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► **To cite this version:**

Horacio Solar, Andrés Ortiz, Victoria Arriagada, Marco Catalán. Argumentation in the mathematical modelling cycle in the classroom. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04412010

**HAL Id: hal-04412010**

**<https://hal.science/hal-04412010>**

Submitted on 23 Jan 2024

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# Argumentation in the mathematical modelling cycle in the classroom

Horacio Solar<sup>1</sup>, Andrés Ortiz<sup>2</sup>, Victoria Arriagada<sup>1</sup> and Marco Catalán<sup>1</sup>

<sup>1</sup>Pontificia Universidad Católica de Chile, Faculty of Education, Santiago, Chile; [hsolar@uc.cl](mailto:hsolar@uc.cl)

<sup>2</sup>Universidad Católica de la Santísima Concepción, Faculty of Education, Concepción, Chile

*Although there is extensive literature in modelling and argumentation, the joint approach to argumentation and modelling is very scarce in the literature. This article accounts for the development of argumentation in the different phases of modelling that students go through in the mathematics classroom. Based on an experience of professional teacher development, the analysis of three cases of teachers is presented to promote modelling and argumentation. The results of this study indicate that an argument structure with rebuttals in modelling tasks does not ensure complex mathematical connections.*

*Keywords: Modelling, argumentation, mathematical connections, mathematics teachers*

## Introduction

Despite the importance of modelling and argumentation, only a few studies have addressed their relationships, which have focused on the construction of arguments in the modelling cycle. (Tekin-Dede, 2019; Güç & Kuleyin, 2021), and therefore, it is necessary to continue deepening the development of the argumentation in the modelling cycle. Based on a study of 2 cases of teachers who implemented a teacher unit to fostering modelling and argumentation in the mathematics classroom, the purpose of this paper is to characterize the argumentation in the mathematical modelling cycle in the classroom.

## Argumentation in the mathematics classroom

There are various approaches to argumentation in the literature, we follow an approach that is commonly accepted by educational research, in which argumentation has as its main purpose to convince both oneself and others of the validity of reasoning (Krummheuer, 1995). In general, the analyses of the statements that contain an argument in the classroom are based on the model of Toulmin (2003). The analysis consists in that the claimed validity of an assertion is established by means of an argument that can consider several elements. Toulmin proposes a model with 6 elements of the argumentative structure (AS): the *data* that correspond to the evidence that is presented to initiate the argumentation. The *conclusion* is the position of which you want to convince the interlocutors. The *warrant* allows the inference of the conclusion from the data. The *refuter* establishes the conditions in which the warrant or conclusion is not valid. The *modal qualifier* that qualifies the conclusion in terms of the certainty that the argument provides, and finally the *backing* that provides legitimacy to the *warrants*. Toulmin's argumentative structure (AS) has been used by various researchers to analyse argumentation in the mathematics classroom. Some of these investigations have reduced the AS by omitting the modal qualifier and rebutter (Krummheuer, 1995), but, in the AS with 6 elements, the persuasive power of rebuttal in collective argumentation can be observed (Reid et al., 2011). The AS can be classified into levels of argumentation, based on the Toulmin model, which goes from level 1 where students respond in a brief and reproductive manner

to level 5 where students base their justifications or refutations by expanding their reasoning to support their arguments. interventions (Peña et al., 2019).

### **Argumentation in the modelling cycle**

Mathematical modelling is a process that relates the extra-mathematical context and mathematics through a model that is built to provide a solution to a problem, for which various cycles have been used to use and create a mathematical model. This research uses the cycle of modelling, simplification, mathematization, working with mathematics, interpretation and validation, proposed by Maaß (2006), which is an adaptation of the cycle by Blum, since this cycle clearly shows the interaction between reality and mathematics during the modelling process, establishing a relationship between them. The literature review for this study revealed that: studies that have addressed the relationships between argumentation with the modeling cycle are scarce and recent, that there are more results on the development of argumentation in the design and resolution of learning tasks. modelling in contexts of initial teacher training (Tekin-Dede, 2019), more than in school contexts (Cho & Jonassen, 2002). In this last cited study, the quality of the argumentation was analysed through a rubric, applied to 6th grade students when performing a modelling task. The findings of this study show that the quality of the argumentation positively affected the students' going through the modelling cycle (Güç & Kuleyin, 2021). However, the quality of the arguments is an aspect that has not been sufficiently studied in the argumentative structures. For the purpose of our research we will relate the quality of the argumentation with the mathematical connections that the students make, understood as a process cognitive process through which a person relates two or more ideas, concepts, definitions, theorems, procedures, representations, and meanings to one another. Mathematical connections emerge when students solve specific tasks and can identify them in their written productions or in the oral arguments or mimes they develop (Campo-Meneses et al., 2021).

Based on this background, the research question of this paper is: how is the argument developed in the different phases of modelling cycle in the classroom? To answer this question, it is relevant to consider the presence of argumentative structure (AS) in the modelling cycle, and the mathematical connections that are established in AS.

### **Methods**

The methodology of this research is qualitative with a multiple case study approach of exploratory scope, whose cases were intentionally selected (Creswell, 2011). The cases correspond to teacher Ángeles, who teaches secondary school students (13- 14 years old), and teacher Soledad, who teaches primary school students (8-9 years old), who implemented a teaching unit designed to promote modelling and argumentation.

### **Context and participants**

A professional development program was carried out to fostering argumentation and modelling in the mathematics classroom in the city of Concepción, Chile, in which nine teachers from primary education and the first two years of secondary education (6-13 years old). The training process was based on the argumentative of the classroom as an essential practice to promote modelling in the mathematics classroom. So, a requirement for participation in the program was that the teachers had

previous experience in argumentation in the mathematics classroom. The training process lasted 8 months and was carried out through 15 sessions of three hours each.

In the second year, four of the nine teachers were selected to monitor their classes, who designed a mathematical task to promote modelling and argumentation with their students. The implementation of the task was 3-4 classes of 45-70 minutes each so that the students had adequate time to go through the modelling cycle. The mathematical tasks were implemented in small groups of students to promote collaborative work and filmed with 3 cameras. One camera recorded all the teacher's movements and the other two cameras recorded all the interventions of two fixed small groups selected by the teacher (group 1 and group 2). In addition, the written productions of the student groups were collected.

In the two cases that shows in this paper, Ángeles and Soledad implemented a sequence of classes in which it was observed that students will go through the complete modelling cycle and argumentative structure with refutation of ideas and/or procedures.

### Data Analysis Strategy

The data corresponds to the videos of the teachers' groups throughout all the classes. The analysis of these videos was carried out in two phases. In the first phase, a set of 5 analysis categories was elaborated that correspond to the transition phases of the modelling cycle (Maaß, 2006): simplification, mathematization, working with mathematics, interpretation and validation. Each of the 5 categories have been characterized by codes entered into the ATLAS.ti software for the analysis of the videos. In a second phase, in the episodes coded with the modelling phases, moments of argumentation were identified (Krummheuer, 1995) with the presence of justifications and/or refutations. In this way, convergence episodes were identified both for modelling and for argumentation.

**Table 1: Level of argumentation (adapted from Toulmin, 2003)**

|                              | Level 1  | Level 2   | Level 3  | Level 4  | Level 5   |
|------------------------------|--|---|--|--|---|
| Argumentative structure (AS) | Students respond with monosyllables or short sentences to the questions of the teacher or their classmates | Students offer answers or positions briefly and without justifying them | Students offer and justify answers and positions | Students offer, justify, and refute responses and positions. | Students offer, justify, and refute responses and positions. In addition, they make clarifications, extensions and reformulations to support their justifications or refutations. |

The moments of argumentation were analysed through argumentation levels that were coded with two dimensions: *Argumentative Structure* (AS) (Table 1) and *Mathematical Connections* (MC) (Table 2). The Argumentative Structure (AS) classifies the interventions of the students in 5 argumentative levels (Peña, et al., 2019). The Mathematical Connections classifies the interventions of the students in terms of the type of connections that the students establish about concepts,

definitions, theorems, procedures, representations and meanings among themselves (Campo-Meneses et. al., 2021), in relation to the justification of their answers or positions or to the rebuttal.

**Table 2: Levels of connections (own elaboration)**

|                                  | Without connections<br>(WC)  | Simple connections<br>(CS)   | Complex<br>connections (CC)   | Mathematical<br>construction (MaC)   |
|----------------------------------|--|--|---|--|
| Mathematical<br>Connections (MC) | Students answer or<br>position themselves<br>without making<br>mathematical<br>connections | Students answer or<br>position themselves<br>through simple<br>mathematical<br>connections | Students answer or<br>position themselves<br>through complex<br>mathematical<br>connections | Students answer or<br>position themselves<br>by building<br>mathematical<br>knowledge. |

To analyse the relationships between the modelling phases and the argumentative structures, episodes with AS level 3 or higher were selected. To analyse the rebuttals, we have selected episodes from level 4. The episodes selected from level 3 of AS were considered to analyse the mathematical connections (MC), at any of its levels.

## Results

For each of the two cases, Ángeles and Soledad, the modeling task, the argumentation levels, and the mathematical connections, in the modeling cycle are presented.

### Ángeles

Ángeles implemented the “Cinema Paraíso” task (Figure 1) in an 8th grade course (13-14 years old). The implementation was carried out in groups of 3-4 students in 3 classes. In addition, the task promotes argumentation since it requests that the students indicate whether or not it is convenient to be a member of the cinema. It should be noted that the task is not based on annual spending, but on annual savings, so the answer is not immediate and invites discussion.

**Cinema Paradiso.** The “Paradiso” movie theater offers a yearly membership card that allows people to buy tickets at a reduced price.

The following table indicates how much a person would spend buying tickets as a member and as a non-member. Unfortunately, the report got wet, and some information was lost.

| Nº de entradas compradas | Dinero gastado<br>sin ser Socio | Dinero gastado<br>Siendo socio |
|--------------------------|---------------------------------|--------------------------------|
| 2                        | 5000                            | 9000                           |
| 3                        | 7500                            | 11000                          |
| 5                        | 12500                           |                                |
| 6                        |                                 |                                |
| 20                       |                                 |                                |

A family says that, by selecting the most affordable choice, they saved \$29,000 last year.  
How many tickets did they buy?

**Figure 1: Task implemented by Ángeles for her 8th grade class**

In the implementation of the mathematical task "Cinema Paraíso" in Ángeles' classes, forty-four argumentative episodes were observed from level 3 or higher in argumentative structures (EA), which have been characterized in relation to the modelling cycle and the type of mathematical connections (CM) (Table 3).

**Table 3: Argumentative levels in modelling cycle in Ángeles**

| Modelling cycle            | AS        |           |          | MC        |          | Total     |
|----------------------------|-----------|-----------|----------|-----------|----------|-----------|
|                            | Level 3   | Level 4   | CS       | CC        | MaC      |           |
| Simplifying                | 1         | 1         | 1        | 1         | 0        | <b>2</b>  |
| Mathematizing              | 1         | 0         | 1        | 0         | 0        | <b>1</b>  |
| Working within mathematics | 17        | 7         | 3        | 21        | 0        | <b>24</b> |
| Interpreting               | 6         | 1         | 0        | 6         | 1        | <b>7</b>  |
| Validating                 | 8         | 2         | 4        | 6         | 0        | <b>10</b> |
| <b>Total</b>               | <b>33</b> | <b>11</b> | <b>9</b> | <b>34</b> | <b>1</b> | <b>44</b> |

The highest levels of argumentation, both for AS, are reached mainly in the modelling stage Working with mathematics, with seventeen episodes at level 3 and seven episodes at level 4. During the search for the mathematical model, various moments were generated where students refute your ideas, answers or procedures. During the second and third class mainly, the students did not agree on how to calculate the value per entry -being and not being a member- and that is why rebuttals occur, since when performing the calculations there were inconsistencies with the data in the table or with the assumption that being a partner at some point is convenient. This was an input for the discussion of the third class during the interpretation and validation stages, since some initial models considered a base value to pay for being a member while others did not, which allowed the students to evaluate the models incorporating the base value of \$5,000 to establish their validity.

Regarding mathematical connections (MC), although episodes with complex connections are observed throughout the entire cycle, these are concentrated in the stage working with mathematics, which presents twenty-one episodes with complex connections (CC) and only three episodes with simple connections (SC). One possible explanation is that in AS, where the students discussed the most about mathematics was, for example, when they positioned themselves regarding the fact that the value of the ticket being a member could not cost \$2,000. In addition, in this phase, they carried out the calculations to establish two very important aspects, first, to establish what was more convenient, to be or not to be a member, and second, how many tickets should be purchased to save \$29,000.

## Soledad

Soledad implemented the task "High Valley pedestrian walkway" (Figure 2) in a 3rd grade (8-9 years old), which was carried out over 4 classes in groups of 4-5 students. The students had to build the catwalk model with multi-base blocks, supported by 2 toy vehicles provided by the teacher: a car and a truck. In class 1 the students were only able to get close for one minute to see the vehicles, in class 2 each group was able to touch and measure it with the technique they wanted and once the models were built (class 3) the teacher agreed that the students They could manipulate them according to their needs. The fourth class, the teacher leads a discussion based on the sum of the sides of the catwalk.

**“Valle Alto” Footbridge.** A new footpath called “Valle Alto” will be built over the 4-lane highway that connects Yumbel with Concepción, which only allows car and bus traffic. The engineer in charge of the project needs help to ensure that the cars and buses that make use of the highway can clear the new footpath, which must be built utilizing as little material as possible.

Get together with your team to design and build a model of the “Valle Alto” footpath with the materials given to you, considering the height of the car and the bus on your teacher's table.



1. How tall is your footpath? Explain how you determined its height.
2. Discuss your answers with the other teams and decide which of all your procedures will be the most useful to the engineer.

**Figure 2: Task implemented by Soledad for her 3rd grade class**

In the implementation of the mathematical task "Pasarela pedestrian Valle alto" in Soledad's classes, twenty-six argumentative episodes were observed from level 3 or higher in argumentative structures (AS), which have been characterized in relation to the modelling cycle and the type of mathematical connections (CM) (Table 4).

The highest levels of AS are found at the end of the cycle, in the validating stage, with one episode at level 3 and twelve at level 4. In the simplifying stage, there is only one episode at Level 3 while there are no argumentative episodes. in the stages of mathematizing or working with mathematics. In interpreting there are nine level 3 episodes and three level 4 episodes. The concentration of argumentative moments at the end of the modelling cycle can be understood because students build a walkway with multi-base blocks based on aspects of size, offering answers or positions of briefly and without justifying them, so that all the episodes of the middle stages of the cycle are of levels below level 3. The appearance of argumentative moments from interpretation is explained because when the students in the second and third class mainly try to pass the auto and the bus through the built walkway, they discuss and realize that not all the walkways are suitable, so they decide to build

the one they have based on the dimensions that they did not initially measure because they could not touch the vehicles.

In relation to mathematical connections (MC), the argumentative episodes that account for mathematical connections are concentrated in the interpreting and validating stages, in which the majority of these episodes, 22 out of 25, show simple connections (SC) on the part of students, and only 3 complex connections (CC).

**Table 4: Argumentative levels in modelling cycle in Soledad**

| Modelling cycle            | AS        |           | MC        |          | Total     |
|----------------------------|-----------|-----------|-----------|----------|-----------|
|                            | Level 3   | Level 4   | SC        | CC       |           |
| Simplifying                | 1         | 0         | 1         | 0        | <b>1</b>  |
| Mathematizing              | 0         | 0         | 0         | 0        | <b>0</b>  |
| Working within mathematics | 0         | 0         | 0         | 0        | <b>0</b>  |
| Interpreting               | 9         | 3         | 11        | 1        | <b>12</b> |
| Validating                 | 1         | 12        | 11        | 2        | <b>13</b> |
| <b>Total</b>               | <b>11</b> | <b>15</b> | <b>23</b> | <b>3</b> | <b>26</b> |

## Conclusions

Based on the results shown in the two cases studied, we can account for how the argumentation develops in the mathematical modelling cycle in the classroom. In the case of Ángeles, it can be seen that the episodes with Argumentative Structure (AS) are significantly higher of level 3 in that they offer answers with warrants, in comparison to the answers in which they also refute level 4, however, in the case of Soledad there are more episodes of AS of level 4 than 3. The AS with more levels of argumentation (level 4) in which the refutation appears is different in each of the two cases: in Ángeles it is concentrated in the stage of working with mathematics, while in Soledad in the validation stage.

Regarding the Mathematical Connections (MC), in the case of Soledad there is a predominance of argumentative episodes with simple mathematical connections, while in the case of Ángeles there are more argumentative episodes with complex mathematical connections (CC).

In the analysis of the two cases of teachers, it can be seen that the increase in the level of argumentation does not ensure complex mathematical connections. Other studies have indicated that the quality of the argumentation positively affects students going through the modelling stages (Güç & Kuleyin, 2021). Based on our study, we can affirm that in the cases studied, the appearance of argumentative moments favours students going through the modelling cycle, but not in terms of the quality of the argumentation. Therefore, it is necessary to carry out more studies on the effect of the

quality of the argumentation to determine if it favourably affects the transit through the modelling stages.

These results are a contribution to the relationships already found between argumentation and modeling that have been described in other studies (Güç & Kuleyin, 2021; Tekin-Dede, 2019), since it delves into how these mathematical competencies interact.

## **Acknowledgment**

This work was supported by ANID Fondecyt 1231303.

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