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Effect of GeoGebra explorations combined with art-based practices on in-service teachers' understanding of geometric transformations

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This study focuses on understanding how GeoGebra explorations combined with art-based practices influenced middle school teachers' understanding of geometric transformations and their perceptions of teaching these topics. The study indicated that in some cases, this integration helped teachers' acquisition of new knowledge and development of deeper conceptual knowledge while addressing misconceptions; in other cases, it helped the retention of knowledge by association with the activity; and in some cases, it facilitated the practical knowledge. Study results also indicated that integration of DGS with art led to a higher level of creative engagement that supported productive struggle and helped teachers appreciate and understand pedagogy differently.

Keywords: Art-based practices, dynamic geometry software (DGS), geometric transformations, teacher knowledge.

Introduction

This study focuses on understanding how GeoGebra explorations combined with art-based practices influenced middle school teachers' understanding of geometric transformations and their perceptions of teaching these topics. In the United States, the importance of geometric transformations has increased significantly with the adoption of the Common Core State Standards in Mathematics that define the concepts of symmetry, congruence, and similarity in terms of geometric transformations (NGA/CCSSO, 2010). At the same time, studies show that middle school teachers have difficulty applying geometric transformations to various tasks (Seago, et al., 2013; Yanik, 2011). Multiple international studies demonstrated that middle school teachers often lack competency and comfort with regard to spatial and visual reasoning for teaching geometry (Gal & Linchevski, 2010; Whiteley, 2004), are hesitant to engage in explorations, and rely on memorization of information when designing spatial and visual tasks (Dietiker, 2015). Moreover, middle school teachers have difficulties recognizing, describing, executing, and representing geometric transformations (Yanik, 2011). Various studies suggest that integrating DGS (Knapp & Moore, 2016) and art-based practices (Nutov, 2021) into teacher training has a potential for enhancing teachers' spatial and visual reasoning that could influence their instructional practices in the geometry classroom.

DGS stimulates development of creative cross-curricular tasks that integrate art, history, and culture into geometry. Karssenberg (2014) demonstrated that integrating analysis and design of Persian mosaics using physical materials and DGS supported high school students in analysis of complex geometrical figures. Voracova (2021) showed that when elementary and middle school students engaged in analysis of real-life symmetric ornaments using GeoGebra, they demonstrated higher engagement, individual creativity, and deeper understanding of symmetry and transformations. Simon (2020) found that high school students deepened their knowledge of transformations while using DGS in graphic design activities. Art infusion into mathematics curricula could support creativity and innovation for both teachers and students (Hunter-Doniger, 2018), leading to

innovative approaches to mathematics education (Goldsmith et al., 2016) that can support students in better understanding of various topics, including geometric transformations (Brezovnik, 2015). This can be achieved through art-based practices such as creating artwork that has individual and personal meaning and relevance. However, no studies were identified that examine such integration. Therefore, this qualitative case study was guided by the following research question: How GeoGebra explorations combined with art-based practices affect teachers' understanding of geometric transformations and their perceptions about teaching this topic?

Theoretical framework

This study combined DGS explorations with art-based practices to design learning experiences that helped teacher-participants develop understanding of geometric transformations. The design of these experiences was informed by merging two theoretical frameworks, Instrumental Genesis (Drijvers & Trouche, 2008) to address the role of GeoGebra and Embodied Cognition (Kantrowitz, 2022) to address the role of art-based practices in the process of knowledge construction by the participants. Dynamic explorations of geometric transformations afforded by a tool, e.g., GeoGebra, provided a basis for the development of mental schemes of using GeoGebra to relate geometric transformations to tessellations. Combining the tool and the scheme led to the genesis of a tool into an instrument. In our study, the instrument is the stencil designed by the teacher-participants in GeoGebra that they wanted to use to create tessellation patterns with physical arts material. Engaging with art materials in a creative process of frieze design provided tangible and embodied experiences that supported teachers' understanding of geometric transformations.

Methods

This study took place in a session of a year-long graduate mathematics course for middle school teachers. The focus of this course was on strengthening and deepening mathematical thinking and quantitative skills of teachers currently teaching mathematics in grades 5–9. Participants were the three teachers who completed the course and agreed to participate in the study. Annie, a 5th grade teacher, taught in elementary school for 18 years as a general education teacher. Katy, a 6th grade mathematics teacher, had 14 years of teaching experience in elementary school and 2 years in middle school. Rita, a 7th grade mathematics teacher, had 10 years of teaching experience in middle school. All three participants did not have prior experience with GeoGebra.

Researchers developed a series of mathematics activities to explore geometric transformations using digital and physical materials (Table 1). The third activity invited participants to use their growing mathematical knowledge to create an artwork that expressed their artistic choices.

Table 1: Examples of activities, materials, and tasks used for the topic of geometric transformations

Activity	Materials	Tasks
1. Exploring rigid transformations	GeoGebra transformations applet	Dynamically explore properties of images by changing attributes of a preimage and each transformation.
2. Transformations in tessellations	Images of six different tessellations	On each image identify and draw attributes of transformations

3. Designing a frieze using tessellations.	GeoGebra tessellations applet. Dotted cardstock, papers and clothes of different texture, rulers, scissors, glue, scotch tape, glitter.	In GeoGebra, transform regular shape to irregular and observe its transformations to produce a tessellations pattern. Create a stencil for this shape to use with different art materials in order to create a frieze pattern using at least 2 different transformations.
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The course session and group interview were video recorded. The semi-structured group interview included open-ended questions related to the course experiences in using DGS explorations and art-based practices. Data also included participant-generated artifacts. The researchers transcribed the videos. All data were analyzed using the grounded theory approach (Strauss & Corbin, 1990) through an iterative process of coding, categorizing, and comparing, with the goal of identifying patterns and themes that emerged from the data. During initial readings of the material, the data were broadly categorized as pertaining to (a) teacher understanding of geometry and (b) pedagogy of teaching geometry. We then identified a number of descriptive codes that emerged from the close reading of the material, i.e., mathematics concepts (MC), art concepts (AC), retention (R), misconceptions (M), mathematics applications (MA), conceptual knowledge (CK), discovery learning (DL), experiential learning (EL), productive struggle (PS), student engagement (SE), and teacher engagement (TE) (Table 2).

Table 2: Classification of codes that emerged from data analysis

Teacher understanding of geometry	Pedagogy of teaching geometry
AC, CK, MC, R, M, MA	MA, SE, TE, EL, DL, PS, SE, TE

Qualitative validity was established through cross-checking data from multiple sources to verify the recurring themes. In order to establish inter-rater reliability, researchers first coded interview transcripts jointly and developed an initial codebook. One excerpt from each session was selected for independent coding by both researchers using this codebook, then some codes were refined, until 90% agreement was reached. The rest of the data were coded independently by each researcher.

Findings and discussion

As a result of the analysis, several themes emerged in each category in relation to the roles of DGS explorations and art-based practices in teaching and learning the topic of geometric transformations. In some cases, DGS explorations and art-based practices helped to address teachers' misconceptions. In other cases, they supported participants' acquisition of new knowledge and development of a deeper understanding of a topic. The study also indicated that these experiences facilitated development of teachers' practical knowledge, i.e., their ability to apply what they learned to solve real-world problems.

Addressing misconceptions. Middle school teachers often have the same misconceptions as their students (Yanik, 2011). When discussing properties of translations, participants of this study believed that two congruent vectors represent two different translations. Another misconception was revealed when they were asked what stays the same and what changes under rotation. Participants suggested

that size and shape stay the same, while orientation changes. Further probing of their understanding of the term ‘orientation’ revealed that they were using everyday meaning for the orientation: “[orientation is] like ah.. like something turned around” (Annie). Therefore, a significant amount was allotted for exploring properties of transformations using GeoGebra (Figure 1).

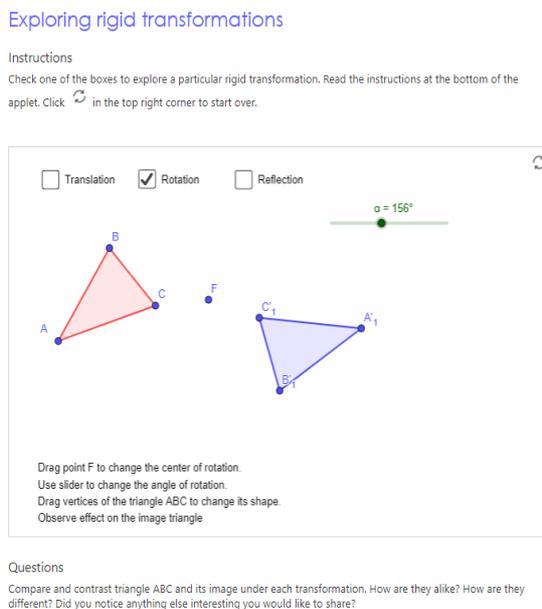


Figure 1: GeoGebra activity 2 Exploring geometric transformations

Seeing that translation by two congruent vectors led to the same image helped address their misconceptions about translation vectors. Comparing relative arrangements of triangle vertices after a transformation helped them recognize that orientation stays the same under translation and rotation, but reverses under reflection. The dynamic nature of GeoGebra supported participants’ development of deeper understanding of rigid transformations in the plane.

Acquisition of new knowledge and development of deeper conceptual knowledge. During the session, participants were asked to examine various artistic tessellations and identify the minimum number of transformations needed to create each pattern (Activity 2). They immediately realized that there is more than one way to do that, and that it “depends on how you are looking at it” (Katy). At the same time, they needed scaffolding to understand that you need to show coverage around a single point only, as reflected in the following conversation about the tessellation shown on Figure 2.



Figure 2: Tessellation from the Alhambra Palace (Spain)

Researcher: Pick a point that you will tessellate about and [explain] what transformations you are using.

Katy: This point [points to the image] and this horizontal line. Then you will need another one below.

Researcher: That is another point, but for this point - how do we get to cover these areas?

Katy: We need another line vertically.

Researcher: Still have this space left [pointing to the image].

Annie: Oh, look at that, it's rotated around a different point. It took me a while to see that.

These results are consistent with research findings that demonstrate similar difficulties for students in going from part (tile) to whole (tessellation) and from whole to part (Aydin-Guc & Hacisalihoglu-Karadeniz, 2020). Analysis of artistic tessellations with irregular shapes and non-geometric images helped participants to deepen their understanding of geometric transformations.

Practical knowledge. The task of frieze design in activity 3 was heavily based on teachers' ability to apply their knowledge of transformations within a specific context of designing an artistic frieze using irregular shapes. Given a premade GeoGebra applet, teachers experimented with tessellating different irregular shapes using translation, rotation, and reflection, in order to design irregular shapes for their own tessellations (Figure 3). Similar to the way she did it on the computer, Annie cut out a small triangle from the kite along its longer side and then rotated it around its vertex to place this triangle on the exterior of the same side of the kite to create a shape for the stencil for her design of a frieze (Figure 4). Using this stencil, Annie cut out pieces for her frieze from different materials. After creating a row of four shapes; however, she was having difficulty creating the second row, as reflected in her comment: "I just actually don't know how to fit this. ... I was first going like this, but then the other ones won't fit in". Her unsuccessful attempts to tessellate are shown on the first three images of Figure 5. This observation is consistent with studies indicating struggles of middle school teachers to recognize and execute geometric transformations (Yanik, 2011). Finally, Annie realized that in her GeoGebra design she used rotation to tessellate this piece and she corrected her approach:

...because this was the original piece when tessellated went like this [Annie rotates her stencil counterclockwise by 90 degrees four times], so I had to think about that, it needs to go like this [Annie places 4 pieces using rotation by 90 degrees around the same point as shown on the 4th image on Figure 5].

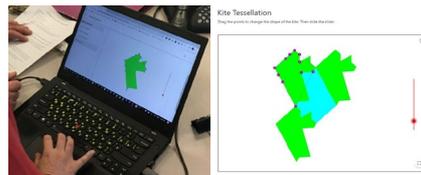


Figure 3: Annie is using GeoGebra to create a tessellation

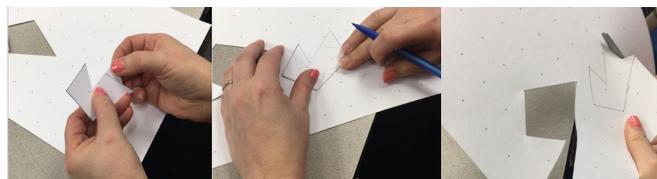


Figure 4: Creating a stencil for a tessellation

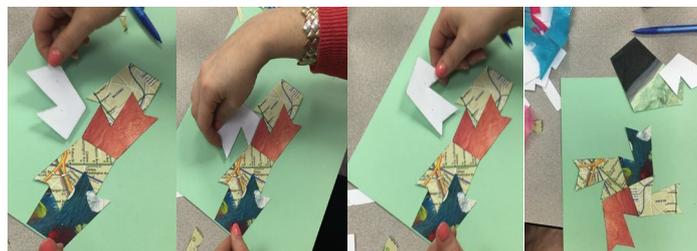


Figure 5: Annie's attempts to create a second row of tessellations and final result

Later during the interview, Annie reflected on this process:

I was going across and like, wait a second, if when I put the other piece on top it's not going to fit, and so I had to remember ... what my original shape was, and then how it moved when I created many more on the screen [GeoGebra applet].

Though participants found this task challenging, all of them emphasized the value of this experience of applying their mathematics knowledge to a practical task as reflected in Katy's interview:

That tessellation, creating and cutting it out ... I had done it across, started going up and I'm like oh no, this is not gonna work, because I have to fit these parts in. How am I gonna fit that into here, and it does work, [be]cause I did it on the [GeoGebra] website ..., but now I have to think of how to make it work, ... but wait a minute I have to keep applying math! I kept saying well okay this is a rectangle and then ... I kept going back to math. So that was really good.

This task helped deepen teachers' understanding of geometric transformations (Brezovnik, 2015) while also encouraging their individual creativity (Hunter-Doniger, 2018).

Even though the course focused on deepening teachers' understanding of challenging mathematics ideas, it was also structured to model effective teaching strategies. Therefore, DGS explorations combined with art-based practices sparked participants' discussions of pedagogy. Based on the data analysis, three recurring themes related to pedagogy emerged as follows: productive struggle and sustained engagement, connecting mathematics to real-world applications, and balancing direct and student-centred instruction.

Productive struggle and sustained engagement. Creating frieze using physical art materials was a challenging activity for the participants that sustained their engagement through productive struggle. Analysis of videos clearly indicated that all of them made several unsuccessful attempts at each stage of the activity, from creating stencils, to putting them together, to designing a pattern using at least two different transformations. During each step of this task, participants referred back to DGS exploration of tessellation (Figure 3) and engaged in discussions with a peer. These results indicate two important findings. First, art-based context that supported individual creativity led to a strong motivation for teachers to complete the challenging task despite the struggles and setbacks. Second, teachers used DGS when reaching roadblocks and the immediate feedback provided by DGS supported sustained engagement. These results are consistent with studies that demonstrated similar effects of using DGS in the art contexts with elementary and middle school students (Voracova, 2021).

Connecting mathematics to real-world applications. The participants recognized that their students also need experiences that help them to connect mathematics to real-world applications:

I think a lot of the trouble that students have in math is they don't think it's real world. It's not applicable, it doesn't exist. I think this [using physical and digital materials] is a way ... to make it fun, to make it real, and make it something that [is] memorable. (Katy)

Balancing direct and student-centered instruction. In the group interview participants reflected that even though they learned about the advantages of the discovery and experiential learning in their teacher preparation and professional development programs, they did not really experience that before, as reflected in Rita's comment: "I feel that we always talked about how we should always model, we should always have manipulatives, we should always, ... but then [this] session I think we had it".

Content coverage is a major challenge for teachers when it comes to planning their lessons. Often, even if teachers recognize the value of student-centred approach, they use lack of time to justify direct instruction necessary to cover required content as evident from this comment by one of the participants. Immersion experiences with DGS applications combined with art-based practices helped participants see that a balance between direct and student-centred instruction supported their learning:

... all the pressures of we can't fit all of these topics in ..., you are running out of time, ... I'm gonna go through this fast. But then ... [course sessions] felt like it was a good balance of both. (Annie)

As a result of this experience, it seems that the participants' attitudes shifted towards exploration as a way of "content learning" versus direct instruction only to deliver content:

... we're on a time crunch but we learn so much in a short amount of time about that, so it's better to do it that way. (Annie)

The exploration time that was earlier perceived as "wasted time" transformed into understanding that learning happens deeper and in different ways when students are allowed to explore mathematics topics through discovery.

Conclusions

This study showed that for participating teachers, the experience of creating artistic products with digital materials and art-based practices was crucial to develop deep understandings not only of geometric concepts but of pedagogy as well. Experiencing learning through using DGS explorations in combination with art materials seems to have taught participants as much about pedagogy and teaching as it did about mathematics - and that is something that merits further research. Although there is no one answer that will fit all curricula and all teachers, this study shows that active explorations with physical and digital materials transformed these teachers' perceptions of what learning means and increased teachers' interest in implementing the same instructional practices with their own students. This fact points to the need for teacher education programs to develop a stronger emphasis on using experiential learning and creative processes in their pedagogy courses. A future and longer study with more time with art and digital materials will be important to understand the extent to which the level of freedom in exploration can influence learning of mathematical (and pedagogical) concepts. Future studies could also explore the unique contribution of each environment (DGS and art materials) on the development of teachers' knowledge and whether combining digital and physical tools in this process is influenced by the instrumental distance (Haspekian, 2005) between them.

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