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Relating the dialogic and dialectic in mathematical argumentation

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The research on mathematical argumentation has mainly adopted a dialectic perspective which focuses on understanding the rationality behind the argumentation. However, this approach may have overlooked other key aspects of mathematical argumentation. It is similarly important to understand mathematical argumentation as a social phenomenon, which aligns with a more dialogical approach. In this paper, a short excerpt of a mathematical argumentation had been analysed from the dialogic and dialectic perspectives respectively. Based on the analysis, I argue for the consideration of both perspectives to help us reconceptualise how mathematical argumentation get actualised, and thus, how it can be learnt and taught in the mathematics classroom.

Keywords: Mathematical argumentation, dialogic, dialectic.

Mathematical argumentation is a complex process with many different aspects that can be studied. The more common ones that have been studied include the cognitive, meta-cognitive and social-cultural aspects of argumentation (Krummheuer, 1995; Schwarz et al., 2010). However, according to Schwarz (2009), there are two important perspectives of analysis in understanding (mathematical) argumentation. He proposed that “argumentation functions in two ways” (p. 104) – dialectically and dialogically. The former emphasises the rationality behind argumentation, focusing on how learners make connections to provide reasons to support or refute their proposed claims, before arriving at an agreement. The latter places emphasis on the rationality that is situated within social-cultural rules and orientations that facilitate the progress and development of the collective argumentation (Schwarz & Shahar, 2017). This paper attempts to analyse mathematical argumentation from these two perspectives. The dialectic perspective focused on the types of reasoning used in argumentation and the structure of the argument, and the dialogic perspective focused on the presence of differences and engagement with the other, that is based on Bakhtin’s dialogism. However, the intent here is not to integrate them in a complementary manner as a single connected whole or argue that one approach is better than the other, which would adopt a more dialectical stance. Rather, the purpose is to present the differences between them and discuss how these two perspectives can be seen as a superimposition of two layers or modes that coexist in a stratigraphical manner. This is analogous to how light exists in superimposition of two modes, a wave and a particle where “only one of which will be actualised at the moment of observation” (Mikulan & Sinclair, 2019, p. 240) or measurement in quantum theory. Similarly, mathematical argumentation can be dialectic and dialogic, where only one gets actualised at the moment of analysis depending on the research question asked.

Dialectic perspectives of argumentation

In mathematics education, it seems that understanding argumentation from the dialectic perspective has been the primary focus of research. This is possibly due to its strong association with proofs, where there is a greater focus on the need to establish truth or validity of mathematical ideas through reasoning. Within this perspective, research generally attended to how arguments and counterarguments are constructed; and how differences between them are synthesized and overcome

to reach a consensus, which are often presented as a “logical story of development” (Wegerif, 2008, p. 350). Such research typically examined two key elements in argumentation – the types of reasoning used in the argumentation, and the structure of the argument which helps in reconstructing the argument in a coherent manner and explaining the underlying rationality of the arguments.

Types of reasoning

Various styles of reasoning can be used to support or oppose an argument. In mathematics, reasoning is often associated with deductive reasoning which preserves truths to establish valid conclusions from premises by following the rules of a well-specified logic (Meyer, 2010). However, other forms of reasoning, such as abductive, analogical, inductive and statistical reasoning, have also been observed to be important and relevant in mathematics. Above all, abductive and inductive reasoning have been highlighted to be more commonly used in mathematics (Feeny & Heit, 2007). Abductive reasoning seeks to provide the best plausible explanations for the claims made from observations; and inductive reasoning infers generalizations from particular observations. While the order of the different forms of reasoning used varies, a typical process may start with abductive reasoning to suggest a tentative explanatory hypothesis, which is tested inductively to support or refute its plausibility. If the hypothesis is refuted, another explanatory hypothesis will be abduced and this process will repeat until the best explanatory hypothesis, supported by inductive evidence, is formulated. After which, a proof can be pursued through deductive reasoning to establish the certainty of the conclusion (Meyer, 2010).

Structure of argument – Toulmin’s model

While there are different models which can be used to analyse the structure of argumentation or argument in education, most mathematics education researchers have used Toulmin’s model to analyse and understand mathematics learners’ argumentation or proving process, as well as the product in terms of the argument or proof. Toulmin (2003) proposed that the validity of an argument can be related to its structure as a certain structure may better exhibit its validity or make the reasons for its validity clearer. According to him, a logically valid argument should contain six elements, with each serving a different function. Firstly, claims should be made based on data, which lays the foundation for the claims. But claims need to be supported by warrants, usually “general, hypothetical statements, which can act as bridges” (p. 91) between the data and the claims. In other words, the warrants provide plausible reasons to explain the validity and soundness of the claims. Nevertheless, the warrants may only support the claims for some conditions and not others. As such, the claims need to be specified further with qualifiers. When the claims are not valid for some conditions, they will then be refuted with exceptions or rebuttals. Lastly, the appropriateness and applicability of the warrants to the claims may need to be defended and elaborated with the necessary backings, which usually take the form of statements that are more absolute in nature. Thus, Toulmin’s model offers a means to analyse the structure of mathematical argumentation through the decomposition of its six elements, which will also reflect the type of reasoning and rationality behind them.

Dialogic perspective of argumentation

Research that takes a dialogic perspective is interested in understanding how the presence of multiple and differing perspectives enhances learning in the argumentation process. The dialogic perspective,

which draws heavily on dialogism, emphasizes the importance of learners' interactions and their engagement with others' ideas and arguments. It focuses on how learners explore the relationship between diverse ideas without the need for a final synthesis or unification (Wegerif et al., 1999). Such a perspective seems to greatly contrast the conventional expectations of mathematical argumentation – which draw on dialectical assumptions and focus on comparing or evaluating the validity and strength of the suggested arguments before converging and deciding unanimously on the best argument (Langer-Osuna & Avalos, 2015).

Presence of differences

Fundamentally, dialogism is based on differences (that are irreducible) rather than identity, where knowledge is deemed to be formed through differences (Bakhtin, 1975/1981; Wegerif, 2008). In other words, object A is not known in and of itself, but in relation to object B (and other objects). It is often contrasted with the notion of dialecticism, which emphasises the independent knowledge of A and B (through their respective properties) and then the logical synthesis that can arise by the overcoming of differences, towards uniformity and certainty. Hence, dialogism and dialecticism are often deemed to be incompatible, in terms of their epistemological and ontological assumptions. As meaning is a product of different perspectives and contexts present in dialogism, it cannot be constructed without the awareness of at least one other possible point of view, thus differing from monologism. The consciousness of the multiple plausible ways of looking at something, the switch between different perspectives, and the context involved, are necessary to develop understanding (Wegerif, 2011). Meaning is also realised through dialogues which bring at least two perspectives together and situate them in a particular context, with no need for agreement. In fact, Bakhtin proposed that if common ground is achieved, the dialogue will be discontinued. There will be no further progress in meaning making as the presence of variation is the spark which opens opportunities for perspectives to shift in the meaning-making process (Wegerif, 2011). Moreover, the presence of differences does not necessarily imply that any one perspective is correct or superior to another. Various perspectives should be equally valued and accepted to reflect the existing multitude of meanings such that “the world of meaning is essentially dialogic” (Wegerif, 2008, p. 349).

Engagement with the other

Another key aspect of Bakhtin's dialogism is the mutual influence or dependency between the two (or more) participants present in the dialogue (Bakhtin, 1975/1981; Wegerif, 2008, 2011). It highlights their interrelatedness where any utterance is dependent on both parties – the speaker, and the addressee. The speaker must take into consideration who the addressee is before formulating the utterance as the message should be conveyed in a manner appropriate for the addressee, in terms of the language and style and keeping in mind how the addressee may react to it. Similarly, the addressee will respond based on a projected view of the speaker. As such, they define each other mutually, where each utterance contains the voices of self and the other; and with each utterance, both the self and the other are being constructed and reconstructed. Eventually, the meaning and understanding constructed is not ascribed to any individual but an integration of the ideas put forth by both. As a result of the interdependency, a metaphorical dialogic space (including position and time) that encompasses both the speaker and the addressee is generated, beyond a fixed and simple connection

between them and their ideas (Wegerif, 2008, 2011). It is a shared and generative space where the two can relate to each other dynamically. Their positions in the space can change as the dialogue progresses (and when time changes too). They may take on each other's perspectives, or they may gradually or totally shift their initial positions as new insights emerge from their interaction.

A stratigraphical approach to analysing argumentation

Although the dialectic and dialogic perspectives have each been used extensively within mathematics education research, recent studies (Schwarz & Shahar, 2017; Wegerif, 2008) seem to be leaning towards a joint approach in analysing and understanding argumentation. Notably, one perspective may not be sufficient in providing a comprehensive understanding of mathematical argumentation and the conditions for it to happen, particularly in the case of collective argumentation. It may also not be possible to separate the dialectic elements from the dialogic elements during argumentation. Thus, I am interested in exploring the use of both dialectic and dialogic in analysing argumentation and examining how they function in argumentation. To do so, this paper is proposing the use of stratigraphy, which is "a way of investigating that actualises the differentiating undercurrents of an event, the layers of the past, present and future that exist simultaneously, sometimes converging and sometimes not" (Mikulan & Sinclair, 2019, p. 243). Instead of pushing for the triangulation of the two perspectives which assumes that there is an underlying group basis common to them, and often ends up reading one in terms of the other (the dialogic elements of what is fundamentally a dialectic enterprise, for example), this stratigraphical approach proposes that the dialectical and dialogical operate at two independent modes in mathematical argumentation. To illustrate this approach, a short excerpt of mathematical argumentation has been analysed to illustrate the unfolding of the dialectic and the dialogic modes independently before relating them in discussion.

The excerpt was drawn from a problem-solving episode which took place in a 12-week university course, focused on problem solving and the investigation of connections across topics. Every week, the students worked on the whiteboard, in random groups of two or three, to solve a variety of mathematical puzzles and problems involving interconnected topics. Selected problem-solving episodes in the course have been videotaped to capture both the discussion (audio) and the work done on the whiteboards (visual) as part of my dissertation research.

Specifically, the problem-solving episode which I have analysed (for the purpose of this paper) is based on the following problem:

A group of 9 pirates are going to disband. They have divided up all their gold, but there remains one giant diamond that cannot be divided. To decide who gets it, the captain brings all the pirates (including himself) into a circle. Then he points at one person to begin, and this person must step out and leave with his gold. The person on the left of that person stays, but the next person steps out. This continues with every second pirate leaving until there is only one left. Who should the captain point at if he wants to ensure that he gets to keep the diamond for himself? What if there were less pirates or more pirates?

The episode involved a pair of students, Jordan and Kobe, working together to solve the problem. After solving for the case of 9 pirates successfully, they tried and solved the problem for the cases of 3 and 4 pirates. It led them to form a conjecture for odd and even number of pirates respectively.

However, their conjecture failed for the case of 6 pirates, so they continued to try for other cases between 5 and 8 pirates to explore possible patterns. Prior to when this argumentation excerpt was drawn, they have also solved for the case of 7 pirates and listed down their results in a table (see Table 1). However, they had not yet reached a general rule or pattern for any number of pirates. The argumentation excerpt that followed has been transcribed for analysis in this paper.

Kobe: But why?
 Jordan: Exactly! [Long pause] Oh! There's a pattern here [pointing to the last column of Table 1].
 Kobe: What pattern do you see?
 Jordan: I see odds going up until you can't anymore, and it goes back to one.
 Kobe: I want to draw Pascal's triangle because usually some pattern is coming from there. [Starts drawing Pascal's triangle]
 Jordan: We should put nine in here too.
 Teacher: Ah, how's that going? Pascal?
 Kobe: There's usually something in this so that's why I was attracted.
 Jordan: But this is true for a pattern, don't you think? Like this is just odds, and then on the left it's going down by ones.
 Kobe: From where?
 Jordan: But from the right, like so nine is one, so ten will probably be three, five, seven and nine if you still keep going.
 Kobe: Ah, I see what you are saying, because it can't do five here because that is five, so it resets and then three and then five and then seven and you can't do nine because that is nine. And then is there something, a similar pattern on the left or is that just a sum?
 Jordan: Wait a minute, does that add up?
 Kobe: And so how does this work? Starts at two then drops down to one and resets and it goes up to four, three, two, one and resets and eight, seven, six, five, four, three, two, one.
 Jordan: Uh huh.
 Kobe: So, let's check thirteen. This says you need to pick four points from the left. [They count together and confirm the result] Okay, we have a rule.

Table 1: Table of results written on whiteboard

Total Number of Pirates	Number of Pirates to the Left	Number of Pirates to the Right
3	2	1
4	1	3
5	4	1
6	3	3
7	2	5
8	1	7
9	8	1

The dialectic mode

To illustrate the dialectic mode, I examine the type of reasoning (abductive, inductive or deductive) the two students were engaged in to construct their arguments. I also identify the different elements (mainly the data, claim and warrant) of Toulmin's model within their argumentation to understand and explain their reasoning. It can be observed that they were engaged in *abductive* and *inductive* reasoning to eventually arrive at a general rule or pattern for the problem. After solving the problem for the case of 9 pirates successfully, they continued to solve the problem for the cases of 3 and 4 pirates. They had abduced that perhaps there would be different solutions for the odd and even cases and went on to search for evidence or data to support their idea. From their solution, they conjectured a solution for the odd and even cases respectively. This resembled some form of inductive reasoning although their argument may be a little weak when based only on two cases. However, as their conjecture was only valid for some cases (as it failed for the case of 6 pirates), they went back to solve the problem for more cases and organised their data in a table form. With a larger set of data to work with, they were able to observe a more complete pattern and generalised the pattern they observed from their results before agreeing on a rule to solve the problem for any number of pirates.

Based on their *data*, they made a *claim* that the captain should always point at an odd number of pirates to his right and this odd number starts from 1 and go up by two each time "until you can't anymore" and it resets to 1 again. Kobe also supported the claim that the odd number cannot increase indefinitely and needs to reset to 1 with the *warrant* that it is restricted by the total number of pirates present. Specifically, when there are 5 pirates, the captain needs to point at the first pirate to his right rather than the fifth pirate to his right (which will be himself) as "it can't do five here because that is five". They also agreed that if you count from the left instead of the right, there is another plausible pattern. From the left, the number goes "down by ones" each time until you reach 1 and it resets again. Furthermore, to test their conjecture, they worked out the solution for the case of 13 pirates and verified that their conjecture is indeed true.

Overall, it is observed that both students employed abductive and inductive reasoning in an alternating manner. The two types of reasoning played complementary roles during their problem-solving process. The cycle of creating conjectures based on data, generating plausible explanations for them, and testing of validity was repeated before they agreed on a generalisation. Although their inductive claims cannot be taken to be absolute, it seems highly probable that the rule that they have formulated is valid.

The dialogic mode

To illustrate the dialogic mode, this analysis now focuses on the presence of different ideas between the two students, and how their perspectives may change, as well as the interrelatedness of their utterances. It is observed that Kobe and Jordan held differing views about the existence of a pattern in their table of results initially. On the one hand, Jordan was deliberating on the odd-numbered pattern that stood out for him from the data. On the other hand, Kobe was attracted to the Pascal's triangle and proposing it as a possible connection to the problem as it has probably been used to solve previous problems. Both were focused on their individual ideas and did not question or challenge each other's ideas. This suggests that they were not quite ready to hear each other out or see the point

of view of the other yet. However, Jordan persisted and tried to convince Kobe to see his perspective by bringing Kobe's attention to the table of results and highlighting the pattern that he observed – that “this is just odds, and then on the left it's going down by ones”. Kobe still seemed confused at that point but was now more open to understanding Jordan's perspective as he sought clarification on where the pattern was. After hearing the elaboration from Jordan, Kobe was able to “see what you [Jordan] are saying” as he reiterated the pattern with additional examples. This switch in perspective was not automatic as Kobe was focused on a different idea initially. However, an alternative way of thinking was brought to his awareness through the discussion. Jordan's explanations prompted Kobe to shift his attention from the Pascal's triangle to the numbers in the table.

In addition, an inter-dependency appeared to exist in the students' utterances. Kobe's description of the pattern “three and then five and then seven” contained the voice of Jordan as it echoed Jordan's preceding explanation of “three, five, seven and nine”. Similarly, in Jordan's subsequent response to Kobe's query on the existence of “a similar pattern on the left or is that just a sum?”, it contained the voice of Kobe where the idea of sum was revoiced by Jordan as “add up”. Thus, this excerpt seems to illustrate an interweaving of voices and a shared dialogic space between the students where they relate and integrate each other's utterances and perspectives dynamically to construct and reconstruct their understanding.

Taking a stratigraphical approach

The independent analysis of the dialectic and dialogic modes shed insights into different aspects of the mathematical argumentation excerpt. The dialectic layer was useful in identifying and making sense of the underlying reasoning in the students' arguments. In this case they had used a combination of abductive and inductive reasoning to generate and test different conjectures before agreeing on their final claims. Separately, the dialogic layer was able to provide insights on how the students' understanding of the problem was influenced by each other. By engaging with each other's ideas in the dialogic space, it enabled possible shifts in perspectives and new meanings to be constructed. While the dialectic layer focuses on reconstructing the development of the argument as one fixed and continuous logical flow towards the conclusion, the dialogic layer clearly illuminates the multiplicity of ways that the argument can unfold and how different ideas and voices interact and relate to each other. Hence, each layer illustrated different yet important emphases in how argumentation can be understood.

Hence, instead of arguing about the compatibility or incompatibility of dialectics and dialogism in mathematical argumentation or reading mathematical argumentation as either dialectical or dialogical, this paper is suggesting that mathematical argumentation is dialectical *and* dialogical, which unfolds at these two modes simultaneously. As such, both dialectic and dialogic aspects of mathematical argumentation are key in the development of students' mathematical argumentation skills. Thus, I see the potential in integrating the learning and teaching of both dialectic and dialogic elements of argumentation in the mathematics classroom. Specifically, a dual focus can be considered in mathematical argumentation – the need to provide reasoning to support individual and collective arguments or counterarguments, and the need to be willing to engage with each other's differing ideas, and switch points of view when necessary.

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