Study of a teacher professional development process in the context of a collaborative work to design a resource for teaching problem solving

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This research deals with a collaborative work carried out with lower secondary teachers in Geneva aiming at developing a resource for teaching problem solving in mathematics. The structure of this resource is based on a characterisation of the problems proposed to the students according to the types of reasoning and approaches involved. The focus is on the interactions between the different actors during the meetings with the purpose of documenting how these interactions can be a means to develop and attest to teachers' professional development, considered in a professionalization approach. The theoretical framework of mathematical knowledge for teaching (Ball et al., 2008) and mathematical problem-solving knowledge for teaching (Chapman, 2015) are used to analyse the data. A conclusion is that teachers' professional development process can be promoted in the context of a collaborative work to design a resource for teaching problem solving.

Keywords: Knowledge base for teaching, mathematical reasoning skills, problem solving, professional development.

The concept of professional development (PD) is polysemous. Moreover, in the French-speaking literature, different expressions are used to refer to it, such as: *formation continue, perfectionnement, croissance professionnelle* (respectively, with our translation: continuous training, improvement, professional growth). On this point, Uwamariya and Mukamurera (2005) identify a possible confusion between the meaning of the concept itself and how it can be achieved. Recently, increasing attention has been paid to mathematics teachers' PD programs (Arzarello et al., 2014; Ball et al., 2008). However, it appears that the study of the effectiveness of training courses is often taking precedence over the analysis of PD processes (Wilkie, 2017). Our research focuses on this last point and takes place in the context of a collaborative work between researchers and lower secondary teachers in Geneva. The main purpose of this collaboration is the production of a resource for teaching mathematical problem solving (PS). Therefore, it doesn't specifically aim to develop teachers' PD. This is the originality of this research since the starting point of our reflection is to know if such a collaboration can nevertheless produce PD for the participating teachers. After the presentation of the theoretical framework used, we follow by the research context, the research questions and the method which is illustrated by the example of the analysis of a teacher's PD process.

Theoretical framework

Professional development

When dealing with teacher PD, a major, conceptual, difficulty emerges: how to define it? Does PD necessarily imply changes in practice? Learning of new professional gestures, knowledge? In that sense, Mukamurera highlights the plurality of definitions and theoretical frameworks about teacher PD. By a work of clarification of this concept, Uwamariya and Mukamurera (2005) identify and

characterise two principal trends. The first, inspired by Piaget's developmental stages, is the developmental approach, which considers the PD in a chronological linearity through successive stages throughout a teacher's career. However, even if Barone et al. (1996), Huberman (1989) or Vonk (1988) have identified such stages, the main criticism of this developmental approach is that, by focusing on changes in the teacher, it neglects the role played by the environment in which the teacher evolves. Another approach, the professionalizing approach, considers PD as a learning process, i.e., "(…) a process of acquiring knowledge that subsequently leads to changes in the teacher and to novelties in his or her practice[1]" (Uwamariya & Mukamurera, 2005, p. 142). We embrace this constructivist approach insofar as the teacher is the actor of his or her PD and the learning process has an individual but also a collective dimension, even if it is difficult to distinguish these two aspects. Therefore, it means that documenting teacher's PD leads to an interest in teacher knowledge. In this study, such knowledge is related to the following question inspired by Ball and colleagues (2008): what do teachers need to know and be able to do in order to teach effectively PS? Indeed, our hypothesis is that it is not sufficient to yourself being capable of solving problems to teach effectively PS. That is why it seems necessary to give a deeper look to the characterisation of the wide range of the mathematical knowledge for teaching (Ball et al., 2008), and then more specifically to the work of Chapman (2015) about mathematical PS knowledge for teaching.

Mathematical knowledge for teaching

The work of Ball and colleagues (2008) to develop a practice-based theory of content knowledge for teaching built on Shulman's notion of pedagogical content knowledge (1986) invites to consider different categories of teacher knowledge (Figure 1). These categories are linked to the two main types identified by Shulman: those related to the discipline itself or to the pedagogical dimension.

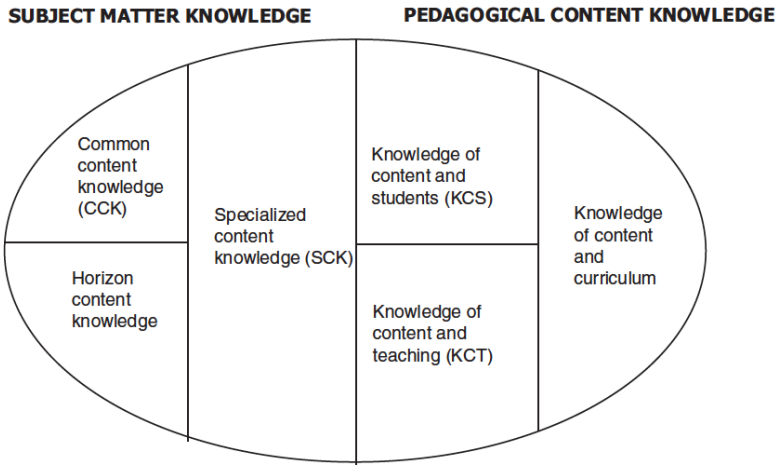

Figure 1: Domains of Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403)

To briefly define the different categories identified by Ball et al.: within pedagogical content knowledge, knowledge of content and students and knowledge of content and teaching are aligned

[1] Our translation. Original quote: « […] un processus d'acquisition des savoirs qui provoque, par la suite, des changements chez l'enseignant ainsi que des nouveautés sur le plan de sa pratique. »

with the definitions of Shulman, respectively as “the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons” and “the ways of representing and formulating the subject that make it comprehensible to others” (1986, p. 9). Knowledge of content and curriculum refers to program and instructional materials. Within subject matter knowledge, horizon content knowledge is “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403). Common content knowledge deals with the mathematical knowledge commonly shared with those who know and use mathematics, that is mathematical knowledge and skills used more widely than in teaching situation. However, authors precise that it is not always easy to distinguish it from specialised content knowledge. This last category is associated to a kind of subject matter knowledge which is not mixed with other categories and specific to mathematics teaching. It is “the mathematical knowledge and skill unique to teaching” (Ibid., p. 400). This theoretical framework seems epistemologically coherent with the professionalizing approach of the PD, especially because Ball et al. (2008) focus on the work of teaching to determine what teachers need to know and be able to do to teach effectively. In the following section, we look more specifically at the mathematical knowledge for teaching specific to PS.

Mathematical problem-solving knowledge for teaching

In her large review of the research literature on PS in mathematics education to characterise the mathematical PS knowledge for teaching, Chapman (2015) identifies and describes seven types of mathematical PS knowledge for teaching: “mathematical PS proficiency (understanding what is needed for successful mathematical PS), mathematical problems (understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners), mathematical PS (being proficient in PS; understanding of mathematical PS as a way of thinking; PS models and the meaning and use of heuristics; how to interpret students’ unusual solutions; and implications of students' different approaches), problem posing (understanding of problem posing before, during and after PS), students as mathematical problem solvers (understanding what a student knows, can do, and is disposed to do), instructional practices for PS (understanding how and what it means to help students to become better problem solvers), and affective factors and belief (understanding nature and impact of productive and unproductive affective factors and beliefs on learning and teaching PS and teaching).” (Chapman, 2015, p. 31).The graphical representation proposed by Clivaz and colleagues (Clivaz et al., 2023, p. 23) provides a better understanding of how these theoretical tools relate to each other (Figure 2).

Before presenting our research questions and the methodology, we describe in the following section the context of the research and the nature of the collaborative work made with teachers.

Context of the research

Even if teacher PD is usually studied in the context of pre-service or in-service training, this research addresses this concept in the context of collaborative work on resource design about PS. On top of that, the context of the research leads to a focus on mathematical knowledge of teachers who do not teach students directly, but who are working on a resource for teaching PS, which therefore aims to help teachers teach it in a relevant way.

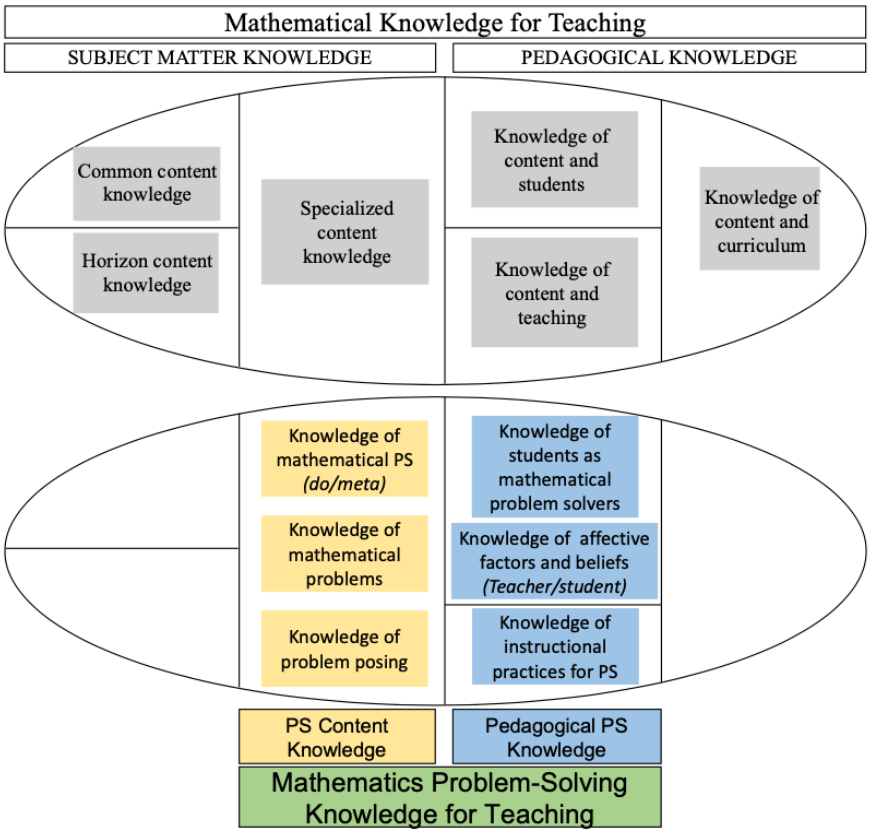

Figure 2. Graphical representation of links between Mathematical Knowledge for Teaching (Ball et al.) and Mathematics Problem-Solving Knowledge for Teaching (Chapman) (Clivaz et al., 2023, p. 23)

The collaborative working group was constituted by four mathematics teachers at the lower secondary and the two authors. It focuses on a problem-solving centred course given in the canton of Geneva in the French-speaking part of Switzerland and called "mathematical and scientific approaches". This annual course is designed for a 45-minute period per week and delivered to 13–14-year-old students (grade 8). Its main goal is to improve students' problem-solving competencies. It is also important to notice that teachers are left very free in the organization of this course (problems proposed to the students, theoretical knowledge, structure of the classes, …). Starting from an existing, institutional resource dealing with PS in the specific context of "mathematical and scientific approaches" course, the aim was to review its contents as well as its form and structure. The choice was made to organise the new resource around the specific types of mathematical approaches and reasoning involved in the problems (Chanudet, 2021; Chanudet & Favier, 2021; Favier & Chanudet, 2021). The group's work consisted of selecting problems to keep from the previous resource, modifying them if needed, proposing new problems to supplement underrepresented types of reasoning or approach, thinking about what additional information should be included for each problem (e.g., solution, right and wrong procedures, possible students' errors, difficulties), and documenting these categories. The group met 11 times between September 2019 and June 2020, each time for approximately 3 hours and all the meetings were audio-recorded. Of the four teachers, three participated in the in-service training that the authors gave on mathematical reasoning and approaches, and PS. Charles is the only one who did not participate. He is also the least experienced teacher in the group (he had just finished his initial training when the collaborative work started).

Research questions

In this context, the study deals with the process of teacher PD and in the teacher knowledge at stake. The interest in PD leads to focus, not on all manifestations of teacher mathematical knowledge but, consistent with the professionalization approach to PD, only on those that attest to newly acquired knowledge. The context makes impossible to investigate effective changes in the teacher and novelties in his/her practice, that is why it focuses on interaction analysis. The research questions are: Is the process of teacher PD encouraged in collaborative work to develop a resource for teaching PS? If so, what types of mathematical knowledge for teaching are developed? To answer these questions, the idea is to identify tracks of PD and the nature of the teacher knowledge mobilised. However, this does not mean that there is no PD, even with no explicit elements in the teachers' discourse.

Methods

Our research draws on qualitative methods. We analysed all the interactions between participants of the group and focused on specific interventions that constitute indications of change in teachers' knowledge (thanks to a comparison with previous or future interventions). In accordance with the theoretical framework presented above and to the different types of mathematical approaches and reasoning at the heart of the PS perspective developed in the resource, the categories of teacher knowledge used to analyse the interactions in this context of PS resource design are:

- Common content knowledge: solving correctly mathematics problems with his/her own knowledge, knowing the main types of mathematical reasoning (inductive, deductive).
- Knowledge of mathematical PS: solving correctly mathematics problems with students' knowledge, anticipating students' specific processes and errors, and possible hints to give to stuck students without "killing" the research potential of the problem (when and how to intervene during students' PS), interpreting unusual students' solutions.
- Knowledge of mathematical problems: identifying and characterizing the specific type(s) of mathematical approaches and reasoning involved in the problems, choosing a problem according to the nature of the reasoning involved and the purpose for the students.
- Knowledge of problem posing: imagining or modifying a problem so that it allows students to work on a specific type of mathematical reasoning
- Knowledge of content and curriculum: identifying intended learning outcomes about PS.
- Knowledge of content and students: choosing adequate problems relatively to students, anticipating students' common processes and errors, knowing students as problem solvers .
- Knowledge of content and teaching: reflecting about a long-term planning of PS instruction, reflecting about different choices related to problems (e.g., statement, mode of presentation).

It is important to notice that we rely on the types of mathematical approaches and reasoning involved in the problems, both to provide finer analyses of teachers' knowledge and to fit the context of this collaborative work.

Analysis and results

To illustrate the study of teachers' PD process, this part focuses on a teacher of the group, Charles. During the first meetings, he seems very interested on the mathematics aspects of the problems, rather

than on the associated PS skills. The first example illustrates the mobilisation of common content knowledge related to general mathematics, to decide if the problem is interesting or not to submit to students. Indeed, Charles refers to the mathematical properties at stake.

Charles: Yes, it is not bad, it is algebraic calculation, they are going to do a lot of trial and error, they are going to find that it is not at all ... yes maybe they are going to succeed with trial and error, I do not know, I have not tried it but with literal calculation it is going well, it is not bad, there is a bit of factoring, I think it is pretty good.

However, he reflets about what students can do or not, related to his knowledge of students as mathematical problem solvers. Later, during the sixth meeting of the group, the discussion is about the different types of reasoning and approaches mobilised in a problem. Charles adds that the second question leads to the hypothetical-deductive reasoning because it involves the notion of parity. When researchers say that there are many things in the same problem, he specifies that "It is is not bad, that it mixes different kinds of reasoning, it shows that the problem is rich.". Charles shows that he developed his knowledge of mathematical problems by identifying the mathematical reasoning and approaches at stake when solving the problem and by considering the richness of a problem, according to the possible associated PS learnings of students.

The following excerpt comes from the discussion that occurs within the group during the eighth meeting, after a long exchange on how to solve the "Motives" problem (see Appendix for the problem statement) and on possible modifications of the statement:

1	Charles:	So that is, uhm, that is what? That is an experimental approach, we agree?
2	Marlene:	Yes.
3	Charles:	Ah I finally understood what it is.
4	Researcher:	So how would you describe it?
5	Charles:	No idea. You have figures, you have to count things, find a formula.
6	Researcher:	When there are squares it is an experimental approach [laughs].
7	Charles:	When you have to find a formula by counting things.

The line 3 constitutes an indication of PD verbalised by Charles. This progression is linked to the identification of the experimental approach, so to his knowledge of mathematical problems. Nevertheless, Charles is not yet able to characterize this approach (line 5). It is as if his knowledge is (only) mobilised in action, in the identification of problems involving an experimental approach.

The next example is an excerpt from the tenth meeting. While the members of the group debate about the type of approach or reasoning involved in the new problem studied, Charles says that he is "not very attached to these names. You can put whatever you want. I leave it to the specialists". He does not give importance to the terms used and seem to consider it useless for his practice. But what he considers as a naming issue is central and has to do with the nature of the types of reasoning or approach involved. The discussion continues as follows:

Researcher: How are you going to use it afterwards, if you are not, if you do not follow it?
Charles: I look at the activity, I do it in my head and then I say to myself, will it be fun to do with my class or not?

Charles does not seem to rely on mathematical criteria to select the problems but rather on their playfulness, for him and the students. He relies on his knowledge of affective factors and beliefs. However, a few minutes later, he links problems according to the type of reasoning to be used:

Charles: "Powers", "decimals" and "ribbon" [titles of 3 problems] are the same thing for me.
Carine: Yes.
Charles: They do a thing, they see something repeated and then...
Carine: Yes for “decimals” I think it is an experimental approach.
Charles: Yes for everything.

Charles connects different problems which mobilise the experimental approach and justifies this link by formulating the beginning of a characterisation of this approach. In the light of all the discussions at the meetings, we identify the development of his knowledge of mathematical problems. Here, it is not the teacher who verbalises a change in his/her knowledge but the researcher who interprets it.

Conclusion

To conclude, our analysis show that Charles has developed his mathematical knowledge for teaching and more specifically his knowledge of mathematical problems. In the context of this research, it is the development of knowledge related to mathematical reasoning that enhances mathematical PS knowledge for teaching. Relatively to the professionalization approach of PD, some of Charles’ interventions illustrate that he has enriched his way of thinking about PS from a teaching perspective, but other times it highlights that prior knowledge is still in play (e.g., major interest in mathematical concepts involved in the problems, rather than in PS). PD does not appear to be a linear process, with new knowledge replacing previous one, but as an enrichment of knowledge, with the possible coexistence of conflicting knowledge. With regard to our research questions, it appears that teacher PD process can be fostered in the context of a collaborative work to design a resource for teaching PS. It means that even a situation not directly focused on PD, as pre-service or in-service training, can strengthen this process. It seems that the need to reach a consensus during the meetings (e.g., on the choice of problems, their statement, the complementary information needed by teachers to implement it efficiently, …), and to take pragmatic decisions involves discussions between actors that seem to be a source PD for teachers. A limit of our research is that the changes observed are inferred from what teachers say in the context of the design of a resource, and not directly connected to their own practice. Finally, a perspective of this research is that looking at the mathematical knowledge for teaching that emerges and is developed during the discussions of a collaborative group around PS resource design, can also enrich the list of mathematical knowledge necessary to teach PS. This provides valuable information about what might be at the core of a teacher training on PS, and what might be specified in the teaching resources for PS.

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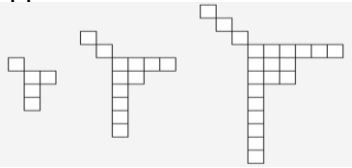
Appendix : "Motives"



1) How many squares are needed to construct Figure 1? To construct figure 2?
2) What is the number of squares needed to construct figure 4?
3) What is the number of squares needed to construct figure 100?