis (H₃) that a stronger tendency towards the framing *to facilitate the student's thinking* may motivate teachers to incorporate the interpretive way in addition to the descriptive and evaluative ways.

Regarding the Deciding aspect, Groups D1 and D2 showed different tendencies in their frequency patterns. Group D1, which had a consistent orientation towards the product-oriented way, most often mentioned the framing *to solve the task*, compared to the framing *to facilitate the student's thinking*, averaging 4.7 versus 3.3 times. Conversely, Group D2, which had a more balanced orientation of using both the product- and process- oriented ways, prioritised the framing *to facilitate the student's thinking* over the framing *to solve the task*, averaging 5.6 versus 4.0 times. It is possible to form yet another hypothesis (H₄) that while the framing *to solve the task* may lead teachers to use the product-oriented way, a stronger tendency towards the framing *to facilitate the student's thinking* may drive them to adopt a more balanced orientation that also incorporates the process-oriented way.

Discussion

Overall, our results show that teachers can be classified into different groups reflecting different orientations towards attending, interpreting, and deciding. Groups A1, I1, and D1 tended to have a consistent orientation towards a single way of noticing, whereas Groups A2, I2, I3 and D2) tended to exhibit a more balanced orientation towards multiple ways of noticing. By comparing the frequency of each group's framing of professional obligations, we formulated four hypotheses (H₁, H₂, H₃, and H₄) based on which we propose some possible mechanisms underlying teachers' orientation of noticing, as presented in Figure 3 and described next.

A stronger tendency in framing the obligation as *to facilitate the student's thinking* may drive teachers to have a more explicit consideration of students' reasoning, which may lead teachers to extend their attention beyond the superficial process of students' reasoning to focus also on the outcome and essential process (H₁). The obligation *to facilitate the student's thinking* may motivate teachers to additionally adopt the interpretive way (e.g., inferring why the student thought in this way) compared to primarily utilising the evaluative and evaluative ways when interpreting students' understandings (H₃). This may also drive teachers to incorporate the process-oriented way, focusing on students experiencing the proving process, instead of consistently orienting them toward the product of

proving (H₄). By contrast, an increased tendency towards framing the obligation as *to solve the task* may lead teachers to prioritise the product-oriented way (H₄), consistently deciding to direct students towards the product of proving as a solution to the proving task. Similarly, an increased tendency in framing the obligation as *to cultivate students' mathematical competence* may prompt teachers to prioritise students' competence in making valid reasoning, driving them to consistently use the evaluative way to interpret students' understandings (H₂).

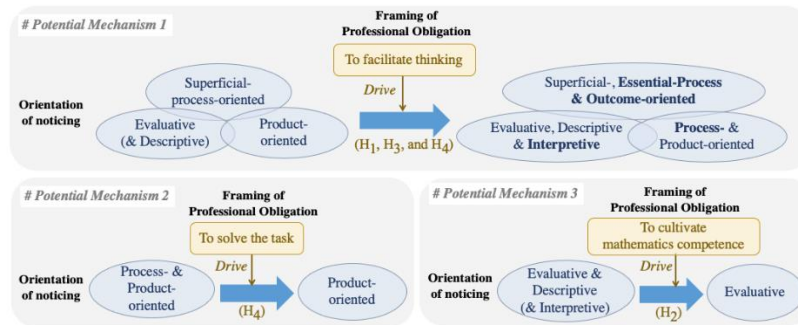


Figure 3: Possible mechanisms underlying the orientation of teacher noticing

It should be noted that our identification of teachers' orientation of noticing does not indicate that they followed their orientation all the time. Instead, our results show that even teachers with a consistent orientation were still able to use other ways, although less frequently. This supports the idea that teachers have the potential to shift from one orientation to another orientation. Teacher professional learning activities often focus on teachers' development of knowledge and beliefs (Opfer & Pedder, 2011). When introducing, for example, a process-oriented practice of teaching proof, teacher training may thus focus on developing teachers' knowledge and beliefs about the practice. This study suggests two alternative approaches. One is to introduce teachers to new ways of noticing, for example, the process-oriented way of deciding. Another is to transform teachers' framing of professional obligation. As illustrated in *Potential Mechanisms 1 and 2* (Figure 3), when teachers' tendency to frame their professional obligations as *to facilitate student thinking* increased or their tendency to frame them as *to solve the task* decreased, they were more likely to exhibit a balanced orientation that utilised both process- and product-oriented ways.

We also acknowledge that different patterns of teacher noticing and associated mechanisms might emerge if the (number of) participants or the research context changed. Nevertheless, the findings of this study enrich the field's current understanding of teachers' perception and cognition in the context of Lakatos-style proving activity, and they lay the foundation for a larger-scale investigation into the underlying mechanisms of teachers' decision-making processes in this context. Indeed, the subsequent strand of our ongoing research has been a survey study whose findings we expect will allow us to explore quantitatively the identified patterns of teacher noticing.

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