



**HAL**  
open science

# The effectiveness of an interactive, autonomous teaching application on the topic of boundedness among pre-service mathematics teachers

Borbála Neogrády-Kiss, Csaba Csapodi, Ádám Besenyei

## ► To cite this version:

Borbála Neogrády-Kiss, Csaba Csapodi, Ádám Besenyei. The effectiveness of an interactive, autonomous teaching application on the topic of boundedness among pre-service mathematics teachers. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04412088

**HAL Id: hal-04412088**

**<https://hal.science/hal-04412088>**

Submitted on 23 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The effectiveness of an interactive, autonomous teaching application on the topic of boundedness among pre-service mathematics teachers

Borbála Neogrády-Kiss<sup>1</sup>, Csaba Csapodi<sup>1,2</sup> and Ádám Besenyei<sup>1</sup>

<sup>1</sup>Eötvös Loránd University, Budapest, Hungary; [nkissborbala@gmail.com](mailto:nkissborbala@gmail.com)

<sup>2</sup>Alfréd Rényi Institute of Mathematics, Budapest, Hungary

*It is often the case in university education that students are unable to attend classes in person, especially since the Covid-19 pandemic. To overcome this problem, we developed an interactive, autonomous teaching application for pre-service mathematics teachers on the topic of boundedness, which teaches both definitions and problem solving. This application is very different from the teaching materials and tools usually used in such cases. We evaluated its efficiency in teaching without any teacher intervention among pre-service mathematics teachers. We found that almost all students had mastered the basics and reached Bloom's level 3, but there was also a relatively high rate of students who had progressed to Bloom's level 6.*

*Keywords: Intelligent tutoring systems, calculus, pre-service teacher education.*

## Introduction

In Hungary, university mathematics teaching takes the form of lectures and seminars. In the lectures, the lecturer explains definitions, statements, theorems and proofs to the whole class (up to several hundred students). The seminars are in smaller groups of about 10-20 students, who solve problems. If someone has a child, is ill, is doing a semester abroad or cannot be present in person for most of the classes for other reasons, there are two cases. The first is if there is a high level of absence in seminars. In this case, the students can request an individual curriculum and they must consult with the teacher about the requirements. However, the monitoring of the students' progress is significantly reduced, as it is not possible to see how they can solve the problems. The other case is absence from lectures. This is allowed in Hungary, so there is no need for an individual curriculum, it is enough to take an exam in the subject, so in this case it is not possible to follow the students' progress, either.

To overcome this problem, we created an online application for use in pre-service mathematics teacher training.<sup>1</sup> It does not involve a teacher, but by answering questions on the computer, the student learns definitions and solves problems, including statements that need to be proven. If the students are familiar with the prior knowledge required to use the application, they can progress through the course without any help from a teacher. If the students answer a question incorrectly, it is essential that, apart from exceptional cases, they are given questions to help them until they can answer the question correctly. One of the great advantages of the application is that it stores the answer to each question and the time spent answering the question, so that the teacher can look at it at any time. The other big advantage is that the student can progress entirely at his or her own pace, as opposed to a face-to-face format, involving a large number of students. Once the application was ready, we assessed how the application could teach the concepts on its own, without any teacher

---

<sup>1</sup> The application is available in Hungarian at <http://www.guessmaths.com>.

assistance. Our goal in creating the application was both to help students who, for one of the reasons listed at the beginning, are unable to attend lectures or seminars, and to make their progress easier for their teacher to follow. On the other hand, we wanted to do this in a way that would give them a tool that is very different from the one they have had so far, and more effective.

Research question: Is the application we developed suitable for teaching and, if so, what level of understanding can be achieved with the help of the application?

## **Theoretical background**

Distance learning can be a great solution for adult education because learners in this age group need flexibility. However, it is not without problems, as motivation may be lost due to lack of personal contact with teachers, and there may also be high start-up costs. In addition, there may be problems, such as inexperience with this type of teaching, which can lead to academic problems. (Galusha, 1998) In contrast, research by Soffer and Nachmias (2018) found that for those who completed an online and face-to-face version of the same course, the online participants had higher grades and there was no difference in the proportion of students who successfully completed the course. Besides the inexperience of students, the inexperience of teachers in using such interfaces may be an issue. Spoel et al. (2020) have shown that previous experience with ICT did not play a significant role in managing the transition to online education, with the suggestion that the new technologies are not a burden for teachers.

Artificial intelligence is also used in education. (Chen et al., 2020) Artificial intelligence in this paper refers to a computer system that can perform a task with certain capabilities and intelligent behaviour that was originally thought to be unique to humans. (Russel et al., 2010) There are many advantages to using artificial intelligence. One of the benefits of using artificial intelligence is that it can help students through problems by diagnosing individual students' learning problems and providing personalized support to maximize their learning performance in mathematics courses (Hwang & Tu, 2021).

The goals of working with artificial intelligence in education can be basically divided into four sub-areas (Chaudhry & Kazim, 2021) The first goal is to reduce teachers' workload without affecting educational outcomes. The second is contextualized learning for students. This is also very important because everyone needs a specific way of learning depending on their previous knowledge, social background, economic well-being and emotional state (Ma et al., 2014). The third is to improve learners' understanding, not just of what they know, but how they learn and what pedagogies work for them. The fourth goal is to provide intelligent learning environments that can interact with learners, provide personalized feedback and improve their understanding of certain topics. Examples of the latter include iTalk2Learn (Grawemeyer et al., 2015), which was originally focused on teaching fractions or ASSISTments, which provides students with instant feedback on whether they have done a task well while working on it, and provides data on this to teachers (Heffernan & Heffernan, 2014). In addition, Pearson has launched a calculus learning tool called AIDA, which gives students step-by-step guidance and helps them complete calculus tasks (Chaudhry & Kazim, 2021). However, there is currently no fully autonomous digital tutoring system (Chaudhry & Kazim, 2021).

Our application focused on the topic of boundedness. As Bloch (2003) writes, the difference between the maximum and the upper bound, and between the minimum and the lower bound, often causes problems for students. In addition, students have had problems with the difference between monotonicity and boundedness. In addition, for a deeper understanding, it is important to explore what it means that a function is not bounded. Also, constructing functions contribute strongly to a deeper understanding of the corresponding concepts.

The research was based on Bloom's taxonomy. This taxonomy was created in 1956 to define and distinguish between different levels of human cognition. Anderson et al. (2001) revised this, and Crompton et al. (2018) used its revised, cognitive-dimensional structure in their research, and we based our research on it as well. The following 6 levels of the cognitive dimension were defined (Table 1):

**Table 1: Anderson et al.'s further categories of Bloom's taxonomy (Crompton et al., 2018, p. 4.)**

Table 1. The cognitive processes dimension — categories, cognitive processes (and alternative names)					
lower order thinking skills			higher order thinking skills		
remember	understand	apply	analyze	evaluate	create
recognizing (identifying)	interpreting (clarifying, paraphrasing, representing, translating)	executing (carrying out)	differentiating (discriminating, distinguishing, focusing, selecting)	checking (coordinating, detecting, monitoring, testing)	generating (hypothesizing)
recalling (retrieving)	exemplifying (illustrating, instantiating)	implementing (using)	organizing (finding coherence, integrating, outlining, parsing, structuring)	critiquing (judging)	planning (designing)
	classifying (categorizing, subsuming)		attributing (deconstructing)		producing (construct)
	summarizing (abstracting, generalizing)				
	inferring (concluding, extrapolating, interpolating, predicting)				
	comparing (contrasting, mapping, matching)				
	explaining (constructing models)				

This differs from Bloom's structure in that the category titles have been changed from nouns to verbs to more clearly identify the specific cognitive processes involved in learning. In addition, the category "synthesis" has been replaced by "evaluation" and renamed "create". Tasks that measure these levels can be defined for this structure.

Bloom's taxonomy, and its revised version, is also effectively used to measure levels of understanding of mathematical concepts (Radmehr & Drake, 2019), and can therefore be used to measure the teaching effectiveness of our application.

## Structure of the application

The application is designed for first-year university pre-service mathematics teachers (mainly 18-19 year olds). It consists of 13 main exercises in which students learn about the concept of boundedness through questions. The motivation for choosing this topic was that, on one hand, it is not part of the secondary school curriculum in Hungary and, on the other hand, we saw it as an area that has not been particularly researched so far. Within a task, students have to answer several questions. First they are asked one question and then, depending on their answer, they are asked another question. If they answer correctly, they are then given the next question needed to understand the material. If they answer incorrectly, they are asked a series of supporting questions, and if they answer these correctly,

they are asked the question they originally answered incorrectly. If they give an incorrect answer to a question that is part of the expected knowledge, the application will highlight the deficiency and ask them to contact their teacher. If they have already managed to make up the gap with the help of a teacher, they can continue from the beginning of the task. To compile the application, we took into consideration Bloch's (2003) findings and interviewed several teachers who teach the subject.<sup>2</sup>

Our application can be seen as an example of using artificial intelligence in education, since if students are aware of the required prior knowledge, the application can work completely independently and teaches boundedness from the basics to more complicated problems, such as rigorously proving that a given function is unbounded. This makes the application definitely unique. We will see later that the application teaches effectively, so since no teacher is needed to complete it, it also contributes to Chaudhry & Kazim's (2021) first goal of using artificial intelligence in education, which is to reduce teachers' workload. However, it also achieves the second goal, because if the students cannot answer something, the application does not just tell them that the answer is incorrect and what the correct answer would be, or give a hint, but continues to ask simpler questions until the students, with the help of their prior knowledge, find the answer to the original question on their own. In addition, it also serves the third purpose, since, as already mentioned, the application stores the answers to each question and the time spent answering, so that conclusions can be drawn about the students' difficulties.

## Methods

This was an experiment using one-group pretest-posttest design, which is used in mathematics education research, e.g. Hidayah et al. (2018). We used this, because the students were not expected to know the concept of boundedness when they entered university, but those, who have been studying mathematics on a higher level might have encountered the concept. The research was carried out as follows. First, we asked students 3 short questions to see if they had encountered the concept of boundedness before and, if they had, whether they could define it or determine whether a simpler function was bounded. This was necessary we wanted to test only those who did not know the concept to see how they would cope using the application alone. Therefore, we did not use the tests of students who already knew the concept of boundedness. A total of 36 students answered these three questions, of which 31 were not familiar with the concept.

After this short survey, the students started to use the application. For these two tasks they had 90 minutes in total, so compared to the amount of time spent in face-to-face teaching in Hungary on teaching boundedness in lecture and seminar combined, we gave them roughly the same amount of time. After this one-and-a-half-hour session, we measured the level of understanding of the concept after about a week. In total, there were 20 students who showed up for both the lesson and the survey who either had not learned the concept or, if they had, had learned it poorly. To measure their understanding of the concept, we created a set of six tasks and gave the students 45 minutes to complete them on paper. Since we could not find a formal set of tasks measuring the level of

---

<sup>2</sup> For more details, see:

<https://docs.google.com/document/d/1gaykua4hwbm1mb7SIfo1WJYkrJyRXqrstuEBo81Viy8/edit>

understanding of the boundedness, we looked for a task for each of the 6 levels in Bloom's taxonomy, with the aim of measuring the level of understanding.

Students were asked to complete the following tasks:

1. Choose the definition of a *function bounded from below* from the following:
  - a. The function  $f$  is bounded from below if there is a real number  $V$  such that the function value  $f(x)$  corresponding to every element  $x$  of the domain is greater than or equal to  $V$ .
  - b. The function  $f$  is bounded from below if it is true for the values  $f(x_1)$  and  $f(x_2)$  belonging to all elements  $x_1, x_2$  of the domain that if  $x_1 < x_2$ , then  $f(x_1) \geq f(x_2)$ .
  - c. The function  $f$  is bounded from below if  $f(x) = f(-x)$  is true for the function value  $f(x)$  belonging to every element  $x$  of the interpretation domain.
2. Decide whether the following functions are bounded from below:
  - a.  $a(x) = 10x, D_a = \mathbb{R}$
  - b.  $b(x) = \left| \frac{1}{x} \right|, D_b = \mathbb{R} \setminus \{0\}$
  - c.  $c(x) = (x - 2)^2 + 3, D_c = [-1, 5]$
  - d.  $d(x) = 2^x, D_d = (2, 8)$
3. Calculate the greatest lower bound of  $f(x) = x^2 + 8x + 15, D_f = \mathbb{R}$ .
4. Continue with the definition of the unbounded function from below without using a negator. A function is unbounded from below if. . .
5. Give a function with a formula whose domain is the set of real numbers, bounded from below and monotonically decreasing.
6. Prove that the function  $f(x) = -2x, D_f = \mathbb{R}$  is unbounded from below.

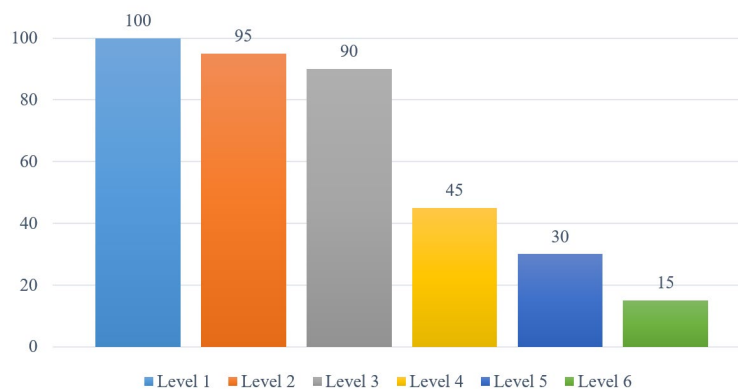
These tasks can be linked, in this order, to the six levels of Bloom's taxonomy. In the following, we have italicised the words that appear in Bloom's table, which link the corresponding tasks to each level. The first task can be linked to the first level because it only requires the students to *recall* the definition that they have already learned through the application. The second task can be linked to the second level because it is necessary to *categorise* given functions according to the lower bound. The third task is related to level three because they have to calculate the maximum lower bound of a given function by *applying* the definition and *carrying out* the appropriate calculations. The fourth task is related to the fourth level and not to the first because the definition of a function unbounded from below is not included in the application, so the students had to find it out on their own. Here, the students had to be able to *distinguish* between unbounded and bounded functions, *find* the common property in unbounded functions and *attribute* the appropriate definition. The fifth task can be linked to the fifth level because it requires the students to independently *detect* the differences between the corresponding concepts, to *check* that the function he has specified does indeed correspond to the parameters given. And the sixth problem can be linked to level six because it requires the students to prove that a given function is unbounded from below, that they have never seen such a proof (only a proof that is unbounded from above), and that they must therefore *design* and *construct* the proof on their own, using the definition.

Students were asked to solve the problems on paper, and their assessment was not automated, but was done manually, using an objective solution key.

## Results and discussion

In the survey, a Bloom's level was accepted as completed if the student had solved all the preceding tasks without error. An exception to this was made for the second task, where it was accepted if someone solved only three of the four sub-tasks correctly.

Taking these into account, the following results were obtained (Figure 1):



**Figure 1: Percentage of students achieving the given level**

The results show that the application performed very well on its own, so the answer to our research question is that the application is indeed a good way to teach the concept, it gets almost all students to Bloom's level 3 easily and the percentage of students who go on to the next levels is relatively high. However, the results also confirm that we can improve the application further and there were several responses that are definitely worth mentioning and in several cases help us to improve the application. Two of these are mentioned here. The first answer worth mentioning, which occurred in 20% of the students, was that in the fourth task, when asked to define a function that is not bounded from below, the answer given was the definition of a function that is bounded from above. We believe that these students may have confused "not bounded from below" with "bounded from above". Although there were examples in the application of a function that was neither bounded from below nor bounded from above, this was a relatively common mistake, so probably not enough emphasis was placed on it in the application. But this is in accordance with the point that, as Bloch (2003) writes, it is important to explore what a not bounded function means. In addition, as we saw in the results, 45% of students solved this problem (none solved it by failing a previous problem), and this is a big change from the 90% who got to the third problem, and certainly shows that more or different attention should be paid to this in the application. For example, it is worth asking a specific question about whether a function being unbounded from above can still be unbounded from below.

The second type of response to note was for Task 6. Although, due to Bloch's (2003) findings, we dealt specifically with the fact that being monotonically increasing does not imply that a function is bounded from above or unbounded from above, many in this problem have tried to prove that the function  $f(x)=-2x$  is unbounded from below merely by showing that the function is strictly monotonically decreasing. While we consider it a very good result that 15% of the students were able

to prove rigorously that the function is unbounded from below only by completing the application (in addition, another 15% started with a good idea, but made minor mistakes, such as not looking at all possible cases), we believe that this mistake has been made to the extent that we should improve this question further in the application. For example, by paying special attention to the fact that being strictly monotonically increasing does not imply that the range of the function will be the set of real numbers, thus it is not certain that the function is not bounded from above even if the set of real numbers is the domain of the function.

## Conclusions

In conclusion, the answer to our research question is that the autonomous application we have developed is suitable for teaching in this topic. In terms of depth of understanding, it takes students to a very high rate (90%) of Bloom's level 3, but some can even reach level 6 with the help of the application alone (15%). However, the application could be developed further and, as explained in the previous section, we have come up with ideas for this based on the results of the research.

Considering the effectiveness of the application, a further step could be to create similar applications for other topics and to see if they remain effective or if they can be taught in roughly the same amount of time, as it may be more difficult to create such applications for other, perhaps less illustrative or more complex definitions, or if the applications we create require more time from students than face-to-face learning.

We could use additional instruments, videos, interviews or questionnaires to further research. These instruments could be used to examine students' attitudes towards this form of teaching and their motivation when they have to learn more concepts using this method. They would also allow an individual assessment of students' understanding. It would also be useful to examine teachers' attitudes towards the application, using these instruments.

Finally, it can be a very useful and interesting question how this material can be used as an aid, how the teacher can make conclusions from seeing the students' paths of completion, whether it helps to give personalized help to a student, or whether the teacher can save time by using this application as an aid and just supplementing the students' knowledge based on the problems they encounter.

## References

- Anderson, L. W. (Ed.), Krathwohl, D. R. (Ed.), Airasian, P. W., Cruikshank, K. A., Mayer, R. E., Pintrich, P. R., Raths, J., & Wittrock, M. C. (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's Taxonomy of educational objectives*. Complete ed., Longman.
- Bloch, I. (2003). Teaching functions in a graphic milieu: What forms of knowledge enable students to conjecture and prove? *Educational Studies in Mathematics*, 52(1), 3–28. <https://doi.org/10.1023/A:1023696731950>
- Chaudhry, M. A., & Kazim, E. (2022). Artificial Intelligence in education (AIEd): A high-level academic and industry note 2021. *AI and Ethics*, 2(1), 157–165. <https://doi.org/10.1007/s43681-021-00074-z>



- Chen, X., Xie, H., Zou, D., & Hwang, G. J. (2020). Application and theory gaps during the rise of artificial intelligence in education. *Computers and Education: Artificial Intelligence*, 1, 100002. <https://doi.org/10.1016/j.caeai.2020.100002>
- Crompton, H., Burke, D., & Lin, Y. C. (2019). Mobile learning and student cognition: A systematic review of PK-12 research using Bloom's Taxonomy. *British Journal of Educational Technology*, 50(2), 684–701. <https://doi.org/10.1111/bjet.12674>
- Galusha, J. M. (1998). Barriers to learning in distance education.
- Grawemeyer, B., Gutierrez-Santos, S., Holmes, W., Mavrikis, M., Rummel, N., Mazziotti, C., & Janning, R. (2015, January). Talk, tutor, explore, learn: Intelligent tutoring and exploration for robust learning. *Proceedings of the 17th International Conference on Artificial Intelligence in Education (AIED)*.
- Heffernan, N. T., & Heffernan, C. L. (2014). The ASSISTments ecosystem: Building a platform that brings scientists and teachers together for minimally invasive research on human learning and teaching. *International Journal of Artificial Intelligence in Education*, 24(4), 470–497. <https://doi.org/10.1007/s40593-014-0024-x>
- Hidayah, I., Dwijanto, & Istiandaru, A. (2018). Manipulatives and question series for elementary school mathematics teaching on solid geometry. *International Journal of Instruction*, 11(3), 649–662. <https://doi.org/10.12973/iji.2018.11344a>
- Hwang, G. J., & Tu, Y. F. (2021). Roles and research trends of artificial intelligence in mathematics education: A bibliometric mapping analysis and systematic review. *Mathematics*, 9(6), 584. <https://doi.org/10.3390/math9060584>
- Ma, W., Adesope, O. O., Nesbit, J. C., & Liu, Q. (2014). Intelligent tutoring systems and learning outcomes: A meta-analysis. *Journal of Educational Psychology*, 106(4), 901. <https://doi.org/10.1037/a0037123>
- Radmehr, F., & Drake, M. (2019). Revised Bloom's taxonomy and major theories and frameworks that influence the teaching, learning, and assessment of mathematics: a comparison. *International Journal of Mathematical Education in Science and Technology*, 50(6), 895–920. <https://doi.org/10.1080/0020739X.2018.1549336>
- Russell, S.J., & Norvig, P. (2010). *Artificial Intelligence: A Modern Approach*. Prentice Hall.
- Soffer, T., & Nachmias, R. (2018). Effectiveness of learning in online academic courses compared with face-to-face courses in higher education. *Journal of Computer Assisted Learning*, 34(5), 534–543. <https://doi.org/10.1111/jcal.12258>
- Van der Spoel, I., Noroozi, O., Schuurink, E., & van Ginkel, S. (2020). Teachers' online teaching expectations and experiences during the Covid19-pandemic in the Netherlands. *European Journal of Teacher Education*, 43(4), 623–638. <https://doi.org/10.1080/02619768.2020.1821185>