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# Teacher-researchers can develop techno-mathematical literacy by posing and teaching problems with digital tools

Tikva Ovadiya

Oranim College, Tivon, Israel; [tikva\\_o@oranim.ac.il](mailto:tikva_o@oranim.ac.il)

*Current research has become interested in how techno-mathematical literacy evolves in teachers. This study of 15 mathematics teacher-researchers suggests that techno-mathematical literacy can evolve when teachers pose problems using digital tools that lead to ‘triggers’ that emerge as a result of contingent events that occur in the classroom. This paper describes four such triggers followed by an analysis of the teachers’ written reflections about the noticeable incidents that took place. The characteristics of these triggers suggest that teacher-researchers may find themselves in contingent situations in which both their knowledge of the digital tools required for problem posing and their mathematical knowledge for teaching these problems is undeveloped.*

*Keywords: Problem posing, teachers as researchers, teaching mathematics with technology, digital literacy, reflection.*

## Introduction

Over the last two decades, research on mathematics education has become interested on the knowledge that teachers develop as a result of using digital tools (DTs), and the research literature abounds with studies that highlight how the introduction of technology into mathematics classrooms adds complexity to the role of the teacher. Although some teachers use digital tools simply as a substitute for paper and pencil, some exploit them for investigating concepts and phenomena, sometimes resulting in contingent events that they must be ready to deal with. Jacinto and Carreira (2023) claim that teachers can develop techno-mathematical literacy (TmL) knowledge. TmL is defined as a combination of mathematical understanding, workplace and ICT knowledge, and communicative skills. The research literature refers to TmL mainly in the context of learners’ skills in software, technical communication, and technical drawing (van der Wal et al., 2019). But how does this knowledge come about? There are many ways to foster such knowledge (e.g., professional development courses), but some studies have shown that having teachers act as researchers can bestow new insights and refine their mathematical knowledge as has been demonstrated by the occasional references to acquired learning that have appeared in the self-reflections of teachers who engaged in research-teaching (Ovadiya, 2019, 2021). In this study, my aim was to explore how such teacher-researchers (TRs) can develop TmL by teaching problem solving using problems they designed with digital tools.

## Theoretical background

### Contingency, noticing, and teachers as researchers

Mason (2002) argued that people notice things insofar as they are unexpected, i.e., contingent. ‘Contingency’ is one of the dimensions in the Knowledge Quartet—a theoretical framework for the analysis of the mathematical knowledge that teachers apply in a classroom. The concept of ‘contingency’ rests upon the consideration that teaching is often a partially improvised activity

(Rowland et al., 2015) and concerns situations in which a teacher encounters an unanticipated event and is challenged to deviate from the prepared agenda. If the teacher is guided to ‘notice’ and reflect upon the contingency they encountered, it may lead to enhanced awareness.

In this paper, I use the term ‘reflection’ to denote the disciplined aspect of such ‘noticing’. This includes careful documentation of the incident and successive introspection with a deep inward gaze. Requiring pre- or in-service teachers to act as TRs and engage in action research during their practical training encourages them to understand and apply teaching practices compatible with learning situations (Ovadiya, 2021).

### **Posing problems with digital tools**

Since the mid-1990s, research on digital technologies in mathematics education has moved from a focus on learners to a focus on teaching practices (Sinclair et al., 2023). Awareness of the importance of integrating digital tools into teaching and into mathematics teacher development programs is widespread, together with the awareness that digital TmL is an acquired skill and that there is a didactic gap between teaching problem solving using paper and pencil and teaching using digital tools (Jacinto & Carreira, 2023). Teachers tend to find problem posing (PP) more difficult than problem solving, and often have difficulty posing complex problems that require extended content and high-level thinking skills (Yao et al., 2021). Ovadiya (2019, 2021) suggested that when teachers design problems for a specific pedagogic or didactic goal, for example, to dismantle potential student misconceptions, demonstrate common algorithmic errors, or provide understanding of specific mathematical concepts, they can improve their pedagogical and didactic knowledge.

In the current study, TRs were asked to use DTs to pose problems that would promote a specific learning process upon which they were focused but that the textbooks and curriculum seemed to underplay. Some teachers used the DT as a tool to promote the enquiry of mathematical or geometric phenomena or concepts; others used it as a visual tool for simplifying data. In this article, I focus on four contingent events that evolved during the teaching process that demonstrate the TRs’ practices of ‘techno-mathematical literacy’.

## **Method**

**Research question:** How does the process of action research enhance TR’s TmL regarding posing curriculum-based mathematics problems using DTs? What do they learn from the PP process? What do they learn from the experience of teaching these problems?

**Participants:** Fifteen in-service teachers who were studying in an M.Ed. program for either elementary or high school mathematics education.

**Procedure:** The TRs were asked to (independently) design a problem in geometry or algebra using DTs and document each step in their research journal. Next, they were to teach the problem to their peers (in their college class) and document their reflections and insights about their teaching experience. If necessary, they were to revise the problem to make it more precisely meet their teaching goal. After making any changes, they were to introduce the problem to their students and document their teaching process and insights. Each of the 15 TRs underwent this procedure six times (i.e., a total of 90 problems).

**Data sources:** Data were gathered from the TRs' journals, video documentation of their teaching (in college and in classes), audio recordings of lessons and mentoring meetings, researcher's (author's) journal.

**Data analysis:** The documents underwent inductive analysis driven by the question, 'How did the events that the TRs noticed spur their development of TmL knowledge for teaching with DTs?' The four-stage analysis (some using ATLAS.ti software) included: (1) analysis of TRs' journals to identify the challenging contingents events they recognized and reported for each problem posed and to obtain clues to the TmL knowledge that they developed; (2) identification (by the author, in her own journal) of technological challenges to the TRs; (3) categorization of all the challenges from (1) and (2); and (4) generalization and production of categories to determine if and how TRs realized they had developed TmL knowledge.

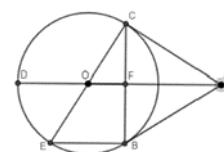
## Findings and discussion

In this article, I focus on one particular aspect of TmL development, that is, the triggers that led the TRs to come up with specific mathematical-geometrical ideas for designing appropriate problems with DTs. Analysis of the data revealed four categories of 'macro triggers': (1) a transition in the nature of their understanding of the concept from logical to ontological as a result of using DTs; (2) grasping how to visually and dynamically present the essence of specific geometric or algebraic theorems or concepts using DTs; (3) insight about how 'adding' and/or 'deleting' specific elements on the dynamic graphs or images can promote new geometric or algebraic knowledge to help solve the problem; and (4) understanding how to design lessons to teach the problems and redesign the problems if necessary according to the teaching enquiry. Each of these triggers stands on its own and an independent article could be written around any of them. In the present article, I present a brief overview and illustration of the four triggers by presenting one example of each as I interpreted it from the journal of one of the TRs (Rebecca).

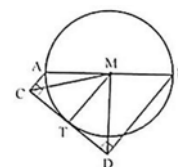
### Trigger 1. Transition in understanding of logical nature of concept to ontological nature

For the first PP task, Rebecca chose the following problem and sketched it using the DT. Her challenge was to adjust the wording of the question to comply with her applet.

Tangents AB and AC to circle O extend from point A. AD is a bisector that passes through centre point O. The continuation of radius CO intersects the circle at point E. AD and BC meet at point F. Prove that a.  $BE \parallel AO$ ; b.  $BE = 2OF$ .



When her colleagues played with the applet, Rebecca noticed that the DT made the answer too obvious. She realized that she needed to identify the benefits that the DT offered and take advantage of it but also understand what shortcomings it presents and how to avoid them. She needed to be able to change a static problem into a dynamic one that has the potential to elicit higher-order thinking and to design the app so that students will have to discover hidden properties and data to form their proof. She realized, in fact, that she had not really posed a problem, since the app answered the problem immediately. She then chose another problem from the textbook:



AB is the diameter of a circle with centre M. Line M is tangential to the circle at point T. Given: BD is perpendicular to CD and CD is perpendicular to AC. Prove that: a.  $TM = (AC+BD)/2$ ; b.  $MC = MD$ .

Rebecca built another applet and considered the dynamic points. By playing with some of them, she discovered some relationships between the geometrical concepts and statements that she had not noticed before. She decided that point T should be dynamic and found that the phenomenon exists for an infinite number of trapezoids and also for a rectangle and a triangle (when points B and D merge). She understood that  $TM = (AC+BD)/2$  is a ratio involving three segments, and that TM is an average (and not a particular case of constant sizes that seems to be implied in the problem in the textbook). This new mathematic reasoning led her pose a new question:

Using radius  $r$ , express the ratio of the area of the circle to the area of quadrilateral ACDB when tangent CD is parallel to diameter AB. (<https://www.geogebra.org/m/epcafpjj>)

Rebecca said, ‘This part of the question provided an opportunity to investigate the relationships of areas in the problem and to understand them in a substantive and non-procedural way. You can also recognize the figures that exist and then change their features by moving point T ...’ However, she quickly realized that using the DT as if it were pencil and paper, as she did initially, did not actually ‘pose’ a problem; it merely allowed one to ‘play’ with the tool and ‘play’ with the mathematics. She thus understood that, instead, she had to create a problem in which the tool allowed visualizing the initial shapes expressed by the definition given, the changes caused by shifting vertex T, and the changes in the relationships between the shapes as a result of the movement. By ‘playing’ with the mathematics and the DT, she observed the movement and the change, and her more in-depth understanding of the objects created allowed her to pose a more suitable problem.

This situation led her to pose a type of problem that could lead students to structural perception. Structural perception is a rather abstract concept, but to speak about mathematical objects, one must be able to deal with the products of some processes without bothering about the processes themselves. A technological tool that dynamically presents different representations of the same concept or mathematical phenomenon is precisely one that can facilitate the development of structural perception because the DT clearly shows the existence of the features. In this case, Rebecca demonstrated techno-mathematical fluency in that she recognized several of the DT’s affordances: interpreting the situation from a techno-mathematical point of view (choose the dynamic point), exploring a conceptual model (express the ratio of the area...), and producing a techno-mathematical problem and solution. Her actions demonstrate how she transitioned to an ontological concept of the shapes and concepts, similar to what Sinclair claimed, ‘The main reason for using digital technologies is neither to concretize a concept nor to excavate the ideas built into objects, but to introduce new actions and therefore: to move. That is, one's senses of shape, quantity, proportion and so on...’ (Sinclair, 2020, p. 105).

### **Trigger 2. Presenting the essence of geometric theorems visually and dynamically**

In this case, Rebecca's challenge was to design an applet to pose problems about the following three geometric theorems:

**Theorem 1:** Given two secants that intersect a circle from a point outside the circle, the product of the first secant and its segment outside the circle will be equal to the product of the second secant and its segment outside the circle.

**Theorem 2:** Given a tangent to a circle from a point outside a circle and a secant that intersects the circle from the same point, the product of the secant and its outer segment will be equal to the square of the tangent.

**Theorem 3:** Given two chords of a circle that intersect each other, the product of the segments of one chord will be equal to the product of the segments of the second chord.

Rebecca decided that her first step (before trying to pose a problem) would be to explore the theorems dynamically. She wrote:

Using GeoGebra, I sketched out the areas of the circles. I played with the application but could not find any relationship or correlation between the areas. So, I tried other directions to find how it is possible for  $AD = AF \times AC \times AB$  to always be true. I didn't have any idea, so I decided to move on to Theorem 2, which has to do with the ratio and proportion in a circle.

I tried to understand the significance of the square of the length of the secant and thought about the double meaning of 'square' (as a shape and as a mathematical operation). I constructed a square on the tangent, which seemed significant. As a result, I understood that the area of the square must be equal to the area of the quadrilateral constructed on the circle's secant, and the area of this quadrilateral will be a continuation of the ratio and will be  $BC \times CE$  (which is the calculation of the area of the rectangle). I tried to construct an appropriate rectangle but discovered that this was impossible because I had formed a dynamic point which contradicted the definition of a rectangle. When I transformed parts of the rectangle into radii of the circle, it created a representation that seemed to illustrate the relationship I was looking for. Once I understood that, I went back to the first theorem and built it proportionately. Then I constructed the sketch for Theorem 3.

(The three applets can be viewed by clicking on the following: [Applet 1](#), [Applet 2](#), [Applet 3](#).)

By playing around with the app, Rebecca changed her understanding of specific facets of the geometric theorems and realized something new and significant about the theorems that she had not thought about before. She now understood that the applets illustrated aspects of the geometric theorem, and that concealed information, such as the 'tangential square', becomes more obvious when one constructs the square. Now that she understood the essence of the theorems, she could be more focused on building an applet; she thus came up with a novel idea for designing problems to lead her students to discover and prove the theorems. This example shows that exploring the dynamic graphical representation of a geometric theorem is a stimulating pre-activity for PP and leads to original ways of thinking about geometry that are not necessarily encouraged by the curriculum, which dictates a specific order in which mathematical and geometrical theorems should be learned. Sinclair claims that fear of violating this order is one reason that teachers' digital literacy is undeveloped (Sinclair et al., 2023). However, here, the TR's activity with the DT led her to understand Theorem 2 before she understood Theorem 1. This was the result of her drive to reach her research goal (to pose a problem by adjusting the DT as required to effectively demonstrate the

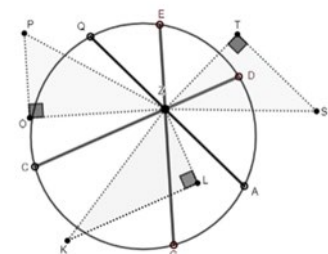
theorem). Experimenting with the DT before actually designing the problem led her to understand the theorem in a digitally dynamic way. In other words, her actions led her to ‘understand’ the theorems in the ‘wrong’ (according to the curriculum) order, but this is precisely what led her to understand the relationships between the elements more precisely. The DT impelled her to progress through ‘micro-cycles’ of understanding composed of several processes (‘I constructed a square ...’, ‘I tried to construct an appropriate...’, ‘I transformed parts of...’), which led to a more thorough understanding in general.

Next, Rebecca’s aim to construct an app to illustrate all three theorems invoked ideas for a problem that would implement understanding it. The insight gained by drawing the elements of the theorem can be referred to as repetitive insight and is an expression of a theorem that seems ‘algorithmic’ or ‘procedural’ ( $AD = AF \times AC \times AB$ ) as a result of structural or ontological understanding. This is similar to Trigger 1, except that in that case, the trigger leading to insight was an error in posing the applet as a problem without delving into its purpose. Here, the trigger is a deliberate and meticulous analysis of a geometric theorem through a dynamic graphical representation that is refined over and over again and that leads to discovering TmL knowledge that includes technical drawing, technical software skills, and greater understanding of the theorems involved, alongside using drawing to express and illustrate attributes, relationships, and calculations.

### Trigger 3. Adding or deleting elements on the applet representations

Rebecca next decided to pose problems that would lead students to discover three theorems of proportion while sketching a representation for the theorem.

Part 1. Given: Three chords in a circle, EG, DC, and AQ, meet at point Z, which is the common vertex for the three right-angled triangles PZO, KZL, and TZS. Investigate the applet and find out what is common to the three triangles mentioned above. You can use the hint box. (<https://www.geogebra.org/m/jkzxnwnw>)



Part 2. Given that  $\Delta QZT$ ,  $\Delta LZD$ , and  $\Delta OZG$  are right-angled isosceles triangles, prove that they are congruent.

Rebecca built an applet that was different from the ones with which she presented the proportion theorems. Playing with the applet gave her the new idea that it was possible to draw ‘half rectangles’ on the chords – that is to draw triangles instead of the rectangles. This innovation to represent the theorems using rectangles and squares led her to another discovery regarding the proportions of triangles that are halves of the rectangles for which proportional relationships have already been proven. She performed this by adding a chord, which although seems unnecessary, created a different visual representation and invited investigation of a new situation. The decision to halve the rectangles would get students to think of a triangle as a half-rectangle, and from there to progress to the proof.

The concept of adding or subtracting an element in dynamic sketching was also displayed by other TRs and this concept triggered discoveries of new geometrical (or mathematical) knowledge for problem posing. This is an TmL knowledge learning process (i.e., it concerns pedagogical and content knowledge), because the TRs had to justify adding or deleting elements, an action that requires deep

geometric thinking regarding the DTs and the mathematics for posing and designing lessons for these problems. The TRs wrote in their journals that one they had discovered this idea, their PP occurred more naturally. These cases are evidence of the TRs' techno-mathematical fluency skills, which include recognizing several of the DTs' affordances: interpreting the situation from a techno-mathematical point of view (crossing a shape), exploring a conceptual model (discovering three theorems mathematically promote understanding of the theorems' sense), and producing a techno-mathematical problem and solution (the problem that was posed).

#### **Trigger 4. Designing a lesson to teach the digital problem, analysing the lesson, and then redesigning the problem**

Rebecca conducted a lesson using the digital problem presented in Trigger 3 (<https://www.geogebra.org/m/jkzxnwn>). In the first stage, she asked her students to solve the problem independently. While solving it, one of the students commented that the sketch seemed to show a parallelogram. Rebecca at first thought this was parallelogram  $DZKL$ , because  $ZD$  is parallel to  $LK$ . This is in fact true based on what is stated in the question (congruent right triangles). Based on her assumption that this concerns a parallelogram, she tried to find other segments with the ratio  $DZ \times ZC = AZ \times ZQ$ . One student noted that  $DL = ZK$  because it seems that they are opposite sides of a parallelogram. But this 'fact' led Rebecca to question if it actually was always a parallelogram, and she asked the student to check this. By dragging point C, it became immediately evident that this was not so. This was the contingent event. Rebecca realized that there was an error in the way she had written the problem. She corrected it and posed it again.

A look at Rebecca's research journal suggests that she posed this problem without solving it herself. Had she solved it before teaching, she would have experienced the process of justifying and explaining the solution, and she might have discovered the error about the type of shape. The act of teaching the problem drew her attention to the inaccuracy embedded in the problem. She now understood that a dynamic drawing can include different representations and different shapes, and that preparing the problem based on only one representation of the drawing and ignoring possible others led to her mistake in the wording of the problem and her anticipation of the solutions she expected to receive. This incident shows that teaching digital problems is necessary to complete and develop techno-mathematical fluency-literacy skills such as technical drawing and technical software skills. TRs can pose problems and reach this literacy in one case yet not develop it in another.

#### **Summary**

'Triggers are conceived as instances of noticing, where an impulse draws a poser's attention and "triggers off" a mathematical re-action, one of the outcomes of which is a creation of a problem...' (Kontorovich, 2020, p. 389). In this research, triggers were also 'instances of noticing' that led to posing problems that were also 'triggers' for promoting some domains of TmL knowledge used for teaching mathematics using digital tools. The present study presents evidence that teachers can develop techno-mathematical literacy to implement the use of non-pencil-and-paper-technology provided that the following three situations occur: one, teachers act as a TR; two, they pose problems using DTs; and, three, they actively teach the problems to their students. In other words, the teachers' TmL knowledge was enriched by the combination of these three aspects. First, by acting as TRs, they



carefully documented (contingent) situations and paid attention to certain events. Second, having to pose the problems created an iterative activity both in determining the goal of the problem mathematically and in representing the problem using the DTs, a process that led both to understanding specific mathematical concepts in a new way and understanding the concept in a new digital (ontological) way. Finally, teaching the problem in class led to interaction with the students who raised new questions about the digital representation or the mathematics or both.

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