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Reasoning and proving tasks on geometry in two distinct textbooks: One topic, two lenses

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This paper presents a comparative analysis of two textbooks, focusing on the topics of line and point symmetry from two distinct perspectives. Firstly, the study examines how these topics are introduced and processed, the learning environment provided by the textbooks, and the roles of both pupils and teachers in the learning process. Secondly, the analysis evaluates the reasoning and proof tasks presented in both textbooks and how they encourage teachers to incorporate these tasks into their teaching. By analysing the textbooks, the study not only identifies technical terms associated with reasoning and proof activities but also explores the interconnections between these activities. This approach offers a more comprehensive understanding of how textbooks encourage specific activities, such as reasoning and proof, and could be useful for teachers, researchers, and textbook authors.

Keywords: Resources, teacher role, pupil role, reasoning and proof, exploration.

1 Introduction

Mathematics textbooks can play an important role in students' opportunities to engage in reasoning and proof (R&P). They are considered to be an important element in educational system (Haggarty & Pepin, 2002) and the content of the textbooks is crucial to student learning. Textbooks can also influence teachers' decision to implement R&P tasks in their classrooms, especially in terms of when and how to do so. In the field of mathematics education, a lot of attention has been paid to R&P (e.g., Stylianides & Ball, 2008; Stacey & Vincent, 2009), and reasoning and proof remain important topics to study (e.g., Herbert & William, 2023; Michal et al., 2022). As stated in Herbert & Williams (2023), reasoning is often challenging to teach, particularly at the primary and lower secondary levels, and authors consider textbooks as important resources of R&P.

Since textbooks are the most common resources used by our teachers (pre- and in-service), as seen in Kiss and Michal (2022), we conducted a comparative study of two series of textbooks in terms of R&P. We looked on the topic of symmetries through two perspectives: (1) how the topic is introduced and processed from a discovery and investigation perspective, and (2) the types of R&P tasks present in 7th-grade textbooks that are still used in Slovak and Czech lower secondary schools. The main difference between the chosen textbooks is their philosophy. One is constructivist-based and focuses on developing problem-solving strategies, while the other focuses on the use of defined (or introduced) concepts and procedures. We intentionally chose these two textbooks to investigate how the philosophy of the textbook influences the R&P tasks within them.

2 Our research

This article is part of a research project that aims to enhance the lesson design capacity of pre-service mathematics teachers in terms of reasoning and proof. Our research is organized as a comparative study of two series of textbooks that contain different types of R&P. Our focus was on answering the following research question: What are the similarities and differences between two philosophically distant textbooks in a selected geometrical topic in terms of R&P? To answer this question, we looked at the chosen textbooks from the two perspectives described above.

2.1 Analytical framework

Several mathematics education researchers have focused their attention on identifying the different modes of reasoning conveyed by textbook problems (e.g., Stacey & Vincent, 2009; Silverman & Even, 2015). For example, Stacey and Vincent (2009) argued that the main purpose of explanations is rule derivation, while Stylianides (2009) developed a framework that can be used as an analysis tool in textbook analysis and as an instructional tool in teacher professional development sessions. Sevinç et al. (2022) presented an integrated framework that can be used to analyse the ways of reasoning in mathematics textbooks. Since this framework was developed for the analysis of solved tasks in textbooks, we chose it as our analytical framework. The framework is briefly introduced in Table 1.

Table 1: Codes for textbook analysis R&P tasks based on the framework Sevinç et al (2022, p. 2085)

	Different ways of reasoning	Short characteristic
1	Appeal to authority	No explanation or reasoning, e.g., Euclid, a textbook, etc. says it is so.
2	Simple (1-step) deduction	a single deduction from one or more premises
3	Mathematizing	the explanation/ justification of transformation/decontextualization of a word problem/ a problem defined in the real world, to a strictly mathematical form
4	Reasoning by analogy	involves making a conjecture based on similarities between two cases, one well known (the source) and another, usually less well understood (the target).
5	Reasoning with empirical arguments/specific cases	Reasoning begins with specific cases and produces a generalization from these cases; testing claims using evidence from examples (sometimes just one example) of direct measurements of quantities, substitutions of specific numbers in algebraic expressions, and so forth
a	Making claims and generalizing	
b	Justification of claim	

6	Developing conclusions/justifying/rejecting through deductive reasoning	conclusions are derived from known information (premises) based on formal logic rules, where conclusions are necessarily derived from the given information and there is no need to validate them by experiments
a	Generic example	
b	Counterexample	
c	Systematic enumeration	
d	Other	
7	Other	e.g., abductive reasoning

On the chosen topic of symmetry, we looked at the approach to introducing new concepts in both textbooks (see Table 2).

Table 2: The list of used codes with description

Code name	Short description of the code
pupils_propose_info	Information was proposed by a pupil
textbook_propose_info	Information was proposed by the textbook
pupils_perform	Solution was performed by a pupil
textbook_perform	Solution was performed by the textbook
pupils_follow_instructions	Textbook provided instructions and pupils should follow them
textbook_follow_instructions	Textbook follow introduced instructions
introduction_of_new_terminology	Textbook introduced new terminology
use_of_introduced_terminology	Textbook used introduced terminology to describe a solution
Use_of_previous_knowledge	Pupils were requested to use previous knowledge (to e.g., find a new math)
Solved	Indicator of solved, or unsolved task
Manipulation_folding	Manipulation by using different activities, materials, and forms
Manipulation_inverting	
Manipulation_measuring	
Manipulation_drawing	

Manipulation_copying	
Manipulation_mirror	
Verbal	Verbal description of a reason
Symbolic	Symbolic description of a reason
Graphical	Graphical representation of a reason (not an illustrative figure)
Digital_technology	Digital technology was required
Other	Other type of representation of an argument

These codes were used as variables for hierarchical analysis using the CHIC software. The goal of using CHIC was to define a way to answer the question: "If an object has a property, does it also have another one?" (Courtier, 2008).

For the first perspective (approaches to the new topic), we used the codes presented in Table 2. For the second perspective (ways of R&P), we used the codes presented in Table 1.

2.2 Material and methods

We decided to limit our considerations to selected areas of geometry through the lenses of two series of textbooks for the 7th grade (age 13-14). We went through the existing series of textbooks in Slovakia and the Czech Republic (the countries we work in) and selected two of them: Novotná et al, 1997 (labeled as Textbook A) and Šedivý et al, 2001 (labeled as Textbook B).

The series of textbooks A is the constructivist one. It is designed in such a way that pupils get to know mathematics from a different perspective, mainly as a suitable method for solving problems from different domains and everyday life, for adopting new concepts in practical activities while experimenting, modelling, gathering, sorting, and processing pieces of information and data (Novotná, 1999). Pupils are directed to solve problems using their previous experiences, using different methods and to look for an optimal solution and an optimal way of finding it. Their active approach to discovery is supported, therefore the used methods and forms assume pupils' own searching, creating, finding connections, and drawing conclusions.

The series of textbooks B is widely used in lower secondary schools, and mathematics teachers like to use it. This textbook is designed differently than Textbook A. Authors at the beginning introduce new concepts usually by short activities, introducing terminology, or procedures. Pupils follow instructions to become familiar with a new concept, and then some questions are raised to support their curiosity. The wording in these textbooks is not explicitly leading to reasoning, but in many cases, the reasoning is implicit.

3 Results

3.1 First perspective

Figure 1 depicts the results of the analysis of the selected textbooks through the utilization of an Implicative graph in CHIC. The analysis focused solely on a line symmetry, as point symmetry did not exhibit any supplementary connections. It can be observed from the visual representations that textbook B exhibits a more direct structure than textbook A, with a discernible pattern being evident:

1. reasoning tasks in the analysed textbooks involved the use of symbolic, verbal, and graphical representations, which we were able to identify from the solved tasks. The tasks also utilized introduced terminology, indicating a strong emphasis on developing students' mathematical language skills.
2. The instructions provided in the textbook require pupils to manipulate slides, including copying, drawing, re-drawing, and folding. Additionally, the textbook emphasizes the use of previously acquired knowledge and introduced notation/terminology.

This pedagogical pattern is widely adopted among teachers and parents in Slovakia due to its ease of implementation, even for a parent or a pupil (e.g., when a pupil is absent due to illness). A closer examination of the pupil activities ("yellow nodes") reveals that the predominant tasks involve copying, redrawing/drawing, and folding. These activities require pupils to follow instructions employing introduced terminology, and later on, their previous knowledge is also employed (e.g., when defining line symmetry after point symmetry).

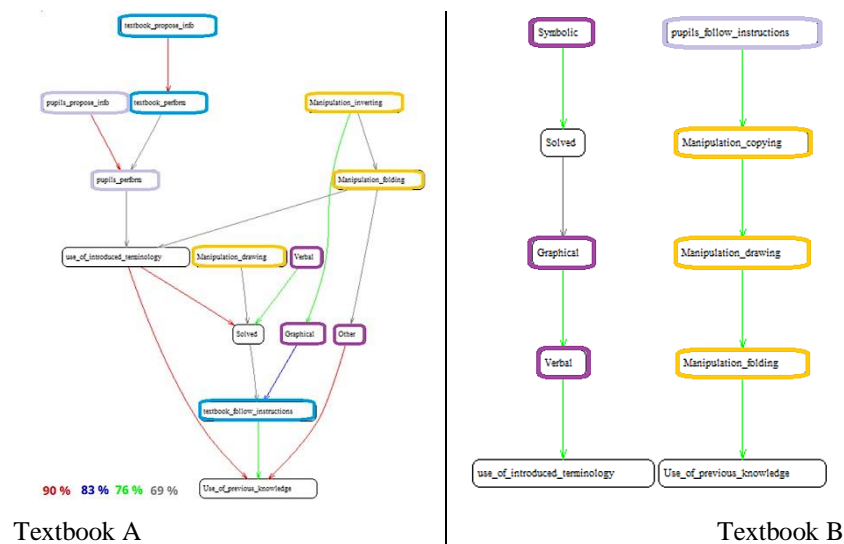


Figure 1: Implicative graph (nodes were re-coloured by the authors)

In contrast to textbook B, there are differences in the use of textbook and pupils' work in textbook A. One notable difference is the presence of multiple subgraphs with strong connections, leading to "richer" graph:

1. Information can be introduced either by the textbook or by the pupils themselves. If it is introduced by the textbook, it is then utilized by both the textbook and the pupils. Conversely, if the information is introduced by a pupil, it is solely used by that pupil. In both scenarios, the pupils are expected to apply the introduced terminology and utilize their prior knowledge.

2. When performing manipulations with objects such as drawing and inverting, the arguments can be represented in graphical or *Other* forms (such as creating a model).
3. In solved tasks, manipulation with drawing and verbal representations is utilized by pupils when they are required to follow instructions.

3.2 Second perspective

In the analysis of the R&P tasks solved in textbook B, we primarily coded for 5a and 6a. As an illustration of the coding process, Figure 2 presents an example of a solved task from textbook B. In this task, pupils are required to identify the object with a line of symmetry and the one with a point of symmetry (depicted in Figure 2). Consequently, we classified this task as 2 (Simple 1-step deduction) since definition of line and point symmetry were introduced earlier in the textbook. The task ends with concluding remark of a symmetrical object possessing a line symmetry.

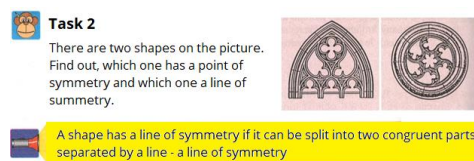


Figure 2: Example of a solved task (own translation)

A significant task in Textbook A is the Car badges task (p. 42-43, Task 7, depicted in Figure 3), which involves manipulating objects to identify their symmetry properties.

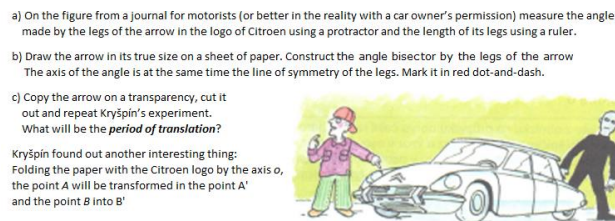


Figure 3: Introductory task suggesting the pupil's own activity (own translation)

The task commences with a focus on manipulation and experimentation, guided by instructions in subtasks *a* and *b*, ultimately leading to the final reasoning in subtask *c*, which involves making claims and generalizations (identified as code 5a). Following the experimentation phase, the task concludes with a confirmation of findings. Our analysis compares the various modes of reasoning employed in the selected topic and textbooks to determine the presence or absence of discernible differences and similarities. In particular, we identified two solved tasks in textbook A that utilize mathematizing (code 3) and four tasks that rely on reasoning with empirical arguments or specific cases to make claims and generalizations (code 5a) in the topic of symmetries.

Textbook A: In the topic of symmetries, our analysis revealed two solved tasks that utilized mathematizing (code 3), while four tasks employed reasoning with empirical arguments (or specific cases) to formulate a claim and generalize (code 5a).

Textbook B: In the topic of symmetry, we identified three tasks that appeal to authority (code 1), four tasks using reasoning with empirical arguments (or specific cases) to make a claim and generalization (code 5a), and one task that develops a conclusion using a generic example (code 6a).

Textbook A contains more activities that encourage pupils to use empirical arguments compared to Textbook B, where "appeal to authority" is used to guide pupils towards both empirical and deductive arguments, using generic examples. These results are consistent with those of the first perspective, which indicates that actively encouraging pupils to engage in independent work leads to the use of empirical arguments.

4 Conclusion

In the article, we presented the methodology and methods of comparing R&P activities in two different set of textbooks focusing on geometry. It is evident that this approach can be utilized for comparing more than two textbook series in different cultural backgrounds. Our experience with the system of codes in other domains of school mathematics demonstrates its suitability and effectiveness. Further comparative research in this direction is planned.

The identified *similarities* in the two textbooks include a tendency to use real-life contexts and emphasize the identification of symmetrical geometrical objects in real life (mostly unsolved tasks). This observation aligns with the findings of Davis et al. (2012), who identified a higher prevalence of opportunities for R&P in a reform-oriented textbook unit compared to a conventional textbook unit.

Silverman & Even (2015) identified *differences* in types of reasoning between the topics, while we identified the most significant difference is in approach (inductive/deductive) which influence the type of the presented tasks and suggested activities to pupils as well as modes of reasoning mentioned in Table 1.

Our results could help not only mathematics teachers to work better with selected textbooks, to search and identify "hidden" R&P and exploratory activities, but also to the authors of the textbooks. They can use our variables in a kind of matrix with characteristics to put stress on when preparing curricular resource. Teacher training is necessary and planned to ensure that both pre- and in-service teachers acquire the skills needed to identify R&P tasks (including those that may be 'hidden') and are able to support their pupils' reasoning skills.

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