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Teachers' critical reflections on digital fabrication for making manipulatives to support mathematical teaching

Henrik Stigberg, Susanne Koch Stigberg, Marianne Maugesten, Odd Tore Kaufmann

Østfold University College, Halden, Norway; hsti@hiof.no

Digital fabrication (DF) is the process of translating a digital design developed on a computer into a physical object. While digital fabrication has been widespread in STEM education and design thinking, we see an opportunity to use DF by mathematics teachers for making manipulatives opening for new learning experiences. We conducted four full-day workshops with eight in-service teachers inquiring into DF tools, techniques, and platforms. Teachers explored 2D and 3D modelling and fabrication using 3D printer, laser- and vinyl-cutter. In this paper, we report on teachers' critical reflections on DF for making manipulatives to support their mathematical teaching. Our findings show that teachers used DF tools to create their own manipulatives and designed customized teaching activities for inquiry-based teaching. They developed a strong sense of ownership over the created manipulatives, crucial for teachers to effectively utilize manipulatives in the long term.

Keywords: Mathematics education, inservice teacher education, digital fabrication, computer assisted design.

Introduction

Concrete manipulatives in mathematics education are physical objects used to reify abstract concepts, such as arithmetic, algebra, geometry, and fractions (Sowell, 1989). The use of manipulatives is promoted by policymakers and national resources alike (e.g., Common Core State Standards - National Council of Teachers of Mathematics, 2017) and there is a strong belief among many teachers that manipulatives are inherently good for mathematics education. The use of manipulatives is advocated by numerous development theorists. Montessori (2013) promoted the use of tactile objects to afford different senses when learning. Piaget describes development from manipulating concrete objects to a more abstract understanding (Huitt & Hummel, 2003). Embodied cognition assumes that features of our cognition are not only formed by our brain but by our body as a whole, including the perceptual and motor system (Wilson, 2002). Using different representations and bodily movements when learning gives more opportunities to retrieve knowledge (Carbonneau et al., 2013; Chu & Kita, 2011; Lindgren, 2014). It is essential for mathematical learning that manipulatives are used cautiously and continuously. To draw maximum benefit from students' use, teachers must continually situate their activities and the material based on students' previous experiences and the teaching context (Brown et al., 2009; Sarama & Clements, 2016; Uttal et al., 1997). There is evidence that successful teachers repeatedly reuse the same manipulative, so the manipulatives become familiar and no longer be viewed as toys or objects in themselves but transparent for the mathematical concept they are supposed to reify (Uttal et al., 1997). Teachers' knowledge gaps and misconceptions about how different representations, such as concrete materials, are translated into mathematical concepts can be problematic when teaching with manipulatives (Stylianou, 2010), since manipulatives do not afford mathematical learning in themselves (Sarama & Clements, 2016). Teachers play a key role in

choosing appropriate mathematics manipulatives and designing learning experiences with them. Therefore, teachers need training to do so. The continuous use of manipulatives in the classroom is related to teachers' convictions (Marshall & Swan, 2008). In summary, successful use of manipulatives for mathematics teaching is tight to teachers' ability to reuse manipulatives and customize teaching activities with manipulatives in their classroom. The main issues for not using manipulatives recognized by teachers are access to manipulatives (money, organisation of materials, space, lack of time) and classroom management (distractions when using manipulatives, classroom size, and behavior in the classroom) (Marshall & Swan, 2008). In this article, we investigate: How do teachers critically reflect on the use of digital fabrication as a tool to create manipulatives in relation to customized teaching activities, reuse of and access to manipulatives as well as classroom management?

Digital fabrication (DF) refers to the use of computer-controlled tools to make physical objects. This includes technologies such as 3D printing, computerized numerical control (CNC) milling, laser cutting, and more. The process typically starts with a digital design, which is then translated into machine-readable code and used to guide the fabrication process. This allows for highly precise and customizable production and has the potential to revolutionize manufacturing by making it faster, cheaper, and more accessible. Blikstein (2013) argues that DF and making can play a major role in education, "bringing powerful ideas, literacies, and expressive tools to children" (p. 2). Previous research on DF and mathematics has focused on STEM and how to teach DF to children. Stigberg (2022) has found scattered research investigating how teachers use 3D printers for creating manipulatives. To our knowledge, there is no previous research that explores the use of multiple DF technologies to support the making of manipulatives for mathematics teaching. In the following, we will present the DF workshops and describe how we collected and analysed the teachers' critical reflections before sharing and discussing our findings.

Methodology

This paper reports on our initial findings from an ongoing study on digital fabrication for making manipulatives for mathematics teaching. Using a Community of Inquiry (Jaworski, 2006) methodology, we (three teacher educators and one DF expert) conducted four monthly full-day workshops with eight in-service elementary teachers with previous experience from using manipulatives (referred to as teachers) to inquire together into DF and its potential to develop practices for making and using manipulatives. In the workshops, we introduced DF tools and techniques including:

- DF sharing platforms (e.g., Thingiverse and Printables),
- 3D modelling and printing using Tinkercad and Prusa printer,
- 2D modelling and cutting using Cricut vinyl-cutter and Glowforge laser cutter,
- design thinking process to support the design of new manipulatives.

Following Koehler and Mishra (2005), we view "teacher knowledge for technology integration as being a transaction between the three factors of content, pedagogy, and technology" when inquiring into DF (p. 95). Our objective for the workshops was to enable teachers to make manipulatives by using DF and reflect and discuss how to use these manipulatives in their mathematics teaching. Table

1 provides an overview of the workshops content. We use the FACS framework (S. K. Stigberg et al., 2022) in the workshops to scaffold DF skills arranged by increasing complexity from finding ready-made manipulatives to designing new manipulatives. FACS stands for finding, adapting, creating, and sharing manipulatives.

	Technology	Pedagogy	Content	Share
Find Workshop 1	Thingiverse Printable Prusa Slicer 3D printer	Representations of mathematical concepts Four principles for working with manipulatives (Svingen, 2018)	Geometrical concepts and properties <i>Example: What object does not fit?</i>	In all workshops teachers shared by: Presenting the manipulative and planned classroom activity at the end of each workshop and Sharing experiences from their teaching with the manipulatives at the beginning of the following workshop.
Adapt Workshop 2	Tinkercad	Inquiry-based teaching Modelling manipulatives for customized teaching	Number concepts and properties <i>Example: Even and uneven numbers pieces</i>	
Adapt/Create Workshop 3		Four stage framework for using manipulatives in teaching (Malmer, 1997) Modelling manipulatives for customized teaching	Position system, positive and negative numbers, spatial geometry	
Adapt/Create Workshop 4	Laser cutter Vinyl cutter Inkscape	Inquiry-based teaching	Using different representations	

Table 1: Overview of workshop content categorized by technology, pedagogy, content, and share

During the fifth workshop, teachers gave individual presentations that reflected critically on the prior DF workshops, one or more selected manipulatives they created, and the classroom activities that were incorporated with the manipulatives. To prepare for the workshop, teachers got a set of questions (<https://shorturl.at/iwGPR>) which they could use to guide their presentation. Critical reflection (Mezirow, 1998) on practice is essential in the process of teacher’s professional growth (Jaworski, 1993) and an important part of a Community of Inquiry. According to Jaworski (1993), this “leads a teacher to ‘give an account of the event which can result in enabling the teacher in some sense to step outside the event, examining questions of what and how. This leads to the ‘distancing’ which is necessary to critically analyse the event” (p. 40).

We collected the data material by videotaping teachers’ presentations of their selected manipulatives, experiences from the classroom and their reflection on DF. During the presentations, we posed follow-up questions on the advantages and disadvantages of DF as a means to make manipulatives. Our goal was to gain a deeper understanding of the teachers' reflections. Teacher presentations lasted in average 17 minutes, adding up to a total of 2 hours and 16 minutes. We analysed teachers’ presentations focusing on access to manipulatives, classroom management, customized teaching activities and reuse of manipulatives. We performed a thematic analysis of the video data individually, using the defined categories and transcribed relevant citations including timestamps. One week after the presentation, we discussed and compared our coding of the data related to each category to come up with common themes. Discrepancies in the coding were discussed and resolved.

We developed thematic maps (Braun & Clarke, 2012) for each category and reviewed relevant segments of the video data to validate our findings. Teachers are coded using T1-T8 in the findings.

Findings

Teachers presented four different manipulatives created using 2D modeling software Inkscape and a laser cutter. Figure 1 presents an overview of the produced manipulatives.



Figure 1: Fractions and geometry; even and odd numbers; number line; multiplication puzzle

Access

Marshall and Swan (2008) describe limited access to manipulatives as one of the major factors for not including manipulatives in teaching activities, including both access to physical storage at school as well as economic limitations to buy manipulatives. We identified three categories related to access in our analysis: physical access, knowledge about using manipulatives, and limited time.

Teachers highlight that manipulatives they fabricated are stored in their classroom, providing access to both students and teachers. They describe that they seldom use manipulatives stored at a common place at school. T4 reflects that she finds manipulatives but does not know how to use them and does not bring them into the classroom. T1 mentions that he has a similar manipulative at school to the one he produced during the workshops. However, he did not think of it before, and his students do not have access to the former manipulative.

All teachers have used the produced manipulatives in teaching activities in the classroom. They point out that there is a difference between taking an off-the-shelf manipulative and a manipulative they have designed for their classroom. “When you are more familiar with it [the manipulative], then it might be easier to use” (T4). Teachers report that they have used their produced manipulatives for various mathematical topics (e.g., the manipulative displayed in Figure 1 to the right is used for both fractions and geometrical concepts). They note that they have designed the manipulatives for multiuse and through reflection on classroom activities, they discover novel ways to incorporate them. “After all, when you start, you see other possibilities. And I think that’s the biggest thing about this.” (T2), referring to how using DF tools to make manipulatives develops her knowledge about teaching mathematics with manipulatives.

Teachers emphasize on the importance of given the opportunity and time to be part in the DF workshops and collaborate with colleagues, teacher educators and DF expert, to inquire into the use of DF for making manipulatives. They express that they otherwise have a high workload at school and limited time to learn DF, make manipulatives, or plan teaching activities for their classroom. They mention that using DF can be time-efficient, since the produced manipulatives are durable and can be used more flexible. T5 reflects “it has become time-efficient in another way, but it requires

that you have time to sit down and do them at the starting point. So, in the long run, I think we save time, but you must have that time first to sit and make things” (T5).

Classroom management

Classroom management is the second-highest reported issue by teachers for not using manipulatives in the classroom (Marshall & Swan, 2008), including distractions when using manipulatives, classroom size, and behavior in the classroom. Our analysis uncovers two facets of teaching strategies to manage these issues: Teaching in small groups and inquiry-based teaching.

Teachers used the created manipulatives for teaching activities in small-group arrangements in their classroom, e.g., in special needs education with three students (T2, T8), to deal with larger classes they implemented station rotation teaching dividing the class into small groups rotating through stations with different teaching activities (T3, T5, T8), and in extracurricular teaching activities with pairs of students (T1, T7).

Teachers planned for inquiry-based teaching activities with the produced manipulatives with different level of instructional support. T5 reports that students found the produced manipulatives in the classroom and started to play with them. She promotes free play with the manipulatives but tries to direct the play toward mathematical content by asking questions. T5 critically reflects: “Even if it is student-led, it does require someone to aim it in a direction. But of course, it can be playful in the beginning so that they become familiar with the manipulatives.”, promoting students’ spontaneous use of the manipulative in the beginning, and a more structured use of the manipulatives later. Teachers T2-T5 and T8 introduced the manipulatives using open-ended questions and then directed questions towards more concrete mathematical content. Teachers report that the use of manipulatives contributes to productive mathematical discussions. Our interpretation is that teachers employ inquiry-based teaching strategies that incorporate playful elements to disarm distractions and encourage active engagement among students, resulting in positive classroom behavior.

Customized teaching activities

To draw maximum benefit from students’ use of manipulatives, teachers need to continually situate their activities and the manipulatives based on students’ previous experiences and the teaching context (Pires et al., 2019; Thompson, 1992). We identified three aspects of how teachers customize manipulatives and teaching activities from the DF workshops. 1) All teachers developed manipulatives that they thought would fit into their teaching context. T2 argues that she came up with the idea for the number line (see Figure 1), since her students have problems with number sequences in previous diagnostic tests. Before, she only had used number lines on paper hand-outs and she said that using the manipulative provided a new way for students to collaborate in number sorting activities, e.g., in turns, students draw a piece and place it on the line. Similar, T3 reflects that she “made this because they believed in it.” 2) Furthermore, teachers planned for inquiry-based teaching using their manipulatives. Students actively engaged with the manipulatives and teachers guided students’ inquiry by asking situated questions, e.g., (T4) “And then, I directed it a bit. Then I asked what happens if zero is in the middle” 3) Finally, teachers report that they used the teaching activities to inquire into their own teaching to develop new activities using the manipulatives. T4 argues that her teaching activity was more “a process than a teaching plan”.

Reuse of the manipulatives

Uttal et al. (1997) describe the need for teachers' continuing use of manipulatives to support students to focus on the mathematical content the manipulatives are supposed to reify. Teachers value DF as a means to afford reusing manipulatives from two facets: durability and flexibility of the produced manipulatives. Despite limited time, teachers believe that knowledge about and access to DF technology affords them to make customized manipulatives that look more professional and are more durable. Manipulatives produced using DF can be used over longer time compared to manipulatives hand-crafted e.g., using laminated paper. Teachers reflect that students take care of the DF produced manipulatives and help to stow away the manipulatives at the end of the class. Teachers consciously made simple-looking manipulatives without distracting features, such as pictures, color, unnecessary patterns, and digits. During the design phase in the workshops, teachers strive to make manipulatives reusable, and they discover new possibilities along the way, e.g., the number line (Figure 1) can be used for numbers, patterns, and the positioning system. Teachers argue that DF provides them an opportunity to adapt and change manipulatives over time.

Discussion and conclusion

In this article, we investigate how teachers critically reflect on the use of digital fabrication as a tool to create manipulatives in relation to customized teaching activities, reuse of and access to manipulatives as well as classroom management. In the workshops, we presented both 3D modeling and printing, as well as 2D modeling and cutting. Interestingly, teachers chose manipulatives that were created using a laser cutter for their presentations. In contrast, previous research has focused on 3D printing (H. Stigberg, 2022). Our findings indicate that teachers value laser cutting as a means for rapid fabrication of professional-looking manipulatives. As laser cutters become more affordable, they are a good choice for making manipulatives for mathematics.

Classroom management was one of the major issues for not using manipulatives reported by Marshall and Swan (2008). Teachers presented two teaching strategies for classroom management: small group settings and an inquiry-based student-centered teaching approach. They highlight that this type of teaching emphasizes students' mathematical discussions and adapted teaching. We argue that our DF workshops combining DF technologies with a focus on inquiry-based teaching in mathematics have empowered teachers to better manage their classrooms.

Based on our findings, we discovered that the DF workshops allowed teachers to not only familiarize themselves with DF technologies but also foster a strong sense of ownership over the manipulatives they created. This ownership entails: 1) storing the manipulatives in their classroom with easy access for them and their students, 2) creating customized manipulatives tailored to their students and classroom, 3) room for further adaptation, modification, or expansion of the manipulatives, 4) and the option to reuse the manipulatives for teaching various mathematical concepts. We believe that this sense of ownership is crucial for teachers to effectively utilize manipulatives in the long term, as it is tied to their convictions and beliefs. Marshall and Swan (2008) describes teachers conviction as one of the primary factors for long-term and effective use of the manipulatives.

Marshall and Swan (2008) argue that "committed teachers still require training in how to make effective use of specific mathematics manipulatives" (p. 339). In line, teachers value the DF

workshops for providing an arena for collaborating with colleagues to make manipulatives, plan and share teaching activities for their classroom. Through our workshops, we facilitate an inquiry process for teachers to explore how DF applies to their practice and reflect on the effective use of mathematics manipulatives in their teaching. As teacher educators and DF expert, we gain valuable insights from teachers' critical reflections, which helps us to refine our approach to integrate DF into teacher training.

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