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Pre-service teachers' use of the word *explain* when working on mathematical reasoning and proving

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In this paper, we look at different ways pre-service teachers use the word explain when evaluating student arguments. The word was not used in the assignment or the lesson, but 20 of 28 groups of PSTs still used it in their responses. This indicates that it is natural for PSTs to use the word explain in the context of mathematical reasoning and proving. However, the word was used in different ways: to address what is done or why the statement is true and also to denote the whole argument or the textual part of the argument. In addition, it was often ambiguous what PSTs wanted to express by using the word. We discuss some possible implications for teaching reasoning and proving.

Keywords: Explanation, reasoning and proving, language use.

Introduction

In the Norwegian dictionary, the word *to explain* means to make something clearer, more understandable, or to give a (more) precise/detailed account of something. Similarly, the Oxford Learners dictionary defines it as “to tell somebody about something in a way that makes it easy to understand”. In mathematics, explaining something can be related to reasoning and proving. Still, the word *explain*¹ is not used in the description of the core element “Reasoning and argumentation” in the Norwegian curriculum, and the word was not used in our introduction of the topic of reasoning and proving in our teacher education. Nevertheless, it was used by 20 out of 28 groups of pre-service teachers (PSTs) in their evaluation of student arguments. This can indicate that the PSTs find it natural to use the word in the context of mathematical reasoning and proving. However, we noticed that the word was used in different ways and with different meanings, and in some cases, the meaning was ambiguous.

Imprecise use of language is not a new phenomenon in mathematics education. The same word can have different meanings in an everyday context compared to a mathematical context, which can lead to imprecise use and challenges in mathematics teaching and learning (Monaghan, 1999). Concerning particularly reasoning and proving, Buchbinder and McCrone (2020) point out that PSTs in their study used somewhat imprecise language concerning argumentation and proof in the proving activities they implemented in classrooms, more imprecise than for other mathematical content in the activities. The researchers suggest that such language use was one of the main challenges in PSTs' implementation of the proving activity. In her study on students in a Norwegian primary school, Iversen (2022) also noticed, as we see with our PSTs, that students used the word *explain* in different ways, resulting in the same argument being chosen as the best and worst.

Since the word *explain* seems to be natural for Norwegian PSTs to use concerning mathematical reasoning and proving, we need awareness and knowledge about the different meanings the word is

¹ When we refer to “the word *explain*” further in the text, we include different derivations of “to explain”, as explanation.

used for in that context. The knowledge can contribute to explicit discussions about the meanings both in teacher education and in school, something that can contribute to easier communication around mathematical reasoning and proving and, more specifically, around qualities in a good mathematical argument. Therefore, our research question is: *How is the word explain used by Norwegian pre-service teachers while working on evaluating different mathematical arguments?*

To answer this research question, we will analyse the PSTs' written evaluations of some mathematical arguments. Before we present our data material and analysis, we will relate our study to other studies on the use of language in mathematics and look at how the word explain can be related to reasoning and proving in mathematics education.

Theoretical framework

In this study, we are interested in the ideational metafunction of language (Halliday, 1985), which is how language is used to communicate an idea. Monaghan (1999) has done a similar study where he analyses the use of the word diagonal in learning resources and identifies an interchanging between the everyday meaning and mathematical meaning of the word, which he points out might be problematic for students' learning. Similar ambiguities in the use of language have been identified in other studies (Dawkins et al., 2019; Herbel-Eisenmann & Otten, 2011). Halliday and Matthiessen (2004) point out that language and the context it is used in are closely related: the context affects the language chosen, and the choice of language contributes to the construction of the context. In a given context, people choose the language, consciously or unconsciously, based on their earlier experiences and understanding of the context. In our study, the context is related to mathematical reasoning and proving. In that context, the PSTs choose to use the word explain, which colours the context and affects the creation of meaning in it.

There are different ways of defining mathematical reasoning and proving. In this study, we view proof as a particular type of argument for or against the validity of a mathematical claim (Stylianides, 2007). Proof has various functions in mathematics, and two of the main functions are widely described as verification and explanation (Hanna, 2000). Verification is showing, beyond any doubt, *that* a given mathematical statement is true or is not true, whereas explanation provides insight into *why* the statement is true or is not true. Hanna (2000) points out that in the educational domain, the primary function of the proof is to explain why the given statement is true or not true and what the properties and relations leading to that are. Therefore, she suggests that the fundamental quality of proof in school should be explanatory. The proofs need to show both that the statement is true and why it is true. There is no well-established definition of what an explanatory proof is in the community of mathematics education, but Lockwood et al. (2020) propose several characteristics of explanatory proofs that are suitable for mathematics education, for example, that proofs that rely on visual reasoning tend to be explanatory, while proofs that rely on purely algebraic manipulations tend not to be explanatory.

It is important to note that word explanation related to a function of proof (as in the studies above) is defined more narrowly than in other studies. For example, Yackel (2001) describes explanations as clarifying aspects of one's mathematical thinking to others. Perry (2000) emphasises two types of explanations: those that clarify how some problem is solved and those focusing on why methods are

used to solve the problem work. Balacheff (2009) uses the term explanation to denote individual work on establishing the validity of a statement from a subject's perspective. An explanation then becomes a proof when the community accepts it as such and further a mathematical proof when it satisfies certain formal standards. However, as discussed above, in the context of reasoning and proving in mathematics education research, explain is mainly used to characterise a given proof's quality in arguing *why* a mathematical hypothesis is true or not true.

Several studies have investigated which properties of an argument PSTs emphasise when asked to evaluate whether arguments are convincing and/or valid, and PSTs do point to explaining why as an important quality (Doğan, 2020; Ko & Hagen, 2013; Morris, 2007). Another quality of arguments that PSTs often emphasise is the use of clarity (Ko & Hagen, 2013). This quality is described by Ko and Hagen (2013) as whether the argument is easy to follow. When an argument is evaluated by PSTs as lacking clarity, when it is hard to understand what a student has done, PSTs tend to classify it as a non-valid proof.

Methods

This study is part of a larger research project on reasoning and proving in primary school and teacher education², with first-year PSTs in Norwegian teacher education for grades 1 to 7. In one of their first lessons in mathematics, the PSTs were introduced to mathematical reasoning and proof as processes of finding out if and why mathematical conjectures are true and the definition of a mathematical proof as given by Stylianides (2007). During the lesson, the PSTs were divided into groups of 3-4 PSTs and evaluated some student arguments (Figure 1). The last sentence in the task points both to the role of verification and the role of explanation of an argument. The data we analyse is the written responses to this task, given in a Padlet, from 28 such groups.

Mira, a 5th grade student, said to the rest of the class:
“The five-times table is easy. When you want to know what something multiplied with five is, you can just times it with ten, then take halve of that”
Several of her classmates were unsure if this was true for all numbers. Some meant it might be true but were unsure about why. So, they examined Mira's conjecture and came up with different arguments.
Your task is to:

- Choose the argument you think is the best and write two reasons why
- Choose the argument you think is the worst and write two reasons why

Remember that the argument should be such that it makes us surer that the conjecture is true for all numbers, and that we understand more of why it is true.

Figure 1: The given task

The PSTs were given five student arguments to evaluate, as shown in Figure 2 (translation from Iversen (2022)). The arguments were created by the researchers and inspired by the different types of arguments presented in G. Stylianides (2008).

² ProPrimEd - Reasoning and Proving in Primary Education. The project is funded by the Norwegian Research Council. For more information about the research project: <https://www.ntnu.edu/ilu/proprimed>

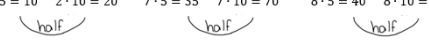



Hannah	Yes, it is correct. $2 \cdot 5 = 10$ $2 \cdot 10 = 20$ $7 \cdot 5 = 35$ $7 \cdot 10 = 70$ $8 \cdot 5 = 40$ $8 \cdot 10 = 80$ 			
Belma	Yes. $6 \cdot 5 = 30$ $6 \cdot 10 = 60$ It happens because 5 is the half of 10. $5 + 5 = 10$ $10 : 2 = 5$	Inga	$4 \cdot 10$  $4 \cdot 5$ $4 \cdot 5$	
Leo	$3 \cdot 10 = 30$ $3 \cdot 5 = 15$	$26 \cdot 20 = 260$ $26 \cdot 5 = 130$	$268 \cdot 10 = 2680$ $268 \cdot 5 = 1340$	
Abi	It is correct, because: For example, if we want to calculate $7 \cdot 5$. That means to find out how much 7 fives are if added together. We can start by working out what 7 tens are:  Next, we can split each ten into two fives:  7 tens = 7 fives + 7 fives $7 \cdot 10 = 7 \cdot 5 + 7 \cdot 5$. Thus, $7 \cdot 5$ is the half of $7 \cdot 10$! If we use another number, not 7, then it will be just the same. It will only be a different number of tens and fives.			

Figure 2: The arguments that were given to PSTs to evaluate

The process of analysing how the word explain was used in the PSTs’ evaluations of the arguments was inductive. The three researchers coded the data material separately by first identifying all cases where it was used and then asking the question, “What is the PST group trying to express here?” We then compared the coding and agreed on the following categories:

- 1) Making clear what, 2) Making clear why, 3) Text, 4) The whole argument, and 5) Ambiguous

We will now present what we have coded to the different categories and why.

Findings

Of the 28 PST groups, 20 used the word explain in their evaluation of the arguments, and the word appears 40 times in the data. The different occurrences are shown in Table 1 below. Some statements were coded to more than one category. For instance, one group of PSTs wrote, “Abi explains better what he does, and why it’s like that”, and in that way use the word *explains* both as making clear what was done and why it is true. Below we will show examples from each category and our analysis of the statements.

Table 1: Number of occurrences of variations of the word explain

Making clear what	Making clear why	Text	The whole argument	Ambiguous
6	10	5	10	12

Making clear what

In six cases, PSTs use the word explain with the meaning to *make clear what is being done*. Statements in this category are recognised by the use of the word “what” or similar formulations. An example of such use is illustrated in this sequence from the data:

We think the procedure was a bit too messy and that it is “missing something” like an explanation for what she does.

Here the PSTs emphasise that writing *what* is being done is an important part of an argument and that explaining what is done is missing in the given argument. So, they emphasise a need for more clarity in what the student who wrote the argument has done.

Making clear why

In some cases, PSTs mean *making clear why* the statement is true when they use the word explain. The main characteristic of statements coded in this category is that they have the word “why”.

Abi’s was the best because he shows with illustrations that the conjecture is correct, as well as explaining why in words.

Using the word explain to point out a need to make clear why occurs more often in the data material than in the previous category, making clear what is done.

Text

The third category is explanation as text. Statements coded to this category all include a formulation of an explanation *and* an illustration. We interpret that the word explanation refers to the text only, the parts of the argument written with words, and that illustrations are not part of the explanation. One example:

Best argument: Abi. Both drawing and explanation. Shows an understanding of why it is like this for all numbers and not just the example.

We have identified five examples of such use in the data. The PSTs use formulations as “both drawing and explanation” or “explanation to the drawing” or similar in all occurrences.

The whole argument

Another way the PSTs use the word is to denote the whole argument, that is, the whole student’s answer to the task. Statements in this category typically refer to his or her explanation or “the explanation” to suggest something more than text. An example of this follows:

His explanation lacks an argument as to why Mira’s hypothesis works. He has only concluded.

The PSTs haven’t connected the word to what or why, but it is more than just text. *His explanation* points to everything the student wrote to argue for the truth of the conjecture and denotes the whole argument.

Ambiguous

The largest category is the one we have chosen to call *ambiguous*. What characterises statements in this category is that the word explain can point to two or more different meanings. Hence it isn’t easy to know what the PSTs mean. Here is an example:

Hannah: Tests the conjecture several times with different numbers. She shows the connection between result and illustrates by drawing a line. Simply explained.

Here we see that the group concludes that it is *simply explained* without clarifying what that really means. Is it the words that are used, or is it that the whole answer is written in an orderly manner? Is it that it is made clear what has been done or why it is true? It is not clear from this answer. Similarly, we can say the same for the following example:

Abi has the best conjecture [sic] because he has a whole explanation that is supported by an illustration.

Here, it is not clear what *a whole explanation* means. Since the group has also written that an illustration supports the argument, it may indicate explanation as text, but it seems to be something more than that.

A final example of a statement that is difficult to place in one of the other categories is this evaluation of Abi's argument:

Because it was very well and thoroughly explained. The student includes drawings in his argument and explains in detail based on that.

Explaining in detail can mean that the student who wrote the argument thoroughly describes what is done or that it is made clear why the statement is true. It is unclear here which of these two meanings the PSTs give the word in their statement.

Discussion

Our research question concerns different meanings of the word explain in Norwegian PSTs' evaluation of different mathematical arguments. In Norwegian, explain means to make something clearer and more understandable, to give a (more) precise/detailed account of something, and this *something* can be different things in everyday life. To our knowledge, the word has a similar, rather broad meaning in English, too. In research on teacher education, *something* can be, for example, aspects of one's mathematical thinking (Yackel, 2001), how some problem is solved, or why the methods used to solve the problem work (Perry, 2000). As presented above, our analysis reveals five different categories of PSTs' use of explain in the evaluation of different mathematical arguments: 1) Making clear what; 2) Making clear why; 3) Text; 4) The whole argument; and 5) Ambiguous, where the word was primarily used in verb form in category 1) and 2), and in noun form in 3) and 4). In research on proving, explaining is usually about *why* a given statement is true or not. Our study shows that PSTs use the word more broadly than this (as also Yackel (2001) and Perry (2000) do). However, as we have found five different categories of the word explain, and the biggest one is *ambiguous*, the result can explain the confusion caused using the word in a classroom in the study of Iversen (2022).

The first two categories we have identified in our data, Making clear what and Making clear why are related to respective categories *Use of clarity* and *Use of explanation* from the study of Ko and Hagen (2013). Ko and Hagen (2013) investigate what PSTs see as qualities in mathematical arguments, and use of clarity and use of explanation is emphasised by PSTs in their study, while PSTs emphasise explanation in the study of Doğan (2020). Our analysis shows that both qualities, clarity and explanation, are valued by our PSTs as well, and the word explain is often used to describe both qualities. In such cases, the meaning is always made clear by using the words "what" or "why". So, the use of the same word is not problematic in these cases: PSTs writing about explaining what is done emphasise what Ko and Hagen (2013) call clarity, and PSTs writing about explaining why the statement is true emphasise what Ko and Hagen (2013) and Doğan (2020) call explanation. As mentioned, the dictionary defines the word explain as making something clearer and more understandable. In cases where PSTs write explicitly what this *something* is (whether it is what is done or why the statement is true), the meaning is clear, and we can relate the use of the word explain to qualities of arguments identified in research. In several of the cases we coded as ambiguous, the problem is that *what* is made clearer and more understandable is not given explicitly, and in the

context of mathematical reasoning and proving, it can mean at least two different things, as shown in our first two categories.

The meaning of the word explain in categories *text*, and *the whole argument* is quite close since both point to something produced by the student during the argumentation. We can relate these uses to the explanatory function of proof Hanna (2000) and explanatory proofs Lockwood et al. (2020). When PSTs call (parts of) an argument an explanation, it can indicate that they see its role more as an explanation than a verification, even though both verification and explanation are promoted in their task. Seeing proof (and argument) in school as primarily an explanation is emphasised as important by Hanna (2000). However, while the category *text* refers to parts of the argument written by words, the category *the whole argument* includes illustrations, calculations, and everything the student wrote to argue for the conjecture. Hence the two uses of the word might complicate the communication around reasoning and proving. Many cases in the *ambiguous* category were coded thus because it was unclear if the PSTs meant text or the whole statement. This shows that the categories are closely related and difficult to separate when given too little information.

As seen above, Norwegian PSTs use the word explain concerning what is done, why the statement is true, and to point to parts of the argument or the whole argument. The use is relatively clear when it's made clear whether it is about explaining what or why and whether the explanation is the whole argument or just the textual part. However, our last category, *ambiguous*, is the largest. As discussed above, it is difficult to interpret what PSTs want to express in these cases.

The word explain can have different meanings in different contexts, and in this study, we have identified its various uses in the context of mathematical reasoning and proving. We suggest that explicit discussion with PSTs and students about the different uses of *explain* in the context of mathematical reasoning and proving can contribute to more precise language use: pointing to a need to be explicit about what is to be explained, what is done or why the statement holds; discuss that we sometimes by explanation can mean just text, other times the whole argument and agree with the class which words to use for what to avoid unnecessary misunderstandings in work on reasoning and proving.

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