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### ► To cite this version:

Stéphane Clivaz, Sara Presutti, Valérie Batteau, Luc-Olivier Bünzli, Audrey Daina, et al.. Mathematical knowledge for teaching problem-solving: Dialogic construction during a lesson study. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04411092

**HAL Id: hal-04411092**

**<https://hal.science/hal-04411092>**

Submitted on 23 Jan 2024

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# Mathematical knowledge for teaching problem-solving: Dialogic construction during a lesson study

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*This research aims to analyse the type of mathematics problem-solving knowledge for teaching used when working collaboratively in a lesson study process and examine how dialogic interactions contribute to knowledge construction. One part of the research is addressed here, using quantitative analysis with Markov chains. The results suggest that participants collectively use their mathematics and their problem-solving content knowledge to focus on pedagogical problem-solving knowledge.*

*Keywords: Teachers' mathematical knowledge, problem-solving, dialogic analysis, lesson study.*

## Introduction

The study of mathematical knowledge needed for teaching has garnered significant attention in mathematics education (as seen in Ball's report, 2017). Lately, problem-solving has been studied from both students' perspectives and through the collective efforts of teachers, researchers, and trainers (Borko & Potari, 2020). Due to its historical roots in Japan and its many adaptations at the international level, lesson study (LS) training and research process is one of the systems at the centre of this interest. The process aims to contribute to the professional development of teachers (Murata, 2011) and, in particular, to the construction and evolution of their mathematical knowledge for teaching (Clivaz & Ni Shuilleabhain, 2019). This study centres on the growth of teachers' professional knowledge through a dialogical process in a LS, as they teach grade 3-4 students problem-solving skills. The research was conducted with a group of eight teachers and two facilitators in Lausanne, Switzerland, over two lesson study cycles. Two papers have already been published about the first part of the study. The first paper outlines the theoretical and methodological aspects (Clivaz, Daina, et al., 2023), while the second presents the results of the analysis of the first part of the LS cycle (covering five out of eight meetings, Clivaz, Batteau, et al., 2023). The theoretical framework will be outlined in Section 2 while Section 3 briefly describes the data and the method. Section 4 summarises one of the research questions and the related main results.

## Theoretical framework

### Lesson Study

“Jugyou Kenkyuu”, or “lesson study” (LS), originated in Japan in the 1890s and gained popularity in the 2000s after international comparisons. LS was introduced in the USA as a professional development approach for improving mathematics teaching and learning (Lewis, 2002). LS has since gained global attention from researchers in educational sciences, particularly in mathematics education.

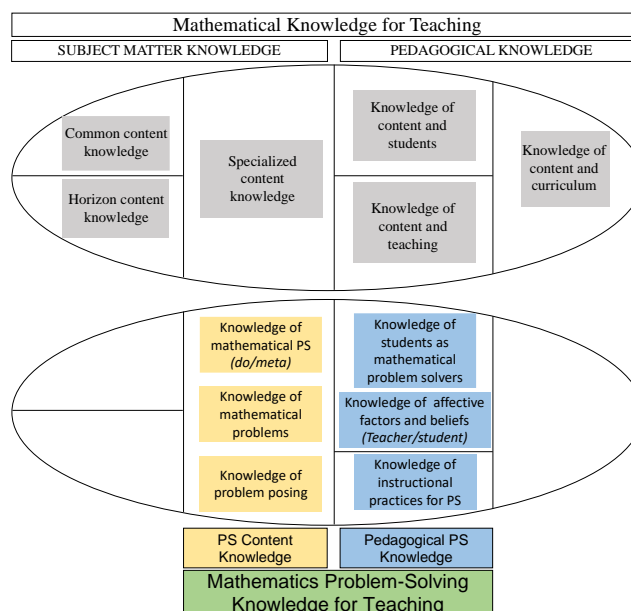
LS is a cyclic process with distinct phases, starting with a group of teachers identifying a difficulty in teaching and learning. They analyse the curriculum, plan a lesson together, implement it in one teacher's classroom, and observe its impact on student learning. They may then plan a modified

version of the lesson and repeat the cycle. The results are disseminated in the form of a lesson plan and professional journal articles.

LS groups are typically led by an experienced teacher or trainer (facilitator) who guides the discussion and involves all participants. In Japan, LS is sometimes facilitated directly by the group members, with the help of a knowledgeable other. In countries where LS is well-established, the roles of facilitators and external experts are well-defined (Clivaz & Takahashi, 2018).

### Mathematical Knowledge for Teaching Problem-Solving (MKTPS)

Research on problem-solving (PS) has mainly focused on students' perspective. However, some studies have explored teachers' understanding of problem-solving and their mathematical knowledge for teaching (Ball et al., 2008). Based on a literature review of studies from 1922 to 2013, Chapman (2015) identified six categories of problem-solving knowledge for teaching, including problem-solving content knowledge and pedagogical problem-solving knowledge. These categories are influenced by teachers' problem-solving proficiency, affective factors, and beliefs. Adopting Chapman's findings, we propose a graphical representation of this categorisation (Figure 1) to bridge the categories of mathematics problem-solving knowledge for teaching and mathematical knowledge for teaching.



**Figure 1: Mathematical Knowledge for Teaching Problem-Solving (MKTPS).**

**Mathematical knowledge for teaching (the upper grey part of the figure) is taken from Ball et al. (2008); mathematics problem-solving knowledge for teaching (coloured categories) is from Chapman (2015). For the description of the categories, refer to (Clivaz, Daina, et al., 2023).**

### Levels of activity, Lesson Study Dialogic Analysis and types of discourses.

Three other analytical tools are part of our conceptual framework: 1. *Knowledge levels* to assess participant knowledge evolution and role differences; 2. *Lesson Study Dialogue Analysis (LSDA)*, for the purpose of determining which type of dialogue would favour the development of knowledge; 3. *Types of speech* in order to find statistical correlations between dialogical characteristics and

teachers' professional development Clivaz, Daina, et al. (2023). These analytical tools are not used in this paper. Nevertheless, they are part of our research, and we will mention their use when relevant.

## **Data and method**

### **Context, data collection and data coding**

A LS group of ten teachers and facilitators from the Lausanne region was formed for this study. The facilitators, one math educator (the first author of this paper) and one teacher, aimed to focus on problem-solving in the classroom, without necessarily focusing on a particular mathematical topic. After discussing difficulties in teaching problem-solving, the group chose to concentrate on helping students understand the problem representation. During the next meeting, they selected a problem from a state test that students struggled with in the past. The group then discussed representation, student approaches, and mathematics involved before planning a research lesson. After teaching and observing the lesson, the group discussed their observations of student performance and the importance of manipulatives, whole-class discussions, mathematics, and support for struggling students. Based on this discussion, they planned a second lesson in a second class observed and discussed it. The last meeting allowed a discussion about the two lessons, about the finalisation of the lesson plan, the assessment of the cycle and the desire for the next cycle.

The meetings were recorded and transcribed using Transana ([www.transana.com](http://www.transana.com)), a qualitative data analysis software. The transcripts consist of 3517 utterances, 2726 of which were coded using LSDA codes and associated with MKTPS and with a unique knowledge level. The coding was performed by a team of the five authors over three years and was based on the analysis grids presented in (Clivaz, Daina, et al., 2023). The software allows for coding the transcript linked to the video and enables descriptive analysis of speaking time per speaker, knowledge types and levels, and LSDA clusters. The data was analysed in French. An English translation is only provided for papers in English.

### **Method**

This research used a mixed analysis method, including qualitative coding of utterances, quantitative analysis with Markov chains (Gagniuc, 2017) modelling knowledge transitions, levels, and interactions, and qualitative analysis at micro-, meso-, and macro-levels. The research aimed to study the construction of mathematical knowledge for teaching problem-solving during the LS process, taking into account temporality. Markov chains were used to analyse the sequences and transitions of knowledge during the LS meetings (for a detailed explanation of the use of Markov chains in our analysis, please refer to Clivaz, Batteau, et al., 2023). The variables analysed using Markov chains were MKTPS, knowledge levels, and dialogic interactions (LSDA). The data was exported from Transana and analysed using custom software.

### **Research question**

Our main research question and sub-questions, and the ones addressed in this paper, are: In what way is MKTPS built collectively during the LS process? Particularly, what is the MKTPS that emerges during each LS meeting? What are the transitions of MKTPS and those most represented in each LS meeting and in all meetings? How does one MKTPS follow on to the next in the dialogue?

## Results

### MKTPS for all meetings and per meeting

During the eight LS meetings analysed, 2096 MKTPS codes were assigned. For the globality of the eight LS meetings (Table 1), the most prevalent MKTPS was knowledge of instructional practices for PS (654) and knowledge of students as mathematical problem solvers (571).

**Table 1: Number of types of knowledge in each meeting**

	Meeting								all
	1	2	3	4	5	6	7	8	
Knowledge of mathematical problems	2	52	83	6	7	5	10	4	169
Knowledge of mathematical PS ( <i>do</i> )	1	18	11	3	7	1	1	0	42
Knowledge of mathematical PS ( <i>meta</i> )	6	10	51	15	39	40	50	20	231
Knowledge of problem posing	2	16	52	58	35	3	7	0	173
Knowledge of students as mathematical problem solvers	12	42	83	72	151	49	138	24	571
Knowledge of instructional practices for PS	15	48	151	98	111	99	81	51	654
Knowledge of affective factors and beliefs <i>teacher</i>	1	8	1	1	0	0	1	0	12
Knowledge of affective factors and beliefs <i>student</i>	11	4	4	2	21	7	35	3	87
Common content knowledge	0	4	5	0	0	0	0	0	9
Specialised content knowledge	6	0	2	13	0	4	8	1	34
Knowledge of content and students	1	3	0	0	0	1	0	0	5
Knowledge of content and teaching	10	1	0	0	1	0	0	0	12
Knowledge of content and curriculum	16	28	37	1	1	2	7	4	95
Horizon content Knowledge	0	0	0	0	0	0	2	0	2
Total	83	234	480	269	373	211	340	107	2096

The high number of these two categories is interpreted per meeting. The first meeting focuses on PS teaching and teachers' representations, with the most prevalent MKTPS being knowledge of content and curriculum (16 utterances), followed by knowledge of instructional practices (12) and knowledge of students as mathematical problem solvers (15). The high frequency of references to textbook tasks in discussions indicates a focus on knowledge of content and curriculum.

Meeting 2 and meeting 3 deal with the study of the curriculum material. The most represented MKTPS is knowledge of instructional practices for PS (48 for meeting 2 and 151 for meeting 3).

In meeting 4, for lesson planning, the two main MKTPS are knowledge of instructional practices for PS (98 utterances) and knowledge of students as mathematical problem solvers (72 utterances), showing that both types of knowledge are used almost equally in the planning process.

Meeting 5, the post-lesson discussion, focuses on student learning and teacher impact with a focus on the knowledge of students as mathematical problem solvers, represented by 151 utterances.

Meetings 6 and 7 encompass the planning and discussion of the second research lesson. Similar to the preceding meetings, 4 and 5 respectively, there is a notable abundance of knowledge of students as mathematical problem solvers and of knowledge of instructional practices for PS, but with a greater frequency of knowledge of mathematical PS (*meta*) and scarcely any occurrences of knowledge of problem posing. Our hypothesis is that there is no rehashing of the problem formulation, but rather an exploration of the consequences of students' various approaches.

In session 8, akin to session 1, there exists relatively limited MKTPS, predominantly involving the same three knowledge as in the two previous meetings. The remaining discussion revolves around LS or the teaching and learning of mathematics in general.

In summary, our study showed fluctuations in the emphasis on the two main teaching/learning MKTPS during meetings. Knowledge of instructional practices for PS was most prominent during the study of the curriculum, while both types of knowledge were used equally in lesson planning. Knowledge of students as mathematical problem solvers was emphasised in the post-lesson discussion. The macro-level analysis of communicative situations allows for the alignment of these MKTPS changes with LS objectives. This echoes similar findings regarding mathematical knowledge for teaching during the numeration-related LS by Clivaz and Ni Shuilleabhain (2019).

### Markov chains for the MKTPS

The Markov chain in Figure 2 allows us to analyse the sequencing of two utterances coded by MKTPS for all LS meetings. The probability of one MKTPS at time  $i + 1$  following MKTPS at time  $i$  is indicated on the arrow. For example, in Figure 2, the percentage shown in red means that, in our data, out of 42 knowledge of mathematical PS (do), 5 were followed in the next utterance by a knowledge of mathematical problems. Therefore, the probability is 11.9%. In the representation of Markov chain diagrams, the edges are depicted with increasingly darker shades of grey as the transition probability increases. The presence of rare transitions can lead to a cluttered and difficult-to-interpret graph. It was decided to only consider transitions with a probability greater than 10% to mitigate this issue, determined through empirical testing as an appropriate threshold value.

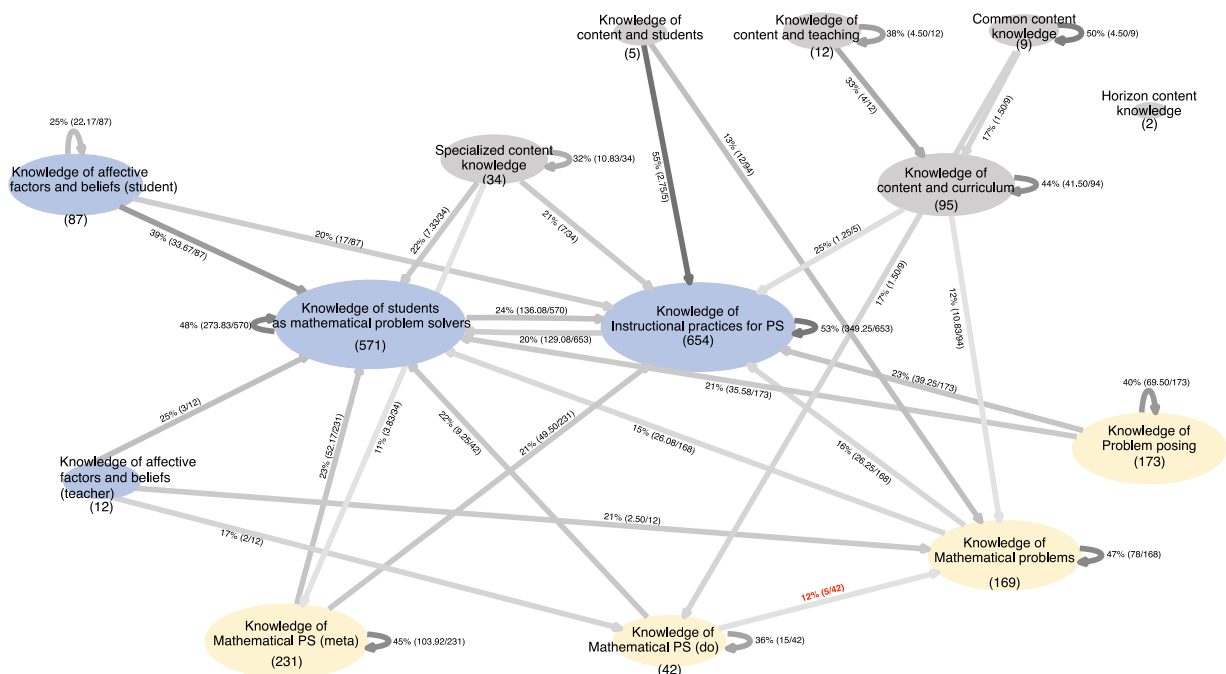


Figure 2: Markov chains for MKTPS for the five LS meetings

One finding is that the probability of a type of MKTPS being the same at time  $i + 1$  as at time  $i$  is around 50% for several MKTPS. This is a consequence of our methodological choice of the turn of speech as a unit of analysis: the same topic is often continued by the next utterer, and the same type

of knowledge is therefore often present in the following utterance. For this reason, we focus our analysis on the changes of MKTPS that are not influenced by this choice of unit of analysis, namely, on the transitions between different MKTPS.

The Markov chain highlights a central hub composed of the two most prominent MKTPS: knowledge of students as mathematical problem solvers and knowledge of instructional practices for PS. This hub exhibits an attractive pattern, with all visible transitions originating from MKTPS outside of the hub. This means that after any utterance coding a different MKTPS, the probability of the next utterance being coded by a MKTPS from the hub is high. During all LS meetings, almost all transitions between utterances were related to or converged towards the knowledge within this hub.

This supports the idea that participants in the LS process used their knowledge to redirect and apply it to their understanding of students as mathematical problem solvers and their knowledge of instructional practices for PS. We illustrate this phenomenon using the knowledge of problem posing. The utterances coded with knowledge of problem posing are associated with utterance coding of the same knowledge (40%, see Figure 2), utterances with knowledge of students as mathematical problem solvers (21%) and with knowledge of instructional practices for PS (23%). When a speaker talks about problem posing, there is a high probability that the next speaker talks about the effect of the choice of problem posing on instructional practices or on students. In the following extract (Table 2), teachers and facilitators deal with the difficulties related to the contexts of problems, and the vocabulary used when addressing problems. T1 and T6 speak about the changes in the wording of the problem (changing “videotape” to “CD”), necessary to make the vocabulary accessible to the students (knowledge of problem posing).

**Table 2: Extract of meeting 3 (1:19:32-1:20:57)**

Time	Utterer	Transcript	LSDA	MKTPS	Level
1:19:32	T1	You put “CD”, or you put...	RJ	K. of problem posing	2
1:19:36	T6	No, I don’t put “CD” but, for me, the objective is really to estimate a duration, and to read a table and then as a result...but it’s true that I take away the cover story of the problem.	PP	K. of instructional practices for PS K. of problem posing	4 4
1:19:45	F2	But do we have to, we...			
1:19:48	F1	But it’s two different objectives [...]. I think we are really discriminating socially when we discuss this kind of problem [...]	EE	K. of problem posing	5
1:20:04	F1	I think there’s a concern [about the problem dealing with compost and tree planting], I mean, we’re first discriminating against the parents’ job. For once, it’s more like the children of farmers who are advantaged, that’s good! I mean, there we’re already separating those who know the context, and those who don’t. I think that...But, in our case, do we want to work more on this question of...yeah, the context, of...maybe solving the problem without knowing the words, and then afterwards there are the difficulties of mathematical solving, or do we work more on mathematical solving... or both at the same time...	EE	K. of instructional practices for PS K. of problem posing	3 4
1:20:35	T6	It’s going to be hard, right?	PA	–	
1:20:36	F2	But I... Do we need to clarify certain starting elements, well, so that, precisely so that they can start. Talking about the vocabulary essentially, but also about the context, without talking about the problem itself, what do we need to clarify so that they can understand?	EIG	K. of instructional practices for PS	3



Likewise, when the speakers study the curriculum or plan the lesson, they use knowledge on problem posing and then discuss the effects of their choices on the instructional practices or students' learning.

Regarding transitions between MKTPS outside the hub, which do not lead to the hub, we can notice the following. Knowledge of mathematical problems follows knowledge of affective factors and beliefs (21%), mathematical PS (do) (12%), and content and curriculum (12%). This highlights two aspects of knowledge of mathematical problems: one focusing on the problem itself (nature, structure, purpose) and one focusing on the effect of problem characteristics on learners. Another two MKTPS, mathematical PS (do) and knowledge of content and curriculum, are more likely to appear after another MKTPS in more than 10% of cases. This supports the concept of a hub, as there are few transitions between two MKTPS outside it. Eight different types of knowledge did not return arrows based on the 10% threshold: the five mathematical knowledge for teaching, knowledge of problem posing, knowledge of mathematical PS (meta), knowledge of affective factors and beliefs (teacher and student). This indicates that other MKTPS do not often appear before the listed eight MKTPS.

## Conclusion

The Markov chains for MKTPS allow for movement from a static to a dynamic viewpoint in relation to knowledge. Not only MKTPS specific to the teaching and learning of problem-solving are the most represented during our LS meetings, but they also pull the focus of the dialogue towards them: participants collectively use their mathematical knowledge for teaching and their PS content knowledge to focus on the knowledge of students as mathematical problem solvers and the knowledge of instructional practices for PS.

The analysis in terms of MKTPS is only the first part of our analysis. One extension is the analysis in terms of levels of knowledge and of LSDA. For this, please refer to Clivaz, Batteau, et al. (2023). As a second addition, we are working on linking the utterances' dialogic features of LSDA to types of talk at the meso-level of longer episodes during LS meetings. We are categorising these types into five categories, as outlined by Dudley: "1. cumulative talk, 2. qualifying or disputational talk, 3. exploratory talk, 4. structuring conversation, 5. managing understanding" (2013, p. 110). Our findings will contribute to understanding the relationship between these discourse types and both MKTPS and knowledge levels (Clivaz, Daina, et al., 2023).

The findings of this case study are specific to the group under examination, and individual factors have impacted the results. Nevertheless, the mixed-method approach provides a comprehensive and interactive perspective on the collective construction of MKTPS. The unique characteristics of both LS and MKTPS stressed in our results reinforce the idea that LS and PS teaching are interdependent and demonstrate how teachers' professional knowledge is shaped through dialogic engagement during LS. This supports the notion that LS and PS teaching are "two wheels of a cart" (Fujii, 2018) and highlights the significance of a dialogic process in developing teachers' professional knowledge

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