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Comparison of “What numbers make sense?” problems solving in the Czech Republic and Lithuania

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The aim of the paper is to investigate fifth-grade pupils' strategies when solving a non-traditional word problem in the Czech Republic and Lithuania. Unlike typical word problems that can be solved by merely extracting numerical data and question words, the approach used in this study involves "What numbers make sense" word problems that demand a deeper understanding of the story of the problem. The purpose of this study is to assess pupils' solution strategies, their argumentation skills, and attention to linguistic limitations.

Keywords: *Argumentation, language, word problem, Singapore math, mathematical discourse.*

Introduction

Word problems are a widely discussed area in mathematics education. In school, it is very common to work with typical word problems, with only one solution, requiring mathematical operations to the numerical data present in the wording of the problem (Verschaffel et al., 2014). The aim of teaching word problems is not only to practice arithmetical operations, but also to discuss the context of the word problem, the possible strategies, the number of solutions, the understanding of the text, etc. Even though we realize that word problems do not primarily aim at argumentation, we agree with Cramer (2019, p. 163) that:

While problem solving is not synonymous to arguing and proving, such tasks almost always rely heavily on logical reasoning, and the results may be taken as an indication for students' attitudes towards argumentation activities.

We consider reasoning and argumentation as an integral part of problem solving. Solving word problems is largely affected by the language in which they are written (Daroczy et al., 2015). The same word problem written in different languages can have different lengths of sentences and words. Moreover, understanding a text is influenced by the used synonyms, the relationship between the sentences and between the conjunctions etc. In some languages, such as Czech or Lithuanian, the number of solutions to a word problem is influenced by the use of prepositions and the declension of nouns and verbs. These influences give space to a new area of research and strengthen the cross-curricular links between mathematics and language.

We used atypical word problems *What numbers make sense* with Czech pupils (Mottlová & Slezáková, 2022). In this paper, we present a study with Czech and Lithuanian pupils. The language systems of both countries are similar. We examine pupils' argumentation and solution strategies, which are influenced by language. Our research questions (RQ) are:

RQ1: What solution strategies and argument types do pupils use when solving the *What numbers make sense* word problems?

RQ2: When solving a word problem, do learners consider its linguistic, mathematical, and contextual components?

Theoretical framework

Word problems

Word problems are typically defined as verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement (Verschaffel et al., 2014). Arbaugh et al. (2018) and Stylianides (2016) underline that “word problem develops mathematical reasoning if solving it includes argumentation, justification or patterns identification and if it uses the statements known to a presenter and a class community” (Kilienė, 2022, p. 217).

Word problems are included in the curricula worldwide, including the Czech Republic and Lithuania. In primary school (Lithuania 1st – 4th grade, Czech 1st – 5th grade), pupils solve word problems with one or two operations, and in the fifth grade, they solve more difficult word problems. There is no prescribed teaching method for word problems in the two countries. Typical word problems requiring an application of previously taught procedures prevail. In Lithuania, after reading a word problem in textbooks (Kilienė, 2021), pupils immediately carry out the calculation and write down the result in number and word form (e.g., 5 balls). Unlike in Lithuania, before creating a mathematical model, Czech pupils are expected to record what is known and what is searched in the word problem and to write a full answer at the end of the solution. In Lithuania, alternative ways of working with word problems can be only encountered in private schools. On the contrary, in the Czech Republic, we can observe the trend of constructivist-led teaching.

‘What numbers make sense’ word problems

Word problems of the type *What numbers make sense?* (WNMS) in the textbook (Kaur & Har, 2009) are the focus of our paper. They originated within the research project (Kaur et al., 2010) which responded to the revision of the Singapore school mathematics curriculum (2012). The last revised part of Mathematics Syllabus Primary One to Three (2020) does not explicitly mention working with word problems but places them into a wider group of real-world problems. At the same time, two new key process skills were introduced in the revisions: communication and mathematical reasoning, which are part of argumentation.

Do the numbers make sense?	6	37	5	7
John buys __ balloons per __ cents and one pacifier for __ cents. He paid __ cents.				

Figure 1: What number makes sense word problem Shopping

WNMS problems do not contain a question. The numerical data are not part of the verbal description of the problem situation but are part of the instruction, the following example is our first word problem (Figure 1). WNMS problems involve an instruction, a selection of data (e.g., numbers) and

a verbal description of a problem situation with gaps where the numbers must be filled in. When solving them, pupils cannot use learned algorithms only but rather use heuristic strategies (Polya, 1945) while considering three aspects characterising WNMS problems:

- a) mathematical aspect: there is a relevant mathematical relation between the data (e.g. $32: 2 \cdot 5 = 80$),
- b) contextual aspect: the problem is set in some context, and it should make sense in real life (e.g. a mother does not give 80 sweets to each child because they would be sick),
- c) linguistic aspect: data must be filled in a way that is grammatically correct (e.g. according to declension).

Prioritizing or omitting any of the above-mentioned aspects affects pupils' choice of solution strategy (Hegarty et al., 1995), the solving process (Reusser, 1985) and, consequently, the argumentation (Krummheuer, 2000) of the correctness of the result. Some pupils neglect the real-life meaning of the task and prefer the mathematical one (De Corte & Verschaffel, 1985). They use, for example, an equation to express the relationship between numerical data as a strategy and the subsequent justification of the correctness of the solution. If numerical or algebraic expression of relation between numbers does not describe the situation of the word problem (nonsensical), then pupils' strategy would be superficial (i.e., direct translation strategy; Nunes et al., 2016). Other pupils, on the other hand, tend to prefer real-life meaning because it is based on their life experience (Van Dooren et al., 2019). This assumption can help pupils with the mathematization process but can also become a "blocked door" (Djepaxhija et al., 2015). Some pupils neglect the linguistic sense, for example, by ignoring declension, which can also be determined by their age (Mottlová & Slezáková, 2022). On the contrary, those who prioritize the linguistic sense might find some tasks easier to solve as they would use linguistic indicators to fill in the numbers correctly.

Linguistic aspect

One of the main components of a word problem is language. We point out to its specificity being aware of its limitations and advantages. In Czech and Lithuanian languages, declensions play an important role and can either help or block when solving WNMS problems. The form of the noun given by declension determines what number is connected with it. The form of a preposition or a verb, etc., can also predetermine numbers. Both countries have similar language systems, numbers are related to nouns in different rules depending on the number value. The following linguistic factors are very specific to both languages and must be taken into account when formulating WNMS problems.

- Czech has a certain word ending for the number 1 (e.g., "1 kniha"). (In formal Czech, 101, 201 etc., have the same ending as number 1 but young pupils might not know it.) The other group of numbers with another ending is 2, 3, 4 (e.g., "2 knihy"). And the third group consists of numbers bigger than 4 (is it 5, 6, ..., 9, 10, 11, ..., 100, 101, 102 ... and more) (e.g., "5 knih").
- In Lithuanian, the following groups of numbers have the same word ending:
numbers 1, 21, 31, 41... (which end with 1 e.g. "1 knyga), numbers 10, 11, 12, ..., 19, 20, 30, 40, 50, ...90 (e.g. "10 knygu", and numbers 2-9, 22-29, 32-39, ..., 92-99 (e.g. "2 knygos").

Methods

Analysis a priori of WNMS problems

In the study, we used two WNMS problems. The first (Figure 1) aimed to introduce a new type of problem to the class. It does not contain any linguistic indicators which would enable pupils to insert numbers without applying the mathematical and contextual aspects. In both Czech and Lithuanian languages, all the numbers can be put in any of the gaps without violating language rules (e.g. if there was number 1, they could not put it in “_ balloons”, because 1 is singular).

The second WNMS problem (Figure 2) was formulated in a way that would allow us to observe how pupils take into account all the three aspects which are the core of our study.

Write the numbers 3, 12, 18, 4 in the correct places and check if you placed them correctly.
Planet D is home to __ aliens. They bought __ boxes of gloves. Each box contained __ pairs of gloves.
Each alien has __ hands. When everyone put a glove on each hand, there were no gloves left.

Figure 2: What number makes sense word problem - Aliens

As shown above, numbers 10-20, and numbers 2, 3, 4 are assigned to the same groups in both Czech and Lithuanian (as they have the same word ending). We wanted numbers from different groups and the same numbers in Czech and in Lithuanian language. Finally, numbers 3, 12, 18 and 4 were used that work well in both languages. If the pupils could notice the linguistic aspect (declensions, see below in bold), the problem would be easier to solve. The possible solutions that consider the linguistic aspects are the following:

Planet D is home to 12/18 aliens (*CZ mimozemšťanů*; *LT ateivių*). They bought 3/4 boxes (*CZ balíky*; *LT dėžės*) of gloves. Each box contained 12/18 pairs (*CZ párů*; *LT porų*) of gloves. Each alien has 3/4 hands (*CZ ruce*; *LT rankas*). When everyone put a glove on each hand, there were no gloves left.

In terms of the contextual aspect, we presumed that a pseudo-real situation involving aliens would be appealing to young pupils. Pupils must use the fact that a pair of gloves has 2 gloves. If pupils notice that numbers 3 and 4 cannot be placed in the first gap for the declension reason, they just have to decide if they choose number 12 or number 18. They can do it by guessing and trying. There is just one possible solution:

Planet D is home to 18 aliens. They bought 3 boxes of gloves. Each box contained 12 pairs of gloves. Each alien has 4 hands. When everyone put a glove on each hand, there were no gloves left.

Experiments

We conducted two experiments in Lithuania and in Czech Republic. Pupils in neither country met WNMS problems before the experiment. Participants were 28 Czech and 20 Lithuanian 5th grade pupils; in primary school in Czech Republic and in lower-secondary school in Lithuania. They were taught the same lesson by the two authors of this paper (each in their country) while the class teachers were observing. Pupils were given some time to solve the two word problems by their own, afterwards, they came to the blackboard one by one, and filled numbers into the gaps of each word problem. They were encouraged to explain their thinking to their classmates.

The lessons were video-recorded and the recordings were transcribed. The parts of the transcripts which consisted of pupils' descriptions of their solving process and their explanation about the correctness of the solution were analysed. About half of the pupils in both classes took an active part in discussions and about one quarter of the pupils in both classes presented their solutions at the blackboard. Each description was coded in terms of the type of justification used. The linguistic aspect was relatively simple to discern in all pupils' statements, while it was not as simple for the mathematical and contextual aspects.

Results

We distinguished two categories of statements supporting pupils' solutions. The first comprises linguistic arguments, and the second consists of arguments based on mathematical or contextual aspects. The categories will be illustrated by examples of pupils' work from both countries. In Figure 3 and Figure 4, the column on the left includes examples from Czech and on the right from the Lithuanian language.

Linguistic aspect

As expected, the linguistic aspects had an impact on pupils' solutions. Both Czech and Lithuanian pupils noticed them and put the first numbers in gaps following language rules (see examples in Figure 3). We can see that pupils in both countries noticed the declension in the word aliens and decided that 3 (or 4) aliens with used ending of the word was not correct. Pupils were not able to express why these numbers did not fit, but they used expressions like: "does not rhyme", "doesn't make sense", "there can't be". When they noticed it, they could only choose between numbers 12 and 18 for aliens.

3 aliens [mimozemšťanů], that doesn't make sense	It cannot be 4 or 3. Does not rhyme...
we thought, like, the mistake is here in this [aliens - mimozemšťanů]	18 pairs would be better.
12 boxes [balíky] or 18 boxes [balíky] doesn't make sense but 3 boxes [balíky] and 4 boxes [balíky] makes sense	It would sound weird [if there were 3 or 4 aliens]. (expressed)
some pairs [párů] in each box, there cannot be 3 pairs [párů] or 4 pairs [párů], so again 12 or 18	There should be aliens[ateiviai], but there are aliens[ateivių] here. (expressed)

Figure 3. Example of Czech (left column) and Lithuanian (right column) statements of linguistic aspects

Both Czech and Lithuanian pupils demonstrated similar linguistic awareness, despite the differences in their respective languages. It is important to note that we chose numbers from the same number groups to ensure that the word problem was identical in both contexts. Overall, the findings suggest that linguistic aspects should be taken into account, when designing WNMS problem-solving tasks.

Solution strategies supported by arguments based on mathematical or contextual aspects

Pupils in both countries understood the mathematical aspect and used it in the solution. As seen in the previous examples, pupils in both countries used mostly heuristic strategies: for example, systematic experimentation or the guess-check-revise strategy. In the solution in Figure 4 (up), a Czech pupil tried to build an equation, while a Lithuanian pupil used a guess-check-revise strategy. These choices may depend on the curriculum and procedures pupils learned in mathematics lessons.

All Lithuanian pupils got the correct answer. Some Czech pupils also reached the solution, but there was no common agreement about the correctness of the solution by the whole class.

I have a piece of advice, one of these two numbers [3/4 in the second gap], must be multiplied by this number [12/18 in the third gap], that I multiply this number [3 in the second gap] by this number [12 in the third gap] and then, or one of these two simply [12/18 in the third gap]... and ... half of the number that comes out, so it must be one of these two numbers [12/18 in the first gap]	Because if there are 18 and they have 4 hands, that's 12... they need 72 gloves. and if there are 3 boxes of 12 each.... [counts in their mind].
If we have 3 hands, it couldn't be pairs because pairs are like 2	Because it's so weird. There must be equal [hands] on each side.
They have 4 hands and there must be a smaller number here [in the second gap], because if there were 4 in the box and they had 3 hands, it wouldn't work out to those pairs.	Because every alien has 4 arms. Because [they have to have] 4, there can't be just a few [aliens].
Pairs and boxes have to, just together they are like you can say relatives.	For example, if 4 hands had 3 boxes, that would be a lot of gloves.
I thought 18 because there must be more than the others.	Because there are too few hands. Then there would be some gloves left.

Figure 4: Czech (left columns) and Lithuanian (right columns) statements: mathematical (up), contextual (middle), and combination aspects (down)

When solving the word problem and justifying their solutions, pupils used their life experience triggered by the context of the word problem (Figure 4; middle). They focused on the word “pair” meaning “two” and argued for an even number of hands. They compared the numbers of hands, boxes, and pairs. Pupils in Lithuania also suggested that the planet is a large area where there will be many people.

In some cases, it was not possible to determine unequivocally whether this was a purely mathematical or a purely contextual aspect (Figure 4; down). For example, pupils tried to justify the choice of numbers and their placement in the text by referring to the size of the numbers. This could stem either from the mathematical calculation or from the image of the situation (situation model).

Discussion and Conclusion

In the paper, we showed how pupils in one Czech and one Lithuanian class solved two WNMS problems, a new type of word problem to them. Pupils were able to solve them and were motivated by them to next working with word problems. Pupils were eager to present their solution to their classmates, and we can say that the use of this type of problem contributed to the development of their communication and reasoning skills. The analysis of pupils’ solutions to the second WNMS problem showed, as expected, that some pupils based their solution on the linguistic aspect, which was included in the WNMS problem by the used declensions. Moreover, most pupils were also able to use the mathematical and contextual aspects for the correct solution.

Our study confirmed that the linguistic aspect of WNMS problems plays an important role in the word problem solution formulated Czech and Lithuanian with a complex linguistic system. Experts who create such problems must consider declensions, so that it is not possible to solve the WNMS problem without considering the mathematical and contextual aspects (one might create a WNMS problem in which there is only one way to fill in the gaps with the given numbers which would lead to a superficial solving strategy).

Our methods were based on the analysis of the lesson transcripts in which pupils described their solving procedures and explained the correctness of the solution to their classmates. One limitation of our study consists of the fact that we presumed (during coding pupils' explanations) that pupils described how they really solved the problem but it could be that they showed additional strategies to explain the correctness of their solution to their classmates that they had not used in the solving the problem independently. In addition, the sample size of the study was small, and the study focused only on two problems. Therefore, further research with a larger sample size and more problems could provide a more comprehensive understanding of how pupils approach WNMS problem solving, the factors that influence their approach and types of their argumentation and justification. As stated above, it is especially important for word problems formulated in languages with many different declensions. Additionally, future research could investigate the role of teachers in promoting effective argumentation and justification in the whole classroom discussion. By understanding the differences and similarities of how pupils approach WNMS problem-solving, teachers could develop teaching strategies that would meet the needs of their pupils to improve their word problem solving performances. By this study, we tried to provide insights on how pupils from two countries approached a WNMS problem and to highlight the importance of considering the cultural and educational context.

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