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# The development of mathematical knowledge of prospective primary teachers in a lesson study

Linda Cardoso<sup>1</sup>, João Pedro da Ponte<sup>1</sup> and Marisa Quaresma<sup>1</sup>

<sup>1</sup>Instituto de Educação da Universidade de Lisboa, Portugal; [lindacardoso28@gmail.com](mailto:lindacardoso28@gmail.com)

*This paper aims to understand how lesson study can promote the development of mathematical knowledge of prospective primary teachers. Following a qualitative approach, this study had the participation of four prospective teachers, two teacher educators and a researcher. The formative process focused on the topic of numbers and operations at grade 4. The results indicate that the prospective teachers may develop aspects related to knowledge of topics, structure of mathematics, and practices in mathematics when they participate in lesson study. This development occurs through the engagement in lesson study activities, such as solving and discussing adaptations of tasks and anticipating students' strategies and their possible difficulties, that allow prospective teachers to deepen their knowledge; but also, through discussions among prospective teachers; and exchanging ideas, discussions, and clarifications with teacher educators.*

*Keywords: Mathematical knowledge, prospective primary teachers, initial teacher education, lesson study.*

## Introduction

Initial teacher education is a field of research that continues to be the subject of many studies, in a constant search for its improvement and quality. However, there are many problems in initial teacher education (Cochran-Smith & Villegas, 2015). Regarding prospective primary teachers, it is well known that there is a strong gap in the mathematical knowledge they have and the mathematical knowledge they need to teach (Strutchens et al., 2016). Thus, it is necessary to look for formative processes that help to minimize some of the problems pointed out in initial teacher education and the difficulties of prospective primary teachers.

Lesson study is a formative process that seeks to improve the teaching and learning of mathematics and constitutes a success factor in changing teachers' teaching practice (Murata, 2011). Over the years, lesson study has been shown to promote the development of prospective teachers' knowledge (Larssen et al., 2018). Given its characteristics, lesson study may enable the development of mathematical knowledge, however, there is still not much in-depth research on this issue (Ponte, 2017). Thus, this research aims to understand how lesson study can promote the development of mathematical knowledge of prospective primary teachers. More specifically, we seek to address the following questions: (i) what aspects of mathematical knowledge can prospective primary teachers develop during lesson study? And (ii) how does the development of prospective primary teachers' mathematical knowledge occur during the lesson study?

## Mathematical knowledge

In their practice, teachers need to use and develop knowledge concerning the content that they teach, the students, the curriculum, and the pedagogical content knowledge. Ponte (2014) states that for a quality teaching of mathematics, the teacher needs to have adequate training in mathematics and as

well as in pedagogical content knowledge (also known as didactical knowledge). This idea is also supported by Ball et al. (2008), who developed a model of teacher knowledge, which include as main domains pedagogical content knowledge and content knowledge. It is also supported by Carrillo-Yañez et al. (2018) that developed a model known as *Mathematics Teacher Specialised Knowledge* (MTSK). This model is specific to the teacher who teaches mathematics, and builds on Ball and colleagues' model, but seeks to improve and overcome some limitations of previous models. In MTSK, teacher knowledge is divided into *Mathematical Knowledge* (MK), *Pedagogical Content Knowledge* (PCK) and with *Beliefs* at the centre. In this model, *Mathematical Knowledge* consists of three domains: *Knowledge of Topics* (KoT), *Knowledge of the Structure of Mathematics* (KSM), and *Knowledge of Practices in Mathematics* (KPM). KoT is knowledge about mathematical concepts and procedures and their meanings. KSM includes knowledge about the structure of the discipline. Finally, KPM refers to ways of working in mathematics, including mathematical communication, mathematical reasoning, knowing how to use definitions, establishing relationships, selecting representations, justifying, generalizing, and exploring.

When they begin their initial training, prospective primary teachers already bring some content knowledge but, during their initial teacher education, this knowledge is developed and deepened (Shulman, 1986). However, a study conducted with 268 students from initial teacher education for primary teachers in Portugal showed that a large proportion of the participants showed difficulties in essential concepts, including in the topic of numbers and operations (Serrazina et al., 2014).

## **Lesson Study**

Lesson study is a collaborative formative process consisting of four main phases: (i) goal setting and preparatory study; (ii) lesson planning; (iii) research lesson; and (iv) post-lesson reflection (Murata, 2011). Throughout this cycle, the working group meets regularly to study, design and plan one or more lessons, which are then put into practice and on which they then reflect and improve. The whole process focuses on students' learning. In the first phase, the group defines the goal of the lesson, starting with a problem in students' learning. In the second phase, they plan the lesson in detail, including the analysis of tasks, teaching strategies, the anticipation of solution strategies and possible students' difficulties, the preparation of the whole-class discussion, and the planning of observation. In the third phase, one of the participants teaches the lesson, with the others observing the student's learning. In the fourth phase, the group discusses and reflects on what was observed and possible improvements (Murata, 2011).

This formative process has characteristics that make it an enabler of knowledge development, namely the fact that it focus on students' learning, is a collaborative and reflective process, and allows a strong connection between theory and practice. Through the different activities developed during lesson study, prospective teachers have the opportunity to develop their knowledge about teaching and learning mathematics (Larssen et al., 2018).

Initial teacher education should help prospective teachers to develop, integrate and deepen their content knowledge and pedagogical content knowledge so that they become specialized teachers in the area they teach. In lesson study, prospective teachers can develop this knowledge when they participate in contextualized discussions about pedagogical content knowledge, a framework is

provided where they can engage in teaching practice, and they can review and develop their knowledge of mathematics and mathematical processes (Hourigan & Leavy, 2019). In a lesson study they developed, throughout the sessions, prospective teachers had the opportunity to wonder about their knowledge at the mathematics level, allowing deeper reflection on what they know and a deepening of other mathematical knowledge (Hourigan & Leavy, 2019).

## Methods

### Participants and context


The lesson study was developed in Portugal, in 2021, in a teacher education institution, at the end of the combined program for primary education (6-10 years) and pre-school education (0-5 years), and integrated into a mathematics pedagogical content knowledge course. The study was developed at this moment because the prospective teachers had already attended the mathematics courses of the initial teacher education program and were attending a supervised teacher practical course in primary school. The course had 25 prospective teachers. Thus, it was possible to have the participation of two teacher educators to guide the prospective teachers in the process. At the time the lesson study was developed, the week of the prospective teachers was divided into two parts: in the first three days they were in a school, in grade 4 class, under the supervision of a cooperating teacher, and guided by a teacher educator; in the other two days, they attended classes at the teacher education institution.

For the prospective teachers this was the first time that they heard about lesson study, but the two teacher educators (Diana and Miriam – all names in this paper are fictitious) already knew about the process. Since the class was divided into small working groups, one of them was selected to be followed by the researcher. A group of four prospective teachers was selected: Maria, Isabel, Jennifer, and Julia. The participants were contacted in advance and indicated their willingness to take part in the study. None of them had teaching experience and they had different academic backgrounds and relationships with the area of mathematics.

### Lesson study structure

The lesson study had eleven sessions, each lasting 60 to 120 minutes. The goal setting and preparatory study phase had one session and focused on selecting the topic and defining the goal of the lesson. It was chosen to address the topic of numbers and operations. Also in this session, the group selected the task (Figure 1). The lesson planning phase had five sessions, and the group worked on solving the task, anticipating possible solution strategies and students' difficulties, building the lesson plan, clarifying questions about mathematical reasoning, and reasoning processes and planning the observation. Then, there were two research lessons,

Solve the following task and don't forget to record every step you took on your solution sheet, even how you thought about it.



For his birthday Vasco prepares shrimp skewered. He hesitates between using 3 or 5 shrimps on each skewered.

1. Can you explain what Vasco is thinking about? What kind of skewered would you prepare? Why would you do it?
2. Vasco counted the shrimps that his mother bought: 52, 54, 58, 60, 61! Think about your choice. Approximately how many skewered could you make? More than 5? More than 10? How could you determine the exact number of skewered?

**Figure 1: Task “shrimp skewers”**

taught by each pair of prospective teachers in the class with which they were working. Finally, there were three post-lesson reflection sessions, in which they shared what they observed in the research lessons, discussed differences and similarities of what happened in the lessons, reflected on what they observed and how they could improve, and possible adaptations to the task. Since there was only one teacher educator in the room at a time, the groups worked autonomously, and the teacher went around the groups. To guide the work, at the beginning of each session, the teacher presented the session goals, and, at the end of the lesson, the whole-class met again for some final considerations.

### **Data collection and analysis**

The research follows a qualitative approach, with a participant observation design (Jorgensen, 1989). Data were collected through video recording of the sessions (Sx) and compiling a research journal, semi-structured interviews with the prospective teachers at the beginning (IE) and end (FE) of the lesson study, and collection of the productions made by the prospective teachers during the sessions and their final written reflections. The data analysis was based on the domains of Carrillo-Yañez et al.'s (2018) model, KoT, KSM, and KPM. We first analyzed the initial interviews, looking for evidence about the prospective teachers' knowledge in the different domains of mathematical knowledge. Then, we analyzed the moments of the lesson study sessions in which aspects related to mathematical knowledge were addressed. Finally, in the final interviews and the final written reflections, we analyzed the prospective teachers' references to the development of knowledge in the three domains.

## **Results**

### **Knowledge of topics (KoT)**

Before the lesson study, the prospective teachers mentioned that they felt difficulties concerning mathematical content:

Isabel: [I] may have more difficulties at grades 3 and 4 than grades 1 and 2 because the contents are a bit more complex... (IE)

Isabel mentioned that starting the practice in a more advanced school year, such as grade 4, brought more challenging content. The other prospective teachers also indicated that they had difficulties with the content, mentioning specific mathematic topics they felt they needed to work on.

During the lesson study, there were several moments that supported the development of KoT. Specifically, the prospective teachers had the opportunity to work on the definition of a prime number. Even though the teacher Miriam guided the discussion and focused attention on the number 61, which was the highlighted number in the task, only when they were working on solving the task, the term "prime number" came up for the first time:

Julia: Girls, regarding the number 61, I was doing the division of the number and every time you divide by any number you always get the approximate number, it's never a certain [whole] number ... Isn't the number 61 a prime number?

Maria: Yes.

Isabel: I wonder if that has something to do with it?

Julia: Yeah, I think so! Because you can't divide it...

Isabel: We can try it with another prime number, to see if that... if it is too.

Maria: What do you mean? Explain yourself. I don't understand.

- Isabel: So, if... if... we can do that example with another prime number and see if that number when divided by another number always gives results with commas.
- Maria: It does because it's a prime number. There it is, it's only divisible by 1 and by itself. (S2)

The prospective teachers had already tried to find divisors of 61, using the calculator to help in the calculations. Julia brought up for discussion the fact that 61 is a prime number. Although Maria agreed right away, Isabel showed difficulties and a lack of knowledge about this definition. The discussion among prospective teachers enabled them to work on their KoT. In the following sessions, they were able to use the definition of prime number correctly.

Students' solution strategies at the research lesson included some strategies that the prospective teachers had not anticipated: "one [student] did drawings too and through the drawings she did the multiplication" (Jennifer, S9). In the planning phase, Maria mentioned that these representations were only used by younger students. The research lesson allowed the development of knowledge about possible students' solving strategies, and also enabled prospective teachers to realize some aspects to improve, such as the importance of not only anticipating possible student difficulties but also anticipating teacher actions, because "We thought about [the difficulties] but we didn't think about a solution, and when we got there... I had no way to answer this question" (Julia, S11).

In the final interview, the prospective teachers indicated the development of knowledge about mathematical content:

- Jennifer: We ended up acquiring some mathematical knowledge, mainly related to the content we were going to develop with the students ... (FE)

The development of KoT of the prospective teachers was visible throughout the sessions of the lesson study and the participants recognize this development.

### **Knowledge of the structure of mathematics (KSM)**

In the initial interview, the prospective teachers did not mention any aspect or difficulty related to KSM. During the lesson study, since they selected the task, they were very focused on addressing prime numbers in the lesson, even putting it in the lesson plan as one of the lesson goals. However, moving forward with the lesson plan analysis, teacher Diana focused on the lesson goals that the prospective teachers wrote down:

- Diana: Did you have as a priority goal in this task to introduce the notion of prime number?
- Maria: Introduce I don't say, because prime numbers are at grade 2.
- Diana: Grade 2? In grade 2, do they already know what a prime number is?
- Maria: I think in the curriculum it's at the end of grade 2...
- Diana: So, let's go to the curriculum...
- Isabel: We looked it up at the beginning. (S6)

The teacher opened the curriculum document and showed that the definition of a prime number is not at grade 2. She also explained that this was not the focus of the task, but to realize that 61 is a special number that only has two divisors, supporting the development of the KSM of prospective teachers. This discussion changed the way the prospective teachers looked at the lesson because until then they were very focused on prime numbers.

Although the prospective teachers did not mention any KSM related aspects in the final interview, as they did initially, it was observed that the discussion with their teacher led to them not giving the emphasis in the research lesson to prime numbers that they had initially planned to give, showing development in the prospective teachers' KSM knowledge.

### **Knowledge of practices in mathematics (KPM)**

In the lessons before the lesson study, the prospective teachers were working on mathematical reasoning and reasoning processes. However, during the initial interview, they mentioned that they had many questions and difficulties related to this:

Julia: I felt very lost [concerning mathematical reasoning] ... I didn't really catch anything ... (IE).

The prospective teachers mentioned that they were confused about mathematical reasoning. Throughout the interview, the participants showed to give great importance to this aspect, possibly, because it was something that had been addressed recently.

In the very first session of the lesson study, when they were looking at different tasks and solving and discussing them, Maria tried to make a generalization.

Isabel: Hm... That one I don't quite understand.

Maria: Ah, we're supposed to erase some values, or 25, or 50, or the 3. For example, here we take out the 3, then ask how much the hat costs. It's kind of like that, I guess.

Jennifer: I guess, it's just going to take it out one at a time.

Maria: Hmm-hmm. For the generalization, it was something like...  $25+3+22=50$ . (S1)

Initially, Isabel showed some difficulty in interpreting the task. Maria tried to explain what was intended and then tried to make a generalization but showed some confusion in understanding what a generalization is. Throughout the sessions, the prospective teachers concluded that it is better to analyze the reasoning processes with the help of the teacher Miriam, dedicating one session only to this aspect. With the teacher's guidance, they ended the session by analyzing the reasoning processes that can arise in the task, considering their solution of the task:

Maria: In justification, in exemplification...

Miriam: ... Perhaps it could also be a generalization, it seems to me that it could arise, but as a learning goal, what is most implicit there is the use of exemplification and the use of justification. (S5)

Maria identified justification and exemplification right away, but she needed help from the teacher to identify a generalization. After the lesson study, during the final interview, the prospective teachers mentioned the importance that the formative process had for deepening their knowledge of mathematical reasoning and reasoning processes:

Isabel: I was able to, let's say, apply ... because in class it was something more abstract. (FE)

In the lesson study, the prospective teachers had the opportunity to deepen the knowledge they had acquired previously, and the practical part was important for this development of knowledge.

### **Discussion and conclusion**

In the lesson study, some moments allowed for discussing aspects of mathematical knowledge related to the three domains identified by Carrillo-Yañez et al. (2018). As in the study conducted by Serrazina

et al. (2014), difficulties in basic knowledge were observed in prospective teachers. For example, the participants in this study showed difficulties and surprise when they identified the divisors of the number 61 which is studied in the very first years of school.

In this lesson study integrated in an initial teacher education course, the prospective teachers developed and deepened their mathematical knowledge (Hourigan & Leavy, 2019), which is expected to happen during initial teacher education (Shulman, 1986). The various phases of the lesson study (Murata, 2011) provided discussions and activities developed by the participants that enhanced this development, as stated by Larssen et al. (2018). However, this study allowed for a deeper exploration of what knowledge the prospective teachers developed and how.

The peer discussion was fundamental not only to clarify questions but also to review concepts and deepen the prospective teachers' mathematical knowledge. However, when the peer discussions were not enough to clarify a certain issue, sometimes the prospective teachers turned to the teachers, as was the case with the reasoning processes. The teachers made an important contribution by guiding prospective teachers to some aspects that they did not even notice.

Thus, in this research that aimed to understand how the lesson study can promote the development of mathematical knowledge of prospective primary teachers, the results indicate that this lesson study provided an opportunity for prospective teachers to develop their mathematical knowledge in the three domains indicated by Carrillo-Yañez et al. (2018). More specifically, the prospective teachers developed KoT when they solved and adapted the task. They developed KSM when they designed the lesson plan, more specifically, when they considered the learning goals. They developed KPM when they solved the task and identified the reasoning processes involved. The lesson study enhanced the development of the participants' mathematical knowledge, as it provided: (i) the realization of activities that require the mobilization of mathematical knowledge; (ii) discussions among prospective teachers with different academic backgrounds, which allowed them to deepen their knowledge and clarify questions; and (iii) the exchange of ideas among themselves and with the teachers to improve lesson planning and reflect on the research lesson. Therefore, lesson study allowed an integrated and collaborative development, providing enriching experiences for the development of these prospective teachers.

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