



HAL
open science

Peirce's and Toulmin's perspectives to examine the informal inferential reasonings of preschool children

Maritza Méndez-Reina, Soledad Estrella

► To cite this version:

Maritza Méndez-Reina, Soledad Estrella. Peirce's and Toulmin's perspectives to examine the informal inferential reasonings of preschool children. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04410978

HAL Id: hal-04410978

<https://hal.science/hal-04410978>

Submitted on 9 Feb 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Peirce's and Toulmin's perspectives to examine the informal inferential reasonings of preschool children

Maritza Méndez-Reina¹ and Soledad Estrella¹

¹Pontificia Universidad Católica de Valparaíso, Chile;

maritzamendez@pensadoresmatematicos.com

This paper reports on the preliminary results of a research that studies the informal inferential reasoning of preschool children that arises when they explore samples and the variability of sampling. For this purpose, some arguments that the children provide when they are required to advise other people in a hypothetical situation are analyzed. In the analysis of the argumentative exercise, the differentiating elements of the types of reasoning proposed by Peirce and the argumentative structure proposed by Toulmin are revealed. The results show how said structural analysis allows describe how the children detect regularities in the samples and declare an informal statistical inference as a conclusion in the decision-making process; in addition, it shows how preschool children are using modal qualifiers to justify generalizations that go beyond the evidence in the data when resolving problems in contexts of uncertainty.

Keywords: Statistical inferential reasoning; informal statistical inference; Peirce; Toulmin; inductive reasoning.

Introduction

Statistical Inference (SI) is the heart of the statistic and reflects the power of its use since it is possible to formulate conclusions under uncertain conditions when only partial data are available (Makar & Rubin, 2018). In everyday life, citizens and researchers from different fields must face predictions, create estimates, and make decisions in the future without having all the data at hand, that is, considering samples of it.

Given this scenario, it is desirable to support the development of early statistical thinking in young children, in particular by promoting the generation of informal statistical inferences from data samples without resorting to the use of formal inference techniques, focusing their reasoning on the behavior of sample distributions and their contrast (Pfannkuch, 2006), in authentic experiences of decision-making that allow coordinating their informal knowledge about the context and the problem data, and making reasonable statements beyond the data they possess (Estrella et al., 2022).

With the interest of studying informal inferential reasoning of preschool children, an analysis of the structural aspects of such reasoning is carried out (Jeanotte & Kieran, 2017), using the ideas of Peirce and Toulmin to characterize elements of the structure of informal statistical inferences (Méndez-Reina & Estrella, 2023). It then uses the combined perspective of both the components of an argument proposed by Toulmin (1958/2003) and the various elements of the reasoning proposed by Peirce (1878) to describe how informal inferential reasoning develops through student conversations when they issue conclusions that go beyond data and when they provide persuasive arguments based on data analysis, the behavior of samples, and the variability of sampling.

Informal statistical inference

Statistical inference, according to de Vetten et al. (2018), can be considered as the outcome and process of creating or testing probabilistic generalizations from data, and it is statistical inferential reasoning that allows conclusions to be drawn from the analysis of the behavior of random samples. Several investigations have pointed out that this reasoning can be developed at an early age, and the term informal inferential reasoning (IIR) has been coined to refer to "the way in which students use their informal statistical knowledge to make arguments to support inferences about unknown populations based on observed samples" (Zieffler et al., 2008, p. 44). However, although IIR has been characterized as a cognitive activity required for formulating generalizations from random samples of data while considering evidence and articulating uncertainty (Aridor & Ben-Zvi, 2017), further research is still required regarding how students establish arguments based on data analysis in situations of uncertainty and methodological tools to describe students' IIR and to classify the different aspects that structure the inferences generated in dealing with situations of uncertainty at the school level. For this report, the combined perspective of Peirce and Toulmin has been chosen as an alternative that could contribute to the characterization of the structural elements of IIR.

Peirce's and Toulmin's perspectives to examine informal inferential reasonings

The types of reasoning according to Peirce and the components it uses to distinguish them (case, rule, and result) and the argument structure proposed by Toulmin from its elements (data, warranty, support, exception, and conclusion) can be used to analyze different structural aspects of reasoning.

Types of reasoning according to Peirce

Peirce points out the distinctive characters ("rule", "case", and "result") that distinguish the structure of different classes of inference, understood as three elementary modes of reasoning: deduction, induction, and abduction. Peirce (1878) uses an example in which all beans are white and clarifies the elements *rule*, *case*, and *result*, explaining the distinctions between types of inference (CP 2.623):

Table 1: Types of elementary inference according to Peirce (1878)

Deduction	Rule	All beans in that bag are white.
	Case	These beans are from that bag.
	**Result	Therefore, these beans are white.
Induction	Case	These beans are from that bag.
	Result	These beans are white.
	** Rule	Therefore, all beans in that bag are white.
Abduction	Rule	These beans are white.
	Result	All beans in that bag are white.
	** Case	Therefore, these beans are from that bag.

In addition, Peirce considers that the reasoning of induction and making a hypothesis are essentially inferences from a sample (later, in 1901, he referred to making a hypothesis through the terms "retroduction" and "abduction").

Toulmin's argumentative model

According to Toulmin the conclusions are stated by the need to address a problem, in which a question has been posed to someone who must state something about it, who then establishes the possible solutions, usually attending to the possibilities that he considers most serious or plausible.

For Toulmin (1958/2003), each argument has three base elements: data (D), warrants (W) and claims (C). However, it may not be enough simply to specify these elements; it may also be necessary to add some explicit reference to the degree of force that the data available to us confer on the assertion made under the warrants. Thus, three secondary elements can appear: modal qualifiers (Q), backing (B) and rebuttal (R), see Figure 1:

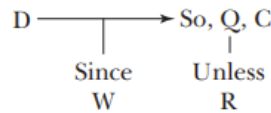


Figure 1. Expanded structure of Toulmin's argumentative model (1958/2003)

The data (D) can be facts, information, figures, or examples that support the conclusion. The warrants (W) can be rules, principles, or statements that allow inferences to be drawn. The claims (C) are the statements that derive from the initial assertion. The backings (B) offer additional evidence or elements that justify the warrants. The rebuttal (R) outlines the circumstances in which the conclusion is false. Finally, the modal qualifiers (Q) qualify the conclusion by expressing degrees of confidence.

Combined perspective of Peirce's types of reasoning and Toulmin's argumentative structure

Various studies have used the combined perspective to analyze mathematical reasoning (Conner et al., 2014; Jeannotte, 2015; Pedemonte & Reid, 2011); however, Méndez-Reina & Estrella (2023) indicate that they have not found research to analyze structural aspects of informal inferential reasoning from the combined perspective of Peirce's types of reasoning and Toulmin's argumentative structure. Based on the position of the elements Case, Rule, and Result in the basic structure of "Data", "Warrant", and "Claim" as shown in Figure 2, the different types of reasoning can be distinguished (Conner et al., 2014; Méndez-Reina & Estrella, 2023).

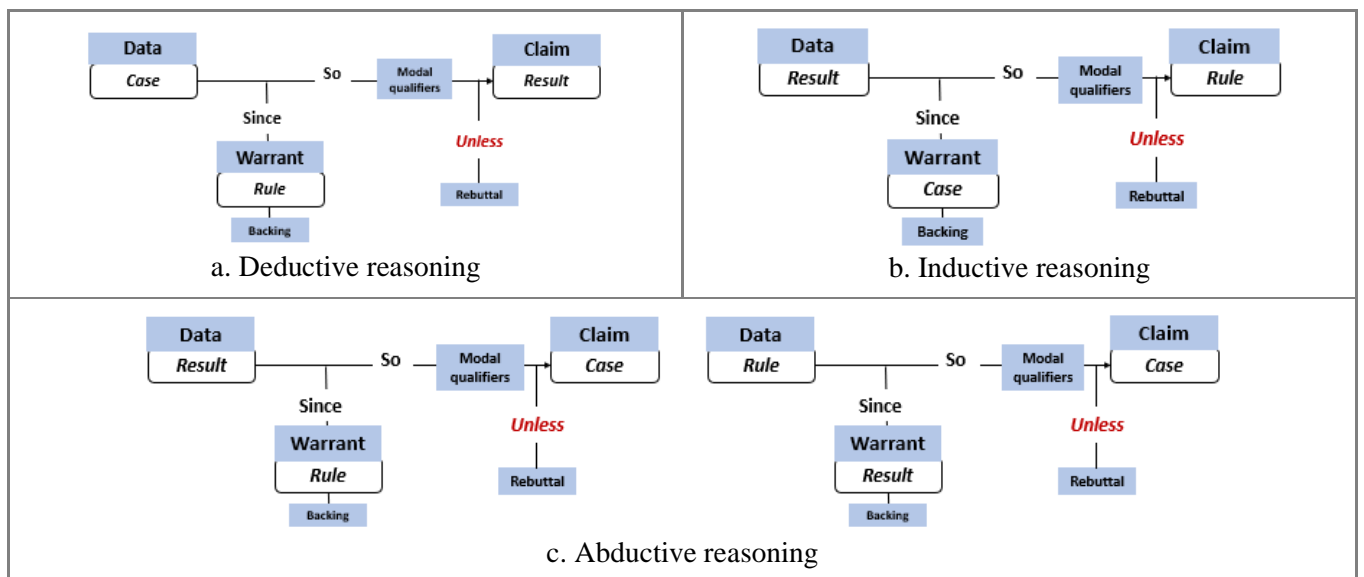


Figure 2. Toulmin-style diagrams of arguments revealing different kinds of reasoning

This report focuses on the action of reasoning in formulating informal statistical inferences and is framed by the arguments generated by preschool children. Considering this, the following research

question is used to structure the current study and the analysis of the collected data: How does the combined perspective of Peirce's types of reasoning and Toulmin's argumentative structure allow the informal inferential reasoning of children to be examined?

Methodology

To answer this question, -and based on consensus among the authors-, a qualitative, descriptive interpretative approach is adopted. Transcriptions of video recordings and the children's written productions were used, and one of the episodes of verbal argumentation was selected based on the manifestation of IIR in different arguments for resolving the task and interactions between the children and the teacher. For the analysis of argumentation episodes, in the first step, the assertions produced by children were identified, and the structure of the argumentative step was reconstructed, specifying data (D), warrants (W), and claims (C). On some occasions, it may also be necessary to use the secondary elements of modal qualifiers (Q), backing (B), and rebuttal (R). In the second step, an analytical, progressive, and consensual review among the authors is conducted of how the structure behaves vis-à-vis the elements proposed by Peirce (case, rule, and outcome) to determine the type of reasoning involved (deductive, inductive, or abductive); and finally, diagrams are designed with the key elements of the ideas of Toulmin and Peirce described, using the combined perspective. In addition, the diagrams show through the colors of the boxes whether the contribution of the elements corresponds to the contribution of the student, the teacher, or whether the teacher and the children worked together. The transcripts and the children's written productions have been translated from Spanish into English.

Participants

The case reported corresponds to one of the work teams: Alicia, Juan, and Francisco; they are identified as Group 2. They belonged to a group of 15 preschool children from a school in the Valparaíso region. The lesson was implemented by the teacher in her usual classroom. It was supported by an assistant and two participating professors-researchers (authors of the work) who were present to conduct the recorded video and stimulate preschool children to work together and talk out loud during the lesson.

Tasks

The implementation of this task is part of a sequence of lessons applied and improved through a Lesson Study process with nine preschool and elementary teachers and three researchers. Children experimented during lesson 1, a task in which they promoted the use of probabilistic language and a random game with concrete material. A detailed summary of this lesson can be found in Estrella and collaborators (2022). The task of lesson 2, called "the treasure of pearls and the pirates", was associated with the black box experiment (Estrella et al., 2023; Van Dijke-Droogers et al., 2020), this was employed in this study because these authors argue that the ideas of repeated sampling with a black box seem to be useful for the introduction of ideas of statistical inference. In this lesson, the black box experiment with three bottles, each of which contained 100 pearls of two different colors (gold and purple), was considered. In the first bottle, the pearls were in a proportion of 25 gold and 75 purple; in the second bottle, the pearls were in a proportion of 15 gold and 85 purple, and finally, in the third bottle, the pearls were in a proportion of 35 gold and 65 purple. In this experiment, the

children shaken each bottle, and viewed it through a small window to investigate which of the 3 bottles can have the most golden pearls inside by collecting data and comparing samples, this means that the children must generate an informal inference. The key concepts were sample and sampling variability. The resources and some examples of these can be seen in Figure 3.

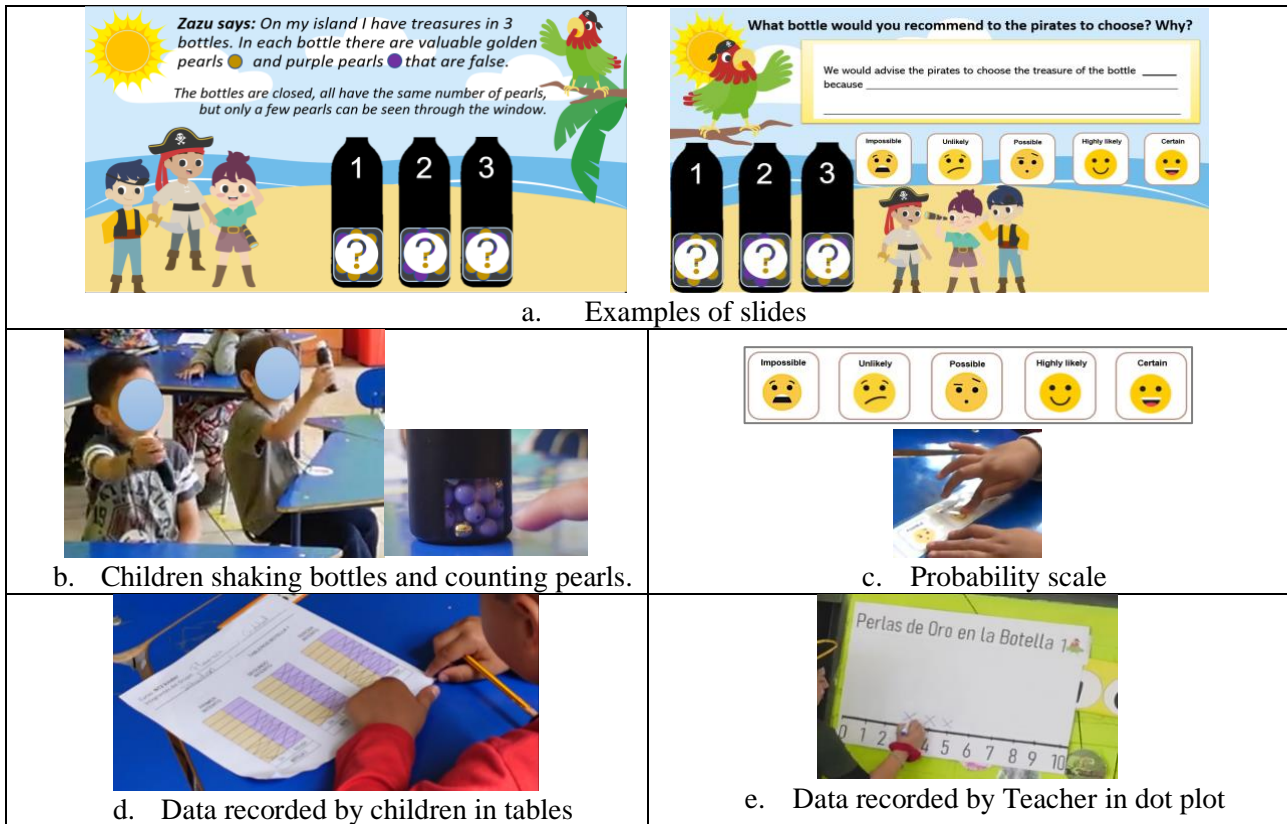


Figure 3. Resources in task "The treasure of pearls and the pirates"

Results

The episodes chosen to illustrate, from the schematic diagrams, the structural elements manifested in the informal inferential reasoning of some preschool children. These episodes clearly show the collective argumentative expression (in the sense of Conner et al., 2014) of the children with the teacher, and according to IRI highlights and concepts typical of inferential statistics: (1) expression of probabilistic language and understanding of variation; (2) evidence based on data and comparison between samples; (3) regular sample behavior by assigning a confidence level to each event; and (4) assertions beyond data using expressions of uncertainty.

After the children shaking each bottle, they observe and record in three tables the results in their worksheets. They then visualize the behavior of the samples through dot plot (one for each Bottle), with this, each of the work teams should give recommendation on the bottle that could have more golden pearls inside. Thus, the teacher asks the full class a question: "What bottle would you recommend to the pirates to choose? Why?" Encouraging children to respond in groups and decide to give advice to pirates. Part of the statements of the children answering this question are presented below.

Teacher: Now, as a group you must decide what you recommend to the pirates

Group 1: The [bottle] 3
 Teacher: Why?
 Group 1: [Bottle] 3, because it is [the bottle] that had more golden pearls than the 1 and 2.
 Teacher: Group 2, did you choose bottle 3? Why?
 Group 2: Because you may find many pearls.
 Teacher: ... is possible, or is highly likely, or is unlikely, to see put the little finger where you believe.



Teacher: And if a pirate comes, you'd tell him it's...
 Group 2: ...highly likely to find many pearls.
 Group 3: This group agrees, the [bottle] 3.
 Teacher: why the bottle 3?
 Group 3: Because it has more golden pearls, it has up to 6 [golden pearls], the [bottle] 3...instead, the other [Bottle 2] reached up to 2 and the other [Bottle 1] reached up to 5, instead this [Bottle 3] reached up to 6”.

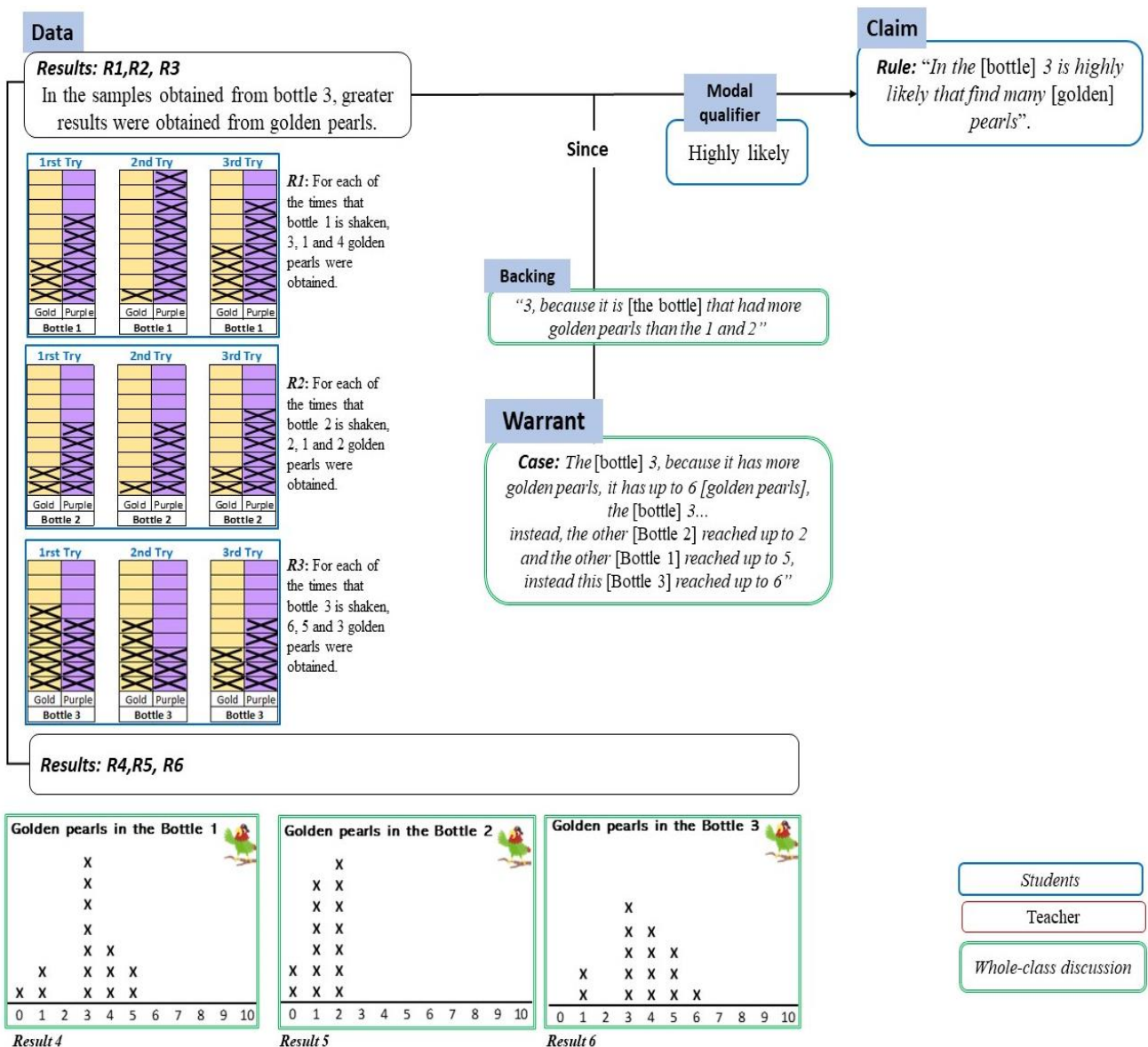


Figure 4. Example of IIR schematized according to combined Peirce-Toulmin perspective

The informal inferential reasoning put into play by Group 2, in a collective argumentative exercise is schematized in Figure 4, this illustrates how preschool children, from the Results (R1 to R6) detect the regularity in the variation of the samples, informally expressing the generalization "*In the [bottle] 3 is highly likely that find many [golden] pearls*".

This generalization is according to the warrant "*The [Bottle] 3, because it has more golden pearls, it has up to 6 [golden pearls], the [Bottle] 3... instead, the other [Bottle 2] reached up to 2 and the other [Bottle 1] reached up to 5, instead this [Bottle 3] reached up to 6*". The Rule expressed by Group 2 is stated as informal statistical inference and accounts for statistical inference in the context of the experiment proposed in the story. The reasoning analyzed, from the Peirce's perspective is inductive, because it has reached to express the generalization of the results and the rule, from the detected regularity.

Discussion

The combination of Peirce's and Toulmin's perspectives allowed us to recognize how informal statistical inferences, were given as a conclusion of the proposed tasks in order to detect regularities in the samples and in the variability of the sampling, either by taking into account the behavior of the samples collected within each group or by including in their warrants descriptions of the behavior of the samples throughout the course, in order to declare widespread ownership over which bottle would have the greatest chance of having golden pearls inside it in the form of informal statistical inference.

The study reported could provide a theoretical framework for understanding ISI and IIR, highlight the usefulness of combining Peirce's and Toulmin's perspectives to analyze the structural aspects of statistical reasoning, and provide insights into how informal inferential reasoning can be examined in young children. The research is still ongoing, and we are currently analyzing in more detail the progressive uses of modal qualifiers in the early years and how they are related to IIR. Some issues need further reflection. One crucial issue is: What are the structures of abductive IIR? Another aspect that needs careful consideration is the procedural aspect of this reasoning, in particular processes like conjecturing, identifying a pattern, generalizing, comparing, and classifying, among others.

Acknowledgment

Support from National Doctorate Scholarship ANID 21210862 and ANID Fondecyt 1200346.

References

- Aridor, K., & Ben-Zvi, D. (2017). The co-emergence of aggregate and modeling reasoning. *Statistics Education Research Journal*, 16(2), 38–63. <https://doi.org/10.52041/serj.v16i2.184>
- Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Identifying kinds of reasoning in a collective argumentation. *Mathematical Thinking and Learning*, 16(3), 181–200. <https://doi.org/10.1080/10986065.2014.921131>
- de Vetten, A., Schoonenboom, J., Keijzer, R., & van Oers, B. (2018). The development of informal statistical inference content knowledge of pre-service primary school teachers during a teacher college intervention. *Educational Studies in Mathematics*, 99(2), 217–234. <https://doi.org/10.1007/s10649-018-9823-6>

- Estrella, S., Méndez-Reina, M., Olfos, R., & Aguilera, J. (2022). Early statistics in kindergarten: analysis of an educator's pedagogical content knowledge in lessons promoting informal inferential reasoning. *International Journal for Lesson and Learning Studies*, 11(1), 1–13. <https://doi.org/10.1108/IJLLS-07-2021-0061>
- Estrella, S., Méndez-Reina, M., Salinas, R., & Rojas, T. (2023). The Mystery of the Black Box: An Experience of Informal Inferential Reasoning. In G. Burrill, L. Oliveria & E. Reston, *Research on Students' Interactions with Data in Teaching Statistics*. Springer. https://doi.org/10.1007/978-3-031-29459-4_16
- Jeannotte, D. (2015). *Mathematical reasoning: a conceptual model for learning and teaching at the primary and secondary levels of schooling*. (Unpublished doctoral dissertation). Université du Québec à Montréal.
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, 96(1), 1–12. <https://doi.org/10.1007/s10649-017-9761-8>
- Makar K., & Rubin A. (2018). Learning About Statistical Inference. In D. Ben-Zvi, K. Makar & J. Garfield (Eds.), *International Handbook of Research in Statistics Education* (pp. 261–294). Springer International Handbooks of Education. Springer. https://doi.org/10.1007/978-3-319-66195-7_8
- Méndez-Reina, M. & Estrella, S. (2023). Razonamiento inferencial en Estadística Temprana: aspectos estructurales desde tipos de razonamiento de Peirce y estructura argumental de Toulmin. In S. Estrella, M. Parraguez & R. Olfos (Eds.), *Pensamiento matemático específico: aportes a la práctica docente desde la Didáctica de la Matemática* [Specific mathematical thinking: approaches to teaching practice from the Didactics of Mathematics]. Editorial GRAÓ.
- Pedemonte, B., & Reid, D. A. (2011). The role of abduction in proving processes. *Educational Studies in Mathematics*, 76(3), 281–303. <https://doi.org/10.1007/s10649-010-9275-0>
- Peirce, C. S. (1878). Deduction, induction, and hypothesis. *Popular science monthly*, 13(August), 470–82. (Compiled in Peirce, C. S., 1960, 2.619–644).
- Pfannkuch, M. (2006). Informal inferential reasoning. In A. Rossman. & B. Chance (Eds.), *Proceedings of the Seventh International Conference on Teaching Statistics*. http://www.stat.auckland.ac.nz/~iase/publications/17/6A2_PFAN.pdf
- Toulmin, S. (2003). *The uses of argument* (updated ed.). New York: Cambridge University Press. Original work published 1958. <http://dx.doi.org/10.1017/CBO9780511840005>
- Van Dijke-Droogers, M., Drijvers, P., & Bakker, A. (2020) Repeated sampling with a black box to make informal statistical inference accessible. *Mathematical Thinking and Learning*, 22(2), 116–138. <https://doi.org/10.1080/10986065.2019.1617025>
- Zieffler, A., Garfield, J., delMas, R., & Reading, C. (2008). A framework to support research on informal inferential reasoning. *Statistics Education Research Journal*, 7(2), 40–58. <https://doi.org/10.52041/serj.v7i2.469>