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BlueBots and irrational numbers: Contingency moments in teaching mathematics and computational thinking in teacher education

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Computational thinking (CT) has recently been introduced as a part of Norwegian mathematics curricula. For that reason, we engaged prospective primary teachers in activities that intended to combine CT and mathematics. In this paper, we explore what kind of mathematical opportunities could be identified during these activities. Focusing on contingency moments, in the sense of Rowland et al. (2005), related to mathematics, we found three such moments in the activity of programming a BlueBot to move in routes resembling geometrical shapes. The constraints of the BlueBot led to unexpected situations in which the opportunity to discuss irrational numbers, angles, shapes, perspective, and the need to prove mathematical concepts could be taken advantage of. There is a potential to combine CT and mathematics for prospective primary teachers in a way that more fully acknowledges the two disciplines.

Keywords: Mathematics, computational thinking, contingency moments, teacher education.

Introduction

In the 2020 update of the Norwegian primary and secondary school curriculum, *computational thinking* (CT) was introduced as an integrated part of the mathematics, natural science, music, and arts and crafts curricula from grade 4 and upwards. This has led to the need to include CT in mathematics courses in teacher education programmes. Since CT in Norwegian curricula is not intended to be a free-standing subject, but rather an integrated part of other subjects. This leads to questions regarding how to teach mathematics and CT in a mutually beneficial way.

This study is part of the larger research project MASCOT¹, which aims to develop knowledge about teaching, learning, and assessment processes of CT in both Nordic schools and Nordic teacher education. MASCOT uses a design-based research approach with several iterations in different institutions in the Nordic countries. This paper reports on parts of one of these interventions conducted in one of the participating institutions.

Hickmott et al. (2018) suggest that research teams should draw on expertise to bridge discipline silos for integrating curricula across subjects. We believe that when combining different subject areas there is both the possibility for an expanded understanding and at the same time there is a potential for unexpected situations. The intervention aims to integrate concepts from CT (as formulated by Shute et al., 2017) with mathematics and generate experiences of abstract concepts from CT in a mathematical context. The focus of this report is opportunities for mathematical exploration that arose during the activities. By identifying and analysing these situations, we aim to uncover moments when the teacher educators came up short, in knowledge or preparation, and see how these situations were managed. This is intended, in turn, to form the basis of an analysis of what steps can and need

¹ <https://uni.oslomet.no/mascot/about-mascot/>

to be taken by teacher educators in order for them to be able to predict and exploit these kinds of mathematical difficulties and opportunities.

The research question of this paper is: *Which opportunities for exploring mathematics can be identified in a lesson that combines mathematics and computational thinking for prospective primary school teachers?*

Previous research

Although there is much literature on CT and education in general, there is considerably less in the intersection of CT and mathematics education, on the one hand, and of CT and teacher education, on the other. Hickmott et al. (2018) presents a systematic review focused on finding links between CT and K-12 mathematics. Nordby et al. (2022) extend this by giving a systematic review of the literature on CT activities in primary mathematics education. While Hickmott et al. (2018) remark on a relatively small number of studies linking the learning of mathematical concepts to CT, Nordby et al. (2022) find that most of their selected studies emphasize mathematics content. In most cases, however, this takes the form of using CT to reinforce students' understanding of mathematical concepts, or as a way for them to demonstrate this understanding.

The combination of CT and mathematics is an area of much active research; involving questions of what precisely the concept of CT encompasses, to what extent and in which sense there are similarities and overlaps between CT and mathematical thinking, and how to integrate mathematics in CT activities. Nordby et al. (2022) argue that “there is a need to help teachers understand how CT can be used productively in mathematics and vice versa” (p. 35). This is, in our experience, also the case for teacher educators, for which the combination of CT activities with mathematics can be an equally new experience. Indeed, in their systematic review on literature concerning mathematics learning in activities aimed at developing CT skills, Barcelos et al. (2018) remark that “most of the described didactic experiences have basic education students as the target audience. However, a relevant amount of experiences was developed with university students. On the other hand, there are not enough reports of experiences developed for the initial and continuous training of teachers” (p. 832). In the few studies we have found, the focus is directed more towards the concept of CT rather than towards the learning of mathematical concepts through CT. For instance, Gadanidis et al. (2017) developed a course for elementary school prospective teachers focusing on CT in mathematics education but focused their questions on what the prospective teachers learned about CT in mathematics education and what attitudes they developed towards CT. Geraniou and Hodgen (2022) researched mathematics teacher educators' CT practices and mathematics teacher educators' views on the link between mathematical thinking and CT but did not focus on issues specifically concerning linking the learning of mathematical concepts to CT. Notably, however, they did conclude with a call to “articulate the nature of Computational Thinking Pedagogical Content Knowledge” (p. 7). In this intervention, we intended to combine the knowledge of CT with mathematics by planning the lesson with teacher educators from both disciplines, with the intent to fully integrate teaching CT and mathematics.

Contingencies in the knowledge quartet

Our research question is which opportunities for exploring mathematics can be identified in a lesson that combines mathematics and computational thinking. The Knowledge Quartet (Huckstep et al., 2011; Rowland et al., 2005) is a framework for the analysis and development of mathematics teaching. The Knowledge Quartet consists of the four dimensions of foundation, connection, transformation, and contingency. We employ an adapted version of this framework to identify situations where the teaching can be improved relative to the recorded lessons, to inform the next intervention.

Since the teacher educators in the intervention were new to teaching CT and mathematics together, we were especially interested in unplanned events and opportunities for working with mathematics in the lessons. This corresponds to the *contingency* dimension of the Knowledge Quartet. This dimension concerns the moments in a lesson where the teacher educator must make an unplanned choice, respond to an unexpected idea in the classroom, or otherwise relate to opportunities to deviate from the planned lesson. The question that must be decided by the teacher within the moment is what to pursue, what to integrate into the lesson and what not to change. They can deviate from the original plan or develop on students' questions related to the original plan, or they can use the opportunities of contingencies to explore mathematic problems not thought of in the original plan. Such situations can be identified during or after the lessons.

Participants, design of intervention and context

The intervention was designed by three members of the MASCOT project, all teacher educators. There was one representative from the mathematics department and two from the department of digital competence. The lesson plan was carried out by the teacher educators of the course, including the members of the project. The teaching was conducted in several parallel classes in a mathematics course for prospective teachers, by different combinations of mathematics teacher educators and digital competence teacher educators. In this paper data from two of these classes are analysed. When teaching the first class there was one mathematics teacher educator (MTE1) and one digital competence teacher educator (DCE1). The same teacher educators participated in the second class, together with an additional mathematics teacher educator (MTE2). The data was collected during a 4-hour lesson conducted in each of the two different classes. We video-recorded the teacher educator(s) work in the front of the classroom in both classes. In addition, in the first class, we recorded the work of two groups of four prospective teachers. In the second class, we recorded the work of one four-person student group in addition to the teacher educators.

The lessons presented students with a sequence of tasks of increasing complexity. These were designed to develop the role of the students from being users of algorithms towards being creators of algorithms. The intention was to model activities they could adapt to their future teaching, while also learning CT and mathematics. The first activity, which took place outdoors, was to follow instructions involving conditions, loops and variables. The students then moved indoors, where they worked on different activities to produce their own algorithms in an analogue environment. The activity we focus on in this paper is from the end of the intervention when the students worked with BlueBots to make algorithms for tracing different geometric shapes (square, rectangle, triangle, and circle). A BlueBot

is a physical digital artefact that can be controlled either by arrows on the BlueBot itself or by a block-based program, run for instance on an iPad. The block-based program has commands where the user can program the BlueBot to move a whole number of steps forward and backwards and to turn in steps of 90- or 45 degrees. The task given to the students was to write an algorithm using step-by-step instructions in the block-based program so the BlueBot moved in a square. When this was accomplished, they could use the concept of loops from CT, to simplify the code. Subsequently, they were to do the same operation to make the BlueBot move in a rectangle and then a triangle.

In the analysis, a preliminary selection from the data material was made based on identified occurrences of mathematical discussion or mathematical problems. This selection primarily consisted of video recordings of both the teacher educators and the student groups. These recordings were then reviewed and analysed for contingency moments. This was done by reviewing the video recordings of both the teacher educators and the student groups. We first reviewed the data intending to discover possibilities for mathematical contingencies. We looked for situations in the classroom where students asked questions or came with comments that could lead to contingencies. When we identified possible relevant episodes we noted the time, cross-checked with the other recordings from the same class and transcribed in detail the situation from the different recordings. We then studied the same part of the lesson in the other class, to compare the outcome from the situations.

Results

From the analysis, we identified three moments of contingencies related to mathematics. They were all a result of tasks involving the BlueBot and geometric shapes. As we pointed out earlier the BlueBot has some constraints regarding angles and whole number steps. These constraints sparked discussions in the student groups when solving the tasks. We saw unexpected moments connected to geometrical shapes (the making of a triangle, a circle, and a star), and the notion of mathematical argumentation and proof in connection to what solutions exist within given conditions. When designing this activity, the idea was that the students would make an algorithm for each geometric shape, debug their algorithm, and hopefully use a loop to make an elegant solution. The planned mathematical content was geometrical shapes. The expected use of mathematics was that the students would need to use definitions of the different geometric shapes and the expected CT was that the students would make use of ideas like conditions, loops and variables we had explored in the previous activities.

Although the intervention was conducted in a mathematics lesson, it is apparent that the main focus was teaching CT. With regards to making a “triangle”, the plan was that the students could use a stepwise approach to make a shape visually similar to a triangle. However, with the block-based program, the students could make the BlueBot turn 45° in addition to 90°. This meant that the students could program the BlueBot to almost make an isosceles right triangle, however, the length of the sides is not possible to make with whole number lengths. These constraints and the student responses sparked some unexpected situations.

The first contingency moment was related to angles and perspective. When looking at a shape, the students took a bird’s eye view and calculated the angles accordingly. The problem arose when programming the BlueBot, which requires a linear perspective relative to the line that defines the edge of the geometrical shape. The students wanted to make the BlueBot turn so that the angle it

made from their perspective was 45° , but from the BlueBot's perspective, the angle it had to turn was 135° ($45^\circ + 90^\circ$). The data material shows similar discussions in many of the groups in both classes. The contingency seems to have appeared because the perspective was different, and because of the constraints of the BlueBot. This was something unexpected when planning the lesson. In our data material, we can see that the students had several interesting discussions about angles and perspective. These possible contingencies were never developed.

The second form of contingency appeared when a group in the first class were challenged to make a circle. This was not in the original plan but was introduced as an extra activity for the groups that finished the other tasks. In the discussion with an observer, the students suggested that they could make a circle by making the BlueBot turn 360° . The observer then points out the difference between a point and a circle in mathematics.

Observer: A point, does it have a size? Is it a circle when it spins around its own axis? Purely practically you may possibly have it, but purely theoretical? Is it then a circle? A point?

The observer uses the moment of contingency to elaborate on the difference between a point and a circle in mathematical terms and how it may differ from what we think of the term in everyday situations. The potential moment of contingency was never picked up by the teacher educators and therefore confined to this group's discussion. The same moment of contingency did not appear in the other class in our data material.

The third contingency moment we discovered was related to the task of programming the BlueBot to make a triangle. After discussing the problem with angles as mentioned before, the students start to notice something about the shape the BlueBot makes. During their work, groups figured out how to make the BlueBot move the required 45° - 45° - 90° for an isosceles triangle. At the same time, we can hear snippets of talk about Pythagoras' theorem, and a group who talked about if it needed to be a right triangle. One student asks the group "But did it get all the way now? New round of debugging".

In the first class, we have the following situation at the end of the lesson. MTE1 asks the students if they have anything fun to share. One student asks about the possibility of making an isosceles triangle and to explore what kinds of triangles are possible to make, with their future pupils.

Student: It is possible to make an isosceles triangle with that one, and in a way find out with pupils, have one of those [...] and then find out what kind of triangles one can make with this?

MTE1 addressed the statement and talked about the need to define different kinds of triangles, then proceeded by addressing an argument about how there was no possible way to make a circle. Another student says: "We worked with square root and Pythagoras to figure it out. To make an exact triangle".

The lesson was then wrapped up with the students' getting instructions for tidying up, without any more talk about the students' suggested themes like square root, Pythagoras theorem and justification of which kind of triangles it is possible to make. This indicates some moments of contingencies that occurred during the lesson, that were not addressed or elaborated on.

The third contingency situation brought with it some interesting developments in the second class. MTE2 starts a conversation focused on the triangle shape, where the students are challenged with

whether the BlueBot gets exactly back to its starting position. One of them answered affirmative and said that the group tried on the floor and that steps of lengths 4, 6 and 7 worked. The interaction continues with students talking about the calculations needed for turning the BlueBot the correct angle, while MTE2 focuses on the length of the sides. These interactions continue with a discussion about whether the BlueBot returns to the starting point or not. MTE2 leads the students towards thinking about which conditions the possible angles and whole-step movement of the BlueBot gives for possible triangles. A student asks if it needs to be whole steps, which is confirmed. The student states: “Then it will be very big because I cannot find it”, referring to the group’s work of test and retest of different possible solutions.

MTE2, with input from a student, calculates on the blackboard what happens in the case of the sides being of length 1. The length of the hypotenuse is calculated to the $\sqrt{2}$, and MTE2 points out that this is not a whole number. The length 12 is then suggested and rejected, and MTE2 continues by asking what would happen if one could scale up the sides of the right isosceles triangle. MTE1 refers back to the students’ previous teaching in mathematics when argumentation and proof were discussed, and says:

- MTE1: Last spring when you had mathematics, we talked about generalising. Is it always this way? For now, we have tested some examples that it does not work on, but because we have tried five examples it is not necessary like number six will work.
(...)
- MTE2: But why don’t we manage [to make the triangle]? Can we choose a whole number at the side such that we get a fraction for the hypotenuse? If I get a fraction on the hypotenuse, then I can just scale up the triangle with the [denominator] n . Then I... (slight pause) then I get all I need to find out if the root of two is a fraction. Is the root of two a fraction?

Here we see that MTE2 utilises the question from the students about whether the BlueBot returns to the starting point. From this, the teacher continued the explanation of why the hypotenuse of a right isosceles triangle never would be a fraction, and therefore be an irrational number. From the same situation, it is also interesting to notice that MTE1 was trying to connect to the student’s previous experiences with argumentation and proof.

As seen, the end of the lesson in the second class played out quite differently than planned, and different from the first class, and here MTE2 used the opportunity to explore an unexpected mathematics situation. The issue of what kind of triangles were possible to make out of the constraints of the BlueBot led to a geometric explanation of the $\sqrt{2}$ being irrational, and thereby it was possible to expand the students’ ideas about irrational numbers. Another important mathematical point that came up is the notion of proof, that testing several examples is not enough to prove the existence or non-existence of solutions.

Discussion and further work

As we have pointed out in the result section, we identified three situation types in the lessons that lead to moments of contingency. The first situation is related to perspective, the second is related to the concepts of point and circle and the third situation is related to irrational numbers. The first situation can bring up an interesting discussion on perspective and angles. The BlueBot has the perspective of the line being “drawn”, while the students have the bird’s eye view. This leads to the

need to connect the different perspectives to program the BlueBot to move in the correct pattern. With the student input about angles and perspectives, the teacher educators have an opportunity to discuss both mathematical issues and the didactical usefulness of exploring geometric shapes in primary classrooms. When students were asked to make a circle, one of the groups suggested making the BlueBot do a 360° turn. It was pointed out by the observer that they then would instead make a point. If this point had been elaborated on for the whole class, students could have explored the difference between mathematical concepts and how they are used in everyday speech.

The constraints of the BlueBot led to unexpected issues when students tried to make it follow a triangular pattern and return to the starting point. From the analysis, we see possible moments of contingency appear in the same parts of the lesson in both classes. However, they played out differently. The students had the same equipment, the same task and were working in groups, but the teacher-educator combination was different. The mathematics teacher educator in the second class took the opportunity to explore argumentation concerning the irrationality of the $\sqrt{2}$ and deviated clearly from the plan for the lesson. The notion of irrational numbers will be an important issue in the planning of the next design cycle. As the BlueBot allows for physically experiencing what is possible side lengths of isosceles triangles we are interested in exploring what this can mean for the teaching of irrational numbers.

Our findings may indicate that the BlueBot with its constraints, the task at hand and the organising in groups gave unexpected opportunities for exploration of mathematics. We know that CT is a new concept for many mathematics teacher educators, and we can see that the teacher educators made different choices when presented with similar opportunities in class. Although the story played out differently, the questions regarding possible triangles, angles, square root, and the Pythagorean theorem were generated by the students in both lessons. The implication for planning lessons when combining CT and mathematics is that the teachers to be aware of the constraints inherent in the CT activities, the mathematical questions and the needs that may arise from those constraints. This requires insights into both mathematical knowledge and competence in the field of CT.

In their review article on CT in mathematics classrooms, Hickmott et al. (2018) points out that most of the studies “involve mathematics but mainly concentrate on teaching programming skills” (p. 48). In hindsight, we see that this was also the case for this iteration, although the intention was to integrate the two subjects in the lesson in a way that teaches both mathematics and CT combined. The planning missed the fact that there was the possibility of a reflective question of what kind of geometrical figures could be drawn by a BlueBot, even though it was conducted jointly by digital competence and mathematics teacher educators. And through analysing the material, we found moments of contingency that if addressed, may have led to a better understanding of mathematics. This is something we need to address in future design cycles.

Using a simple artefact such as BlueBot to explore geometrical shapes generated these moments of contingency related to exploring angles from different perspectives, discussing the difference between points and circles and the proof of irrational numbers. From our data, we can see that there is a possibility for exploring geometrical shapes with a focus on perspective and angles to spark mathematical discussions. There is a possibility for addressing the difference between everyday

language and mathematical concepts by exploring the difference between a point and a circle. There is a potential to elaborate on these identified situations in lessons that combine CT and mathematics in a way that more fully acknowledges the two disciplines. This is something we will explore further in the next iteration of MASCOT. This study was conducted on a small number of prospective teachers and restricted to the tasks identified to generate mathematical discussion. Further research is needed to be able to examine the applicability to other students and situations.

References

- Barcelos, T. S., Munoz, R., Villarroel, R., Merino, E., & Silveira, I. F. (2018). Mathematics learning through computational thinking activities: A systematic literature review. *Journal of Universal Computer Science*, 24(7), 815–845.
- Gadanidis, G., Cendros, R., Floyd, L., & Namukasa, I. (2017). Computational thinking in mathematics teacher education. *Contemporary Issues in Technology and Teacher Education*, 17(4), 458–477.
- Geraniou, E., & Hodgen, J. (2022). An exploratory study on mathematics teacher educators' beliefs and understandings about computational thinking. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of European Research in Mathematics Education (CERME12)* (pp.1932–1940). <https://hal.science/hal-03748436>
- Hickmott, D., Prieto-Rodriguez, E., & Holmes, K. (2018). A scoping review of studies on computational thinking in K-12 mathematics classrooms. *Digital Experiences in Mathematics Education*, 4(1), 48–69. <https://doi.org/10.1007/s40751-017-0038-8>
- Huckstep, P., Thwaites, A., Turner, F., & Rowland, T. (2011). *Developing primary mathematics teaching with the knowledge quartet*. Sage Publications.
- Nordby, S. K., Bjerke, A. H., & Mifsud, L. (2022). Computational thinking in the primary mathematics classroom: A systematic review. *Digital Experiences in Mathematics Education*, 8(1), 27–49. <https://doi.org/10.1007/s40751-022-00102-5>
- Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, 8(3), 255–281. <https://doi.org/10.1007/s10857-005-0853-5>
- Shute, V. J., Sun, C., & Asbell-Clarke, J. (2017). Demystifying computational thinking. *Educational Research Review*, 22, 142–158. <https://doi.org/10.1016/j.edurev.2017.09.003>